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Comparative cheap talk☆

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Abstract

When are comparative statements credible? We show that simple complementarity conditions ensure that an expert with private information about multiple issues can credibly rank the issues for a decision maker. By restricting the expert's ability to exaggerate, multidimensional cheap talk of this form permits communication when it would not be credible in a single dimension. The communication gains can be substantial with even a couple of dimensions, and the complete ranking is asymptotically equivalent to full revelation as the number of issues becomes large. Nevertheless, partial rankings are sometimes more credible and/or more profitable for the expert than the complete ranking. Comparative cheap talk is robust to asymmetries that are not too large. Consequently, for sufficiently many independent issues, there are always some issues sufficiently symmetric to permit comparative cheap talk.

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1. Introduction

Simple comparative statements such as "that one is better" or "this one is best" are widely used and often believed. More formal communication frequently relies on comparisons as well. For instance, a professor ranks different students for an employer, a stock analyst ranks different stocks for a client, or a website ranks different products for a buyer. It is also common to reveal comparative information implicitly. For instance, a lobbyist discusses one bill rather than another with a senator, or a newspaper emphasizes one story over another.

Despite their widespread use, little is known about the role of comparative statements in strategic communication. Are such statements more credible than claims such as "they both look great" or "every student is excellent"? How much information can comparative statements convey? When does it make sense to withhold comparative information? And, are comparative statements still credible when the speaker is not impartial, e.g. when a professor has a favorite student, or a salesperson receives a larger commission on a particular product?

To answer these questions we follow the standard Crawford–Sobel [13] model of costless, unverifiable "cheap talk" in which an expert with an incentive to exaggerate has private information of interest to a decision maker. For instance, a lobbyist wants to oversell the merits of a bill, or a stock analyst profits from overestimating the value of a stock. The Crawford–Sobel model is used to analyze communication in fields from accounting [19] to political science [20] to zoology [16], but it assumes that there is only one dimension in which the expert has information and in which the decision maker acts. To understand the role of comparative statements, we extend the model to allow for multiple dimensions, e.g. a lobbyist knows the merits of multiple bills, a stock analyst knows the values of multiple stocks, or a professor knows the abilities of multiple students.

In this multidimensional cheap talk model, we find that a strong conflict of interest *within* dimensions still permits enough commonality of interest to allow comparative communication *across* dimensions. This contrasts with the standard one-dimensional result that cheap talk breaks down when the expert's incentive to exaggerate is too strong. For instance, even if a professor is so inclined to exaggerate the abilities of her students that cheap talk regarding an individual student is not credible, we show that it is often credible for a professor to rank one student above another. Such "comparative cheap talk" can be an equilibrium because it simultaneously sends both favorable and unfavorable information, thereby eliminating the ability to exaggerate.

We find that simple complementarity (i.e., supermodularity) conditions on preferences capture the notion of commonality of interest across dimensions. They imply that the expert wants the decision maker to take a higher action when the variable known by the expert is higher, and that the decision maker wants to take a higher action when this variable is expected to be higher. Consequently, both parties agree on the ranking of desired actions in each state, even though interests may differ strongly on the magnitude of the desired actions. The complementarity conditions appear to fit many real world situations and are satisfied under the basic assumptions of the Crawford–Sobel model. They are sufficient for the credibility of the complete ranking of all the variables, and for the credibility of less informative partial rankings in which the variables are sorted into categories, e.g. "buy" or "sell" ratings used by an analyst following multiple stocks.

Even though comparative cheap talk only provides rankings of the issues, the communication gains can be substantial. As the number of dimensions grows, the expert's ranking becomes an increasingly accurate signal of each variable's value. For instance, the class rank of a student can be quite informative of ability when the class size is very large. More generally, for any $p \in (0, 1)$, as the number N of independently distributed variables increases the pNth variable becomes very likely to be very close to the pth quantile of the original distribution. As a result, revealing the expert's information through a complete ranking is asymptotically equivalent to revealing *all* of the expert's private information.

These results are for symmetric "apples to apples" comparisons in which the expert has the same preference weight on each issue and the distributions of the variables are identical, so that the game is an N-dimensional replication of a standard one-dimensional cheap talk game.[1] We also consider the asymmetric "apples to oranges" case in which it is common knowledge that the players' preferences and/or distributions are different for each issue. Such asymmetries can counteract the incentive generated by preference complementarities to provide a truthful ranking. For instance, if a stock analyst receives higher compensation for promoting a particular stock, the analyst's ranking of the stock might be suspect. Or if one student is already expected to be quite good, a professor might be tempted to rank another less appreciated student higher. We find that a comparative cheap talk equilibrium between any two issues still exists if the asymmetries are sufficiently small, but that it often breaks down when the asymmetries become larger.

Even with arbitrary asymmetries, we find that possibilities for influential communication are generated simply by adding enough independently distributed dimensions. Provided utility functions and distributions are chosen from a compact set, as the number of issues increases some of the issues must be distributed similarly and each of the expert's and the decision maker's preferences across these issues must also be similar. Whenever this happens, comparative cheap talk between these issues becomes credible and influential. For instance, as long as a liberal and conservative who disagree on every issue have enough issues to discuss, there must be some issues across which each has preferences that are sufficiently similar to permit meaningful communication.

We apply our results to two types of games. The first is what we call valuation games and includes the canonical game with quadratic preferences first introduced as an example by Crawford and Sobel [13] for the one-dimensional case. In this game a biased expert tries to influence a decision maker's estimate of the situation when the two sides have partial common interests in each dimension. Valuation games also include cases where the expert wants to increase the receiver's estimate as much as possible, e.g. a stock analyst tries to push up stock market valuations, or an auction house tries to push up auction prices. Despite these strong incentives to exaggerate within each dimension, comparative cheap talk across dimensions is credible.

The second type is what we call recommendation games and covers situations where the expert effectively recommends one of two possible actions, e.g. whether or not to hire a

[1] We continue to follow other assumptions from the Crawford–Sobel framework that have been relaxed elsewhere in the literature. For instance, we do not consider situations where the game is repeated [30,32,27], there are multiple stages of cheap talk [2,22], the expert and decision-maker are not necessarily fully rational [12,29], or there is some uncertainty over the expert's incentive to exaggerate [26,15].

student, to buy a product, to grant a license, or to vote for a proposal. In recommendation games we find that the expert prefers ex ante to reveal a partial ranking rather than the complete ranking. For instance, if there are three students being recommended by a professor and the middle student is unlikely to receive a job based on the complete ranking, an alternative is to put the top two students in a group and not differentiate between them. As the number of issues increases, such groupings can be used more and more effectively to maximize the expert's payoffs. The gains from partial rankings may explain why highly ranked schools often obscure the relative quality of their graduates, either by grade inflation as in Ivy League undergraduate programs, or by withholding grades from employers as in some elite M.B.A. programs.[2]

In the Crawford–Sobel model of communication within a single dimension, even a slight incentive to exaggerate makes it impossible for the expert to reveal the exact value of the unknown variable, but if the incentive is not too strong the expert can communicate coarse information about the range of the variable by partitioning the variable space into different intervals reflecting qualitative information such as "good or bad" or "low, medium, or high". Comparative cheap talk across dimensions partitions the variable space differently than does such "interval cheap talk", but the two forms of cheap talk can appear quite similar. For instance, if a stock analyst places stocks into buy or sell categories, the analyst could be engaged in interval cheap talk regarding each stock, or in comparative cheap talk that provides a ranking of the stocks. The key difference is that under interval cheap talk the credibility of a buy ranking depends on a stock analyst not wanting to push up the value of a very bad stock, while under comparative cheap talk the credibility of a buy ranking depends on the presence of other stocks in the sell category.

The potential for communication across multiple dimensions offers insight into how organizational structures are affected by the relative advantages of delegation versus cheap talk. In the one-dimensional Crawford–Sobel model the decision maker often prefers to give the expert complete freedom to make an informed but biased decision rather than rely on the expert to provide noisy and biased cheap talk [14]. For instance, an upper manager might fully delegate authority to better-informed lower managers rather than rely on them for biased advice about a project.[3] In our multidimensional model the benefits to the decision maker from cheap talk increase with extra dimensions, but there is no corresponding increase in the benefits of full delegation. For instance, if there are two projects that a lower manager has information on and she prefers the maximal action on each project, comparative cheap talk about their relative merits can improve the decision-making process, but fully delegating the decisions to the lower manager will just result in the maximal action being chosen on each project.[4]

[2] For related analyses, see Costrell [11], Chan et al. [10], and Ostrovsky and Schwarz [28].

[3] Of course, delegation may not be feasible in certain contexts, e.g., when the sender is a hypochondriac and the receiver is a doctor.

[4] An alternative to full delegation is partial delegation in which the sender is given a limited set of actions to choose from and the receiver commits to following the sender's choices. For the same reasons we examine, the benefits of partial delegation also increase with multiple dimensions. Because of the commitment assumption, partial delegation (or screening) always weakly beats cheap talk from the receiver's perspective. Closely related is the mechanism design case where the receiver can also reward or penalize the sender for different choices. See Jackson and Sonnenschein [21] for results of a similar flavor in a mechanism design context.

The idea that additional dimensions can facilitate communication in standard cheap talk games is investigated by Battaglini [5] for the case of multiple experts.[5] He finds that a decision maker can structure competition between experts to induce full information revelation, and that in special cases a single expert can reveal full information in one of two dimensions. The question of when cheap talk with a single expert breaks down is examined by Levy and Razin [23] in a model where preferences are defined by the expert's bias, i.e., the distance between the expert's and the decision maker's ideal actions. They show that, in the presence of asymmetries and correlation, sufficiently large biases preclude the possibility of informative cheap talk. We find the complementary result that, regardless of correlation or the magnitude of the biases, informative equilibria always exist when asymmetries are not too large. We further find that, regardless of any asymmetries, informative equilibria always exist when biases and distributions are chosen from compact sets and there are a sufficient number of independently distributed dimensions.

The rest of the paper is organized as follows. In Section 2 we consider the symmetric model in which the distribution functions and utility functions are the same across dimensions. In Section 3 we use these results to examine the general case. Section 4 shows how the model can be used to analyze two types of games with a wide range of practical applications, Section 5 discusses extensions of the model, and Section 6 concludes.

2. The symmetric model

Consider a multidimensional game in which player S (the sender or expert) possesses private information about $N \geqslant 2$ different issues that player R (the receiver or decision-maker) takes actions on. The sender's private information about issue $k = 1, \dots, N$ is represented by a random variable $\theta_k \in \Theta = [0, 1]$. Let $\theta = (\theta_1, \dots, \theta_N)$ and let F denote the joint distribution of θ with support on Θ^N. We assume that F has a strictly positive continuous density f. We will denote by F_k and f_k the marginal distribution and the marginal density of θ_k respectively. Throughout this section we assume that f is symmetric, i.e., it is invariant to permutations of its arguments. On occasion, we will also consider the more restrictive case in which f also displays independence, i.e., $f(\theta) = \Pi_k f_k(\theta_k)$ and all the $f_k(\cdot)$ are identical.

At the beginning of the game the sender sends a message m from a set M that is heard by player R.[6] Subsequently, for each issue k player R chooses an action a_k from a set A that is independent of k. We assume that A is a compact convex subset of $\mathbb{R}$ that we will also identify with the unit interval [0, 1]. Let $a = (a_1, \dots, a_N)$ denote the action profile chosen by the receiver.[7]

[5] See also Austen-Smith [3] for a model with multiple experts and Spector [31] for a model in which players with divergent priors learn from each other the true state of the world.

[6] The results are robust to allowing different receivers for each issue, as long as the sender's message is a public message.

[7] Note that the receiver can take actions independently on each issue. In some cases bundling the issues so that actions are interdependent can encourage communication as shown in Chakraborty and Harbaugh [9].

The payoff from issue k to player $i \in \{S, R\}$ is given by a function $u^i : \Theta \times A \to \mathbb{R}$ that is continuous in each argument. In this section we assume that the utility functions are symmetric, i.e. u^i is independent of k. For each θ_k let $a^*(\theta_k)$ be the unique maximand of $u^R(\theta_k, a_k)$ with respect to a_k. We denote by $U^i(\theta, a)$ the total payoff to player i from an action profile a in a state of the world θ, and assume that it is additive across issues so that $U^i(\theta, a) = \sum_k u^i(\theta_k, a_k)$. While the additive form is restrictive, it rules out direct preference spillovers across dimensions and so highlights that any expansion in the benefits of communication with added dimensions does not arise out of exploiting such spillovers. Note also that the payoffs of either player do not directly depend on the message m that is sent by player S. In other words, the sender's message is pure cheap talk.

For any set X, let $\Delta(X)$ denote the set of probability distributions on X. A strategy for the sender is a function $\mu : \Theta^N \to \Delta(M)$ and a strategy for the receiver is a function $\sigma : M \to \Delta(A^N)$. Beliefs of the receiver over Θ^N (inferred from a message m) are given by a function $\phi : M \to \Delta(\Theta^N)$. We use the standard notion of a weak perfect Bayesian equilibrium.[8]

Let $\mu(m) = \Pr[\{\theta \in \Theta | \mu(m|\theta) > 0\}]$ be the probability with which a message m is chosen by S given a strategy μ. Cheap talk equilibria are interesting when communication by the sender influences the probability distribution of actions chosen by the receiver. Formally, an equilibrium (μ, σ, ϕ) is said to be *influential* if there are two messages m and m' with $\mu(m), \mu(m') > 0$ such that $\sigma(\cdot|m) \neq \sigma(\cdot|m')$. In looking for influential equilibria, we follow the usual practice in the literature on cheap talk games and rule out the possibility of out-of-equilibrium messages. That is, we assume that for each message $m \in M$, $\mu(m) > 0$. This is without loss of generality since any time an equilibrium exists where some messages are not chosen in equilibrium, an outcome equivalent equilibrium exists where all messages are chosen on the equilibrium path.

We focus on the existence of influential cheap talk equilibria where the sender's message consists of disclosing a partial or complete ranking of her private information $\theta_1, \ldots, \theta_N$ about the N issues. Such a message contains information about each issue that is not independent of the information it contains about other issues. As we show below, this implies that even when there is a strong conflict of interest between the sender and receiver with regard to the optimal action that should be taken on each issue, informative communication is still possible. We call such strategies comparative cheap talk strategies.

Formally, let $\theta_{i:N}$ indicate the ith smallest realization of the N different θ_k. Let $C = (c_1, \ldots, c_{|C|})$ denote a ranking, i.e., a partition of the set of indices $\{1, \ldots, N\}$ of $\{\theta_{1:N}, \ldots, \theta_{N:N}\}$ into $|C| \leqslant N$ elements or categories, such that the jth category c_j has $|c_j| \geqslant 1$ elements with $\sum_{j=1}^{|C|} |c_j| = N$. That is, the first category $c_1 = \{1, \ldots, |c_1|\}$ denotes a set identifying the lowest $|c_1|$ of the θ's, $\{\theta_{1:N}, \ldots, \theta_{|c_1|:N}\}$, the second category $c_2 = \{|c_1| + 1, \ldots, |c_2|\}$ denotes a set identifying the next set of the $|c_2|$ lowest θ's, $\{\theta_{|c_1|+1:N}, \ldots, \theta_{|c_1|+|c_2|:N}\}$, and so on.

A comparative cheap talk strategy is represented by a ranking or categorization C which is fixed and does not depend on the realization of θ. The strategy is described as follows.

[8] Since R's action has to be optimal given his inference about θ upon hearing m, this distinguishes our cheap talk model from a screening problem where R first commits to a menu of actions for each message and S chooses among them.

For each realization of θ, the sender announces that the $|c_1|$ issues with the lowest values of θ_k are in category c_1, the next $|c_2|$ issues are in category c_2 and so on. If there are ties between some of the θ_k's, the sender uniformly randomizes when she sorts those issues into different categories. Consequently, the receiver knows that for issues in higher categories the sender's private information has a weakly higher value and cannot distinguish between issues within a category. The finest possible ranking $C = (c_1, \ldots, c_N)$ with $c_j = \{j\}$ for all $j = 1, \ldots, N$, corresponds to the strategy where the sender completely ranks the N issues. On the other hand, the coarsest possible ranking, $C = (\{c_1\})$ with $c_1 = \{1, \ldots, N\}$, corresponds to an uninformative babbling strategy. We will use the term partial ranking to denote rankings that are coarser than the complete ranking and finer than the babbling strategy.[9]

Note next that since f is symmetric, for any candidate equilibrium ranking C, the distribution $F_{c_j:N}$ of θ_k given that the sender has announced that it belongs to category j does not depend on the index k. That is, $F_{c_j:N}$ can be defined to be the distribution function for $\theta_{c_j:N}$, where the latter is a random variable that is equally likely to be one of the $\theta_{i:N}$'s that belong to category c_j. Therefore, for any comparative cheap talk strategy C, the symmetry of f implies that the possible equilibrium beliefs of the receiver with respect to θ_k are summarized by the collection $\{F_{c_j:N}\}_{j=1}^{|C|}$ with corresponding densities $\{f_{c_j:N}\}_{j=1}^{|C|}$.

Observe that if an action profile $a = (a_1, \ldots, a_N)$ maximizes R's expected total payoff given a message that issue k belongs to category c_j, it must be that

$$a_k \in \arg\max_{a'_k} \int_{\Theta} u^R(\theta_k, a'_k)\, dF_{c_j:N}(\theta_k). \tag{1}$$

Our assumptions on $u^R(\theta_k, a_k)$ and A imply that the maximization in (1) has a solution, which we denote as $a_{c_j:N}$.

Our first result provides sufficient conditions on preferences for comparative cheap talk to be an equilibrium in the symmetric model. These complementarity conditions take the form of a supermodularity condition. We adapt from Athey [1], the definition.

Definition 1. For $i \in \{S, R\}$, u^i satisfies supermodularity (respectively, strict supermodularity) if, for all $a_k > a'_k$, the difference $u^i(\theta_k, a_k) - u^i(\theta_k, a'_k)$ as a function of θ_k is non-decreasing (respectively, increasing) in θ_k.

These complementary conditions on preferences imply that the receiver will take higher actions for issues announced to be in higher categories and, given this, the sender has no incentive to misreport the ranking. Comparative cheap talk is therefore an equilibrium. Theorem 1 makes this precise.

Theorem 1. *Suppose f is symmetric and u^S and u^R are supermodular. Then the complete ranking and every partial ranking are comparative cheap talk equilibria.*

[9] In cheap talk games it is well known that there always exists an equilibrium (that is not influential) where the sender uses the babbling strategy. Because of this it is not possible to use traditional fixed-point type arguments to demonstrate the existence of influential equilibria.

Proof. Consider any ranking $C = (c_1, \ldots, c_{|C|})$ and note that for any message that puts issue k in category c_j, a best-response for R is to choose $a_k = a_{c_j:N}$, the solution to (1), for all $k = 1, \ldots, N$ and $j = 1, \ldots, |C|$. Notice next that since marginals for order statistics are first order stochastically rankable, the $F_{c_j:N}$ are stochastically ordered in j. Since u^R is supermodular, it follows from Theorem 3.10.1 in Topkis [33] that the $a_{c_j:N}$ are non-decreasing in j.

We show now that this implies that the sender has no incentive to misreport the correct ranking. Let k', k be such that $\theta_{k'} \leqslant \theta_k$. Since u^S is supermodular, for any j

$$u^S(\theta_{k'}, a_{c_1:N}) + u^S(\theta_k, a_{c_j:N}) \geqslant u^S(\theta_{k'}, a_{c_j:N}) + u^S(\theta_k, a_{c_1:N}).$$

Consequently, for any realization of θ the sender can do no better than to announce the lowest category for the issue with the lowest value, regardless of her announcements for the other issues. Given this, it follows that the seller can do no better than to announce the lowest category still available for the issue with the second-lowest value. Continuing this logic, for every realization of θ, the sender can do no better than to announce that the $|c_1|$ issues with the lowest values belong to category 1, the next $|c_2|$ issues to category 2 and so on, until all of the issues are ranked correctly. In other words, truthfully announcing the ranking is a best-response for the sender.[10] $\square$

The complementarity conditions in Theorem 1 provide our basic measure of commonality of interest with respect to communication across dimensions. Under these conditions, sender and receiver interests coincide on the rankings of the actions so comparative cheap talk is an equilibrium. Comparative cheap talk is not credible when such complementarity conditions are not shared by the sender and the receiver, for instance, when u^S is strictly submodular and u^R is strictly supermodular. Note that the equilibria characterized by Theorem 1 always convey information but are not guaranteed to be influential. They are influential if the information in the ranking is of sufficient importance to the receiver that not all the actions $a_{c_j:N}$ are equal to each other (e.g., corner solutions). Simple sufficient conditions to ensure that no two actions are the same are that $u^R(\theta_k, a_k)$ be strictly supermodular and differentiable, with the derivative with respect to a_k being negative at $a_k = 1$ and positive at $a_k = 0$ for almost every θ_k.

Supermodularity of u^S and u^R is also assumed in the Crawford–Sobel model in order to generate communication within a single dimension. Therefore the idea that shared complementarities are a measure of commonality of interest applies both to communication across as well as within dimensions. However, since these conditions are sufficient for comparative cheap talk, communication is in a sense easier when there are multiple dimensions. In particular, the Crawford–Sobel model makes the additional assumptions that u^S and u^R are concave and have interior maximands. In contrast, Theorem 1 allows u^S to be strictly increasing in a_k for each θ_k, a condition that rules out the possibility of influential cheap talk in a single dimension.

[10] In the special case where f in addition displays independence, the condition on u^R in the statement of Theorem 1 can be weakened to satisfy single-crossing. In such cases, the $\{f_{c_j:N}\}$ can be shown to satisfy the monotone likelihood ratio property and so, from Theorem 2 in Athey [1] it follows that the $a_{c_j:N}$ will be non-decreasing. The rest of the arguments in the proof above then follow.

Complementarities are sufficient for communication in part because of the symmetry assumptions adopted in this section. Asymmetries can arise in the distribution functions and also in the utility functions. Since asymmetries can limit the potential for comparative cheap talk, the degree of symmetry across dimensions can be thought of as a measure of the similarity of interest of each player in multidimensional environments. We investigate the effect of asymmetries on comparative cheap talk in the next section.

Theorem 1 can be immediately extended to demonstrate the existence of other influential equilibria such as ranking a subset of the N variables and being "silent" on the remaining variables. Similarly, sorting the issues into disjoint groups and engaging in comparative cheap talk within but not across groups is also an equilibrium. We refer to such strategies as partial rankings as well and consider them in the next section.

Note finally that other influential equilibria may coexist with comparative cheap talk equilibria under the conditions of Theorem 1. For instance, in the canonical quadratic version of the Crawford–Sobel model (see Section 4.1) with independence, interval cheap talk in each dimension is still an equilibrium as long as the parameter b is small enough. In this paper, our focus is on comparative cheap talk and we do not attempt a full characterization of the set of influential equilibria.

Can comparative cheap talk be very informative? Theorem 2 uses the Glivenko–Cantelli theorem to show that when f is i.i.d. comparative information is essentially all information for sufficiently many issues.

Theorem 2. *Suppose f displays symmetry and independence. Then per-issue sender and receiver payoffs under the complete ranking asymptotically approach their expected full information values as the number of issues N increases*:

$$\lim_{N\to\infty} \frac{1}{N}\sum_{j=1}^{N} u^i(\theta_{j:N}, a_{j:N}) = E[u^i(\theta_k, a^*(\theta_k))],\ a.s.$$

Proof. For each $q \in (0, 1)$, by the Glivenko–Cantelli theorem (see, e.g., Billingsley [6]),

$$\lim_{N\to\infty} \theta_{\lceil qN\rceil:N} = F_k^{-1}(q) \text{ a.s.} \tag{2}$$

where $\lceil x\rceil$ denotes the smallest integer at least as large as x and, since f is i.i.d., F_k is the same for all k. Recalling the definition $a^*(\theta_k) \equiv \arg\max_a\ u^S(\theta_k, a)$, we obtain

$$\lim_{N\to\infty} a_{\lceil qN\rceil:N} = a^*(F_k^{-1}(q)). \tag{3}$$

To see this, suppose that (3) does not hold. Since the sequence $a_{\lceil qN\rceil:N}$ is in a closed set A, it has a subsequence converging to a point $a' \neq a^*(F_k^{-1}(q))$. By the continuity of u^R and the definition of $a^*(F_k^{-1}(q))$, it follows that for N large enough, $E[u^R(\theta_{\lceil qN\rceil:N}, a^*(F_k^{-1}(q)))] > E[u^R(\theta_{\lceil qN\rceil:N}, a_{\lceil qN\rceil:N})]$, a contradiction with the definition of $a_{\lceil qN\rceil:N}$. By continuity of u^R and u^S, (3) implies

$$\begin{aligned}&\lim_{N\to\infty} u^i(\theta_{\lceil qN\rceil:N}, a_{\lceil qN\rceil:N})\\ &\quad = u^i(F_k^{-1}(q), a^*(F_k^{-1}(q))),\ \text{a.s.}\quad \text{for all } i \in \{S, R\}.\end{aligned} \tag{4}$$

Using this, the limit of the average payoff for $i \in \{S, R\}$ under the complete ranking is given by

$$\begin{aligned}
\lim_{N\to\infty} \frac{1}{N}\sum_{j=1}^{N} u^i(\theta_{j:N}, a_{j:N}) &\equiv \lim_{N\to\infty}\int_0^1 u^i(\theta_{\lceil qN\rceil:N}, a_{\lceil qN\rceil:N})\,dq \\
&= \int_0^1 \lim_{N\to\infty} u^i(\theta_{\lceil qN\rceil:N}, a_{\lceil qN\rceil:N})\,dq \\
&= \int_0^1 u^i(F_k^{-1}(q), a^*(F_k^{-1}(q)))\,dq, \text{ a.s.} \\
&\equiv E[u^i(\theta_k, a^*(\theta_k))]. \qquad \square
\end{aligned}$$

Theorem 2 uses a law of large numbers type argument and so relies on independence. Full independence of the θ_k is not needed in order to obtain weaker but analogous results. For example, if an analyst believes the value of a stock is $\theta_k = \omega + \varepsilon_k$, where ω is a common factor capturing the overall direction of the market and the ε_k's are i.i.d. idiosyncratic factors, then the complete ranking of stocks by an analyst will asymptotically reveal all the ε_k's but not ω.

Since the receiver can always choose the same actions when he has more information, he prefers the more informative complete ranking to any partial ranking. In the limit as the number of issues increases, the receiver attains his full information payoff under the complete ranking by Theorem 2, which is the best the receiver can do under any mechanism, including partial or full delegation. For the sender such general conclusions about finer versus coarser rankings are not possible. We will discuss the impact on the sender's payoffs in the context of our applications in Section 4. But first we turn to a consideration of influential cheap talk in multiple dimensions under asymmetries.

3. The general model

So far we have assumed that the multidimensional game is a straightforward replication of the one-dimensional game. This assumption highlights our main finding that adding dimensions allows for cheap talk possibilities not present in a single dimension. But clearly there are situations where the sender or receiver is known to care differently about some dimensions or where the distributions are not identical. We will show in this section that our main finding is robust to such asymmetries.

Note first that comparative statements are fully robust to some types of asymmetries. For instance, it might be common knowledge that a professor favors students of a particular gender, nationality, or field with no asymmetries among students within a type. In this case even if comparisons across favored and unfavored types of students are suspect, rankings within the same type are clearly still credible.[11] In this section we investigate the more

[11] The same holds if it is common knowledge that there are differences in ability distributions across types. In either case, by a simple extension of Theorem 2, sender and receiver payoffs are asymptotically equivalent to full information revelation as the number of each type of student increases.

interesting case where issues with different preferences and distributions are compared so that the receiver has good reason to be suspicious whether a higher ranked θ_k is really larger. In such situations it is natural to consider comparative cheap talk that adjusts for such favoritism, i.e., cases where the sender does not directly rank the θ_k but instead ranks monotonic transformations of them. For instance, if the sender is known to be very biased in favor of issue 1, it might be credible for the sender to state whether or not $2\theta_1 \geqslant \theta_2$ instead of whether or not $\theta_1 \geqslant \theta_2$. These adjustments could be explicit in the ranking or they could just be implicit in that the receiver knows that the ranking of the issues should be "taken with a grain of salt" and adjusted to reflect the sender's biases or other asymmetries.

The model with possible asymmetries in the utility functions and the distribution function is specified by a continuous joint density f for θ with support on $[0, 1]^N$ that is not necessarily symmetric and, for $i \in \{S, R\}$, by the preference specification

$$U^i = \sum_{k=1}^{N} u_k^i(\theta_k, a_k),$$

where $\mathbf{u}^i = (u_1^i, \ldots, u_N^i)$ is a vector of utility functions for i, one for each issue.

We will utilize the Implicit Function Theorem for the results in this section. In order to do so we will impose some regularity conditions on preferences in addition to the supermodularity conditions of the previous section. The conditions are sufficient for all proofs to follow, although not necessary for all the conclusions, as examples in Section 4 demonstrate. Specifically, we assume that for each $k = 1, \ldots, N$ and $i \in \{S, R\}$, u_k^i is $\mathcal{C}^2$. Let $\mathbf{C}^2$ be the set of $\mathcal{C}^2$ functions with domain in $[0, 1]^2$ and range in $\mathbb{R}$. Let $\mathbf{U}_S \subset \mathbf{C}^2$ be the subset of such functions that are strictly supermodular (i.e., have strictly positive second cross-partials; see, e.g., Topkis [33]). For each k, we will allow u_k^S to be any element of $\mathbf{U}_S$. For the receiver, in order to guarantee that first order conditions for an interior maximum are necessary and sufficient to characterize the solution to the receiver's problem, we assume in addition that, for each $k = 1, \ldots, N$, u_k^R is strictly concave in the action and that, for a.e. $\theta_k \in [0, 1]$, $\partial u_k^R(\theta_k, 0)/\partial a_k > 0 > \partial u_k^R(\theta_k, 1)/\partial a_k$. Let $\mathbf{U}_R \subset \mathbf{U}_S$ be the subset of utility functions that satisfy these additional regularity conditions. For each k, we will assume that u_k^R is an element of $\mathbf{U}_R$.

Let $\mathbf{D}_N$ be the set of continuous joint densities with support on $[0, 1]^N$ and $\mathbf{D}_N^*$ be the subset consisting of symmetric densities. Let $\mathbf{P} = \mathbf{U}_S^N \times \mathbf{U}_R^N \times \mathbf{D}_N$. A general N-dimensional game $\Gamma(p, N)$ is specified by the number of dimensions N and primitives $p = (\mathbf{u}^S, \mathbf{u}^R, f) \in \mathbf{P}$ that specify the preferences and the distribution. We will denote a symmetric version of such a game by primitives $p^* = (\mathbf{u}^{S*}, \mathbf{u}^{R*}, f^*)$ where $\mathbf{u}^{i*} = (u^{i*}, \ldots, u^{i*})$ for some $u^{i*} \in \mathbf{U}_i$, $i \in \{S, R\}$, and $f^* \in \mathbf{D}_N^*$ is a symmetric density. Let $\mathbf{P}^*$ be the set of symmetric primitives. Let $\mathbf{C}^2$ have the $\mathcal{C}^2$-uniform convergence norm and $\mathbf{D}_N$ the sup (i.e., $\mathcal{C}^0$-uniform convergence) norm. All subspaces and product spaces have associated relative and product topologies.

Theorem 3 below perturbs the symmetric model around a symmetric solution to show the generic robustness of comparative cheap talk to small asymmetries (i.e., their regularity).[12]

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[12] We thank Joel Sobel for suggesting a robustness proof based on this approach.

The proof is a straightforward application of the implicit function theorem and we present it in the appendix.

Theorem 3. *Suppose $N = 2$. Comparative cheap talk is generically robust to small asymmetries: there exists an open and dense subset $\mathbf{P}_{gen}$ of $\mathbf{P}^*$ such that for each $p^* \in \mathbf{P}_{gen}$, there exists a neighborhood $B(p^*)$ in $\mathbf{P}$ such that for each $p \in B(p^*)$ an influential comparative cheap talk equilibrium exists.*

Proof. See appendix. □

Crawford and Sobel [13] show that small conflicts of interest within dimensions do not destroy the viability of influential cheap talk. Theorem 3 is an analog of this result for multidimensional cheap talk when asymmetries are thought of as creating a conflict of interest with respect to communication across dimensions. Note that since the equilibria characterized above are close to the equilibria of the symmetric case, the receiver chooses every equilibrium action profile with roughly equal ex ante probability, i.e. the equilibria are quite "informative". In a complementary result that differs in the order of quantifiers, Levy and Razin [23] show conditions under which asymmetries cause the receiver to take one of the possible equilibrium action profiles with arbitrarily large probability as the conflict within dimensions increases.

Theorem 3 is stated for the case $N = 2$. We now apply it to prove our next result for the case of $N \geqslant 2$ independently distributed dimensions (i.e., games where f displays independence but is not necessarily symmetric). Cheap talk games with independently distributed dimensions have two nice properties from our perspective. The first is that if an influential equilibrium exists in an N-dimensional game, it exists for every N' dimensional game, $N' > N$, that has a subset of dimensions identical to that of the N-dimensional game. The second is that under independence the space of primitives has a simple product structure. We will use both of these properties of cheap talk games with independent dimensions for our next result.

To do so let $\mathbf{D}$ be the set of continuous densities with support on $[0, 1]$ and, abusing notation slightly, let $\mathbf{P} = \mathbf{U}_S \times \mathbf{U}_R \times \mathbf{D}$. Notice that an N-dimensional game with independence across dimensions is defined by a tuple $(\mathbf{u}^S, \mathbf{u}^R, f) \in \mathbf{P}^N$, where f is now a vector $(f_1, \ldots, f_N)$ of densities one for each dimension. Theorem 3 applied to this case asserts the existence of an open and dense subset $\mathbf{P}_{gen}$ of $\mathbf{P}$ such that for every $p \in \mathbf{P}_{gen}$ there exists a neighborhood $B(p)$ such that an influential equilibrium exists in the N-dimensional game as long as the primitives for at least two of the dimensions are chosen from $B(p)$. Consider now an N-dimensional game where primitives in each dimension are chosen from some non-empty set $\mathbf{Q} \subset \mathbf{P}_{gen}$. For $p \in \mathbf{Q}^N$, $\Gamma(p, N)$ is the multi-dimensional cheap talk game with primitives p and N independent dimensions. We have the following result regarding the existence of an influential equilibrium in such a game.

Theorem 4. *For each compact $\mathbf{Q}$ there exists a number of dimensions $N^*(\mathbf{Q})$ such that, for each $N \geqslant N^*(\mathbf{Q})$, and every $p \in \mathbf{Q}^N$, an influential equilibrium exists in the game $\Gamma(p, N)$ with N independently distributed dimensions.*

Proof. Consider the collection $\{B(p)\}_{p\in\mathbf{Q}}$ where $B(p)$ are the open sets whose existence is asserted by Theorem 3. This is an open cover of $\mathbf{Q}$. Since $\mathbf{Q}$ is compact it has a finite subcover. Let $N^*(\mathbf{Q}) - 1$ be the number of elements in this finite subcover. When $N \geqslant N^*(\mathbf{Q})$, the primitives for at least two of the dimensions must lie in the same element of the subcover, at which point we can apply Theorem 3 to create influential comparative cheap talk across those two dimensions. □

Note first that the number $N^*(\mathbf{Q})$ depends on the set $\mathbf{Q}$ and not on the particular choice of p in $\mathbf{Q}^N$. Note also from the proof that as N becomes large, we must have at least $N/2N^*$ pairs of issues, each of which allow comparative cheap talk across those two issues. It follows that as N becomes large, there exists a sequence of equilibria with the property that the number of messages with distinct meanings (equivalently, the number of distinct action profiles induced in equilibrium) must go to infinity along this sequence. In the limit, there will be pair-wise influential cheap talk involving a proportion of at least $1/N^*$ of the issues. Since the existence of each of these pair-wise comparisons is obtained from the arguments in Theorem 3, they are "close" to the equilibria of the symmetric case and hence informative in the ex ante sense discussed earlier.

4. Examples

We now consider two types of games that illustrate key aspects of our results. The first type includes the canonical quadratic preferences example introduced by Crawford and Sobel [13], while the second type covers cases where the receiver has a binary agenda.

4.1. Valuation games

First consider games in which the receiver's equilibrium action equals the expected value of θ_k given all available information. For instance, the receiver wants to make an accurate estimate of a situation, or the receiver's action is the outcome of a competitive valuation process, e.g. a stock market price, a wage, or an auction price. To capture this behavioral assumption, let the receiver's payoff for each issue be

$$u^R(\theta_k, a_k) = -(a_k - \theta_k)^2. \tag{5}$$

Note that (5) is the quadratic loss function used in the standard application of the Crawford–Sobel model in the cheap talk literature. Since the receiver's utility function is supermodular, $u^R_{12} = 2 > 0$, comparative cheap talk is an equilibrium for symmetric f if the sender's utility function is also supermodular and the symmetry assumptions of Theorem 1 hold.

In the following, we consider several different functional forms for the sender's utility function. All of these meet the supermodularity condition, but generate different functional forms for comparative cheap talk equilibria when the game is not symmetric. The different functional forms also imply different ex ante incentives for the sender to reveal more information through finer rankings or to reveal less information or even no information at all. Finer rankings increase the correlation between each θ_k and the receiver's estimate of θ_k, which benefits the sender when the sender's utility function is supermodular. However, finer rankings also imply a mean-preserving spread in the ex ante distribution of actions taken by the receiver in valuation games, which hurts the sender if her utility function is concave and helps the sender if her utility function is convex.

4.1.1. Quadratic sender preferences

For the sender's utility function, consider first the quadratic loss function used in the standard application of the Crawford–Sobel model. In this example, the sender's utility function differs from the receiver's only by a bias parameter b. We will consider the multi-dimensional generalization of this model in which there is a separate bias b_k in each dimension, and in which the sender weights each issue by a parameter $\lambda_k > 0$:

$$u_k^S(\theta_k, a_k; \lambda_k, b_k) = -\lambda_k(a_k - (\theta_k + b_k))^2. \tag{6}$$

On each issue the receiver's ideal action is θ_k while the sender's ideal action is $\theta_k + b_k$. Therefore, there is some commonality of interest in each dimension since both players' ideal actions are increasing in θ_k, but also some conflict of interest since the sender's ideal action is always b_k higher. Even for an arbitrarily small bias $b_k > 0$, the sender cannot credibly state the true value of θ_k, but Crawford and Sobel show that communication involving coarse statements within a dimension is still possible if b_k is not too large. For instance, with the uniform distribution, if $b_k = \frac{1}{10}$ the sender can credibly state whether or not $\theta_k \in [0, \frac{3}{10})$,[13] and as b_k becomes smaller more partitions become possible. However, when the conflict of interest becomes too large ($b_k > \frac{1}{4}$ for the uniform distribution), the incentive to exaggerate is too strong for interval cheap talk within a dimension to be credible.

Since u_k^S is supermodular, Theorem 1 implies that, no matter how strong the conflict of interest within each dimension, comparative cheap talk across dimensions is an influential equilibrium in the symmetric case where $b_k = b$ and $\lambda_k = \lambda$ for all k, and f is symmetric. Regarding the relative payoffs from finer and coarser rankings, with quadratic preferences the positive impact of supermodularity dominates the negative effect of concavity so the sender is better off with the complete ranking rather than no ranking or any partial ranking. Fig. 1 depicts the sender's maximum per-issue ex ante expected payoff from interval cheap talk in each dimension and from comparative cheap talk across dimensions as a function of b for the cases $N = 2$ and $N = 5$ with the i.i.d. uniform distribution.[14] As N increases comparative cheap talk approaches the case of full information revelation as shown in Theorem 2, so for any given $b > 0$ the sender's per-issue expected payoff is higher under comparative cheap talk if N is allowed to become sufficiently large. However, for any given N the sender's per-issue expected payoff is higher under interval cheap talk if b is sufficiently small.

[13] The statements imply actions of $a_k = \frac{3}{20}$ and $a_k = \frac{13}{20}$, respectively. From (6), if $\theta_k = \frac{3}{10}$ the sender is indifferent between these two actions, if $\theta_k < \frac{3}{10}$ the smaller action is preferred, and if $\theta_k > \frac{3}{10}$ the larger action is preferred. So interval cheap talk of this form is credible. Details of all assertions throughout this section are available upon request.

[14] Note that if f is not independent then messages in one dimension affect estimates in other dimensions, so interval cheap talk in a multidimensional model may not simply be "products" of one-dimensional interval cheap talk.

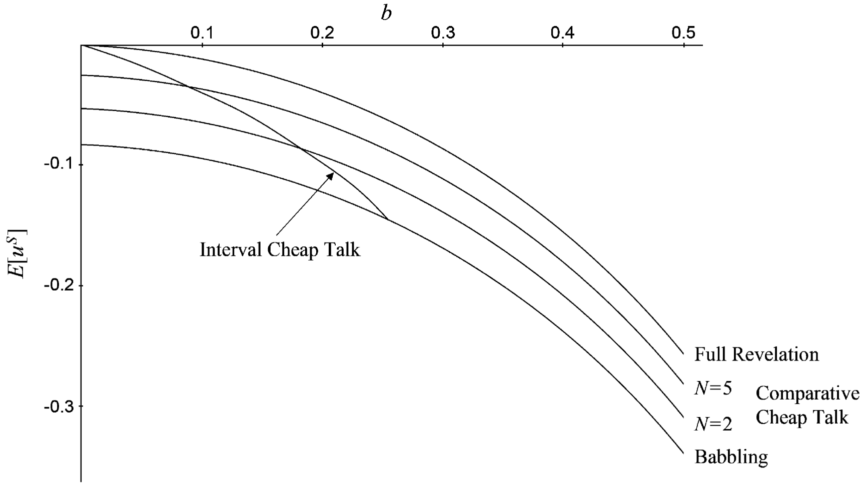

Fig. 1. Expected sender per-issue payoffs in uniform-quadratic valuation game.

Now turn to the asymmetric case. For any two distinct action profiles $a = (a_1, a_2)$ and $a' = (a'_1, a'_2)$ consider the sender types who are indifferent between them, i.e., the (θ_1, θ_2) satisfying the indifference condition

$$u_1^S(\theta_1, a_1; \lambda_1, b_1) + u_2^S(\theta_2, a_2; \lambda_2, b_2) = u_1^S(\theta_1, a'_1; \lambda_1, b_1) + u_2^S(\theta_2, a'_2; \lambda_2, b_2). \tag{7}$$

With quadratic preferences this takes the simple affine form, $\theta_k = \alpha + \beta\theta_{k'}$ for constants α, β. Supermodularity then implies that senders on one side of this line will strictly prefer one profile while those on the other side will strictly prefer the other.

Let the sender's messages in a candidate equilibrium be $m = \{\theta_2 \leqslant \alpha^* + \beta^*\theta_1\}$ and $m' = \{\theta_2 > \alpha^* + \beta^*\theta_1\}$, for some constants α^*, β^* with $\beta^* > 0$. Let the receiver's optimal actions in response be $a_k = E[\theta_k|m]$ and $a'_k = E[\theta_k|m']$ for $k = 1, 2$. These messages and actions constitute an equilibrium if (7) holds exactly for those θ_1, θ_2 such that $\theta_2 = \alpha^* + \beta^*\theta_1$. While we present our results in this section in terms of the sender explicitly adjusting her ranking strategy to reflect the asymmetries, all announcement strategies can be equally well understood in terms of the receiver adjusting his interpretation of the messages.[15]

From Theorem 3 we know that for sufficiently small asymmetries such an equilibrium exists with α^* close to 0 and β^* close to 1. In fact, comparative cheap talk is often robust to substantial asymmetries in this quadratic expert game. To illustrate this, assume that *f* is i.i.d. uniform in what follows. Theorem 3 tells us (and computations indicate) that similar conclusions also obtain for general asymmetric *f*.

[15] A similar situation arises in interval cheap talk. For instance, the sender could explicitly state that the variable is in the range $[0, c]$ or $(c, 1]$, or the sender could just state that the variable is high or low and leave it to the receiver to make the appropriate inference of c based on all available information.

First consider asymmetries in the biases while holding the weights constant, $\lambda_1 = \lambda_2$. In this case, for all $0 \leqslant b_1 - b_2 < \frac{1}{2}$, an influential equilibrium exists where $\beta^* = 1$ and $\alpha^* > 0$, with α^* the solution to

$$\frac{\alpha(\alpha^2 - 3\alpha - 1)}{3(\alpha^2 - 2\alpha - 1)} = b_1 - b_2.$$

In such equilibria, the sender adjusts her pronouncements in favor of θ_1 by the constant amount α^*. For larger values of $b_1 - b_2$, balancing the sender's incentives requires both the constant α^* and slope β^* to be adjusted. For instance, when $b_2 = \frac{1}{4}$ and $b_1 = 1$, an equilibrium exists with $\alpha^* = 0.16$ and $\beta^* = 0.11$, with corresponding actions $a = (0.54, 0.11)$ and $a' = (0.49, 0.61)$. Recall that the value of $\frac{1}{4}$ for b_2 is just enough to prevent informative communication on issue 2 in isolation. However, if issue 1, about which the expert is known to be extremely biased, is also brought into the discussion, considerable information about issue 2 can be revealed. With quadratic preferences, the sender's marginal utility for a higher action on any issue is increasing in the bias. The large bias on issue 1 relative to 2 allows the sender to balance small changes in the receiver's action on issue 1 against large changes in the receiver's action on issue 2. Equivalently, the receiver is more skeptical of the sender's pronouncements in favor of issue 1 than those in favor of issue 2.

As with biases, comparative cheap talk is robust to asymmetries in the weights. For instance, if we let the weights λ_k vary with the biases held constant, $b_1 = b_2 = b$, an influential equilibrium exists for $\frac{1}{3} < \lambda_2/\lambda_1 < 3$ if $b = \frac{1}{2}$, and for $\frac{3}{5} < \lambda_2/\lambda_1 < \frac{5}{3}$ if $b = 1$. Even for arbitrary λ_k or b_k, Theorem 4 indicates that an equilibrium exists in the quadratic case as long as the possible biases b_k and weights λ_k lie in compact sets in $\mathbb{R}$. To generate an explicit expression for N^*, consider the case of equal biases $b_k = 1$ and normalize the weights so that $1 = \lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_N > 0$. When $N = 2$, an influential equilibrium exists provided $\lambda_2 > r \equiv \frac{3}{5}$. It follows that when $N = 3$ an influential equilibrium exists when $\lambda_3 > r^2$ since either $\lambda_2 > r$, in which case comparative cheap talk on issues 1 and 2 is influential, or $\lambda_2 \leqslant r$, in which case we must have $\lambda_3/\lambda_2 > r$ so comparative cheap talk on issues 2 and 3 is influential. Generalizing, we see that an influential equilibrium exists for the N-dimensional game if $\lambda_N > r^{N-1}$. It follows that when the λ_k lie in a compact set $[x, 1]$ with $x \in (0, 1)$, an influential equilibrium exists as long as $N \geqslant N^* = \lceil 1 + \frac{\ln x}{\ln r} \rceil$.

Observe that it is possible in the last example to construct a sequence $\{\lambda_k\}$ with $\lambda_k \leqslant r^{k-1}$ for each k, such that an influential equilibrium does not exist for any N. Since such a sequence cannot lie in a compact set in $(0, 1]$, the relevant space for the λ_k, a "converse" of Theorem 4 can be said to obtain in this case. Since first derivatives of quadratic preferences are unbounded in b_k, a set of quadratic preferences with unboundedly large b_k's is itself not (uniformly) bounded under the $\mathcal{C}^2$-uniform convergence norm and so not compact (see, e.g., [24]). This suggests that a similar converse may be obtained by choosing a suitable unbounded sequence of biases.

4.1.2. Multiplicatively separable sender preferences

The quadratic sender utility function in (6) is typically used in applications of one-dimensional cheap talk because of its relative ease of analysis and its potential to generate credible communication. However, in a multidimensional environment the potential for communication exists in even simpler models where the sender's preferences are monotonic so that the sender always wants to exaggerate the maximum amount. To see this consider a valuation game where u^R has the same quadratic form as above, but the sender's utility function takes the multiplicatively separable form

$$u_k^S = \lambda_k v(\theta_k) w(a_k), \tag{8}$$

where v is a continuous, positive, increasing function and w is a continuous, increasing function. Note that cheap talk is not influential in a single dimension since the sender always wants the receiver to take the maximum action. However, $u_{12}^S = v'w' > 0$ so the sender's utility function is supermodular and, by Theorem 1, comparative cheap talk is an equilibrium in the symmetric case. Regarding sender payoffs, if w is linear or convex then supermodularity implies the sender is necessarily better off from revealing finer information, but if w is sufficiently concave the sender will prefer to withhold such information.

An attractive feature of this specification of sender preferences is that in many cases comparative cheap talk is fully robust to asymmetries, no matter how large. For instance, if $v(0) = 0$, and $N = 2$, the set of sender types who are indifferent between two different action profiles takes the particularly simple form $v(\theta_2) = \beta v(\theta_1)$ for some constant $\beta > 0$. Such an equilibrium exists when $N = 2$, *regardless* of the asymmetries in f or the λ_k's.[16] Intuitively, the slope term β has enough room to move around in the unit box to adjust for any degree of asymmetries.[17] However, when asymmetries are very large, β will be close to 0 or ∞, and so the informativeness of such a ranking will be limited. Such large asymmetries essentially reduce a multi-dimensional game into a one-dimensional game. Nevertheless, with our next example of a valuation game we show the existence of influential equilibria whose informativeness is not undermined by arbitrary asymmetries.

4.1.3. State-independent sender preferences

Now consider an even simpler specification of sender preferences that also captures situations where the sender has a maximal incentive to exaggerate,

$$u_k^S = \lambda_k w(a_k),$$

where w is a continuous, increasing function as before. Since the sender's payoff depends only on the receiver's valuation a_k, this model is especially applicable to situations where only monetary values are important. For instance a sell-side stock analyst reports on the value θ_k of different stocks to investors and only cares about pushing up the market valuations regardless of the actual value of the assets. Or an auction house provides price estimates to competing buyers and cares only about maximizing revenue.

[16] Details of this result, which requires some mild regularity conditions on the correlation between the θ_k's, are available on request. Correlation can create an incentive to always rank one issue higher for large asymmetries. For instance, saying that $v(\theta_2) \geqslant \beta v(\theta_1)$ for large β implies that θ_2 is quite large, but strong correlation then implies that θ_1 is also quite large. So the receiver's estimates of both θ_1 and θ_2 can be higher if the sender claims that $v(\theta_2) \geqslant \beta v(\theta_1)$.

[17] Note that when $v(0) > 0$ the model resembles that of quadratic sender preferences in that the indifference locus also has an intercept term α. As with quadratic sender preferences, such an intercept term may not have enough room in the unit box to adjust for arbitrarily large asymmetries.

Because of this state-independence, the utility function is only weakly supermodular, but this is still sufficient for comparative cheap talk to be credible under the assumptions of Theorem 1. Therefore, comparative cheap talk may play a role in situations such as the provision of estimated sales prices to buyers by auction houses. Even if the incentive to exaggerate undermines the credibility of the absolute information in such price estimates, buyers might still believe the relative information in the estimates. Regarding sender payoffs, weak supermodularity implies that the sender is ex ante strictly better off revealing coarser information if w is concave and revealing finer information if w is convex.[18] Under the stock analyst interpretation, this implies that the analyst is better off from finer rankings only if she is rewarded disproportionately for pushing up the price of stocks or is risk loving.

Regarding asymmetries, a key regulatory issue in the securities industry is whether sell-side analysts favor stocks of companies that do business with the analyst's investment bank. Clearly such favoritism will undermine the usefulness of analyst reports, but it is often argued that investors can see through such biases and still garner some information from an analyst's ratings. Since u_k^S is not strictly supermodular one cannot directly apply Theorem 3 to analyze this question, but it can be shown that the conclusions of Theorem 3 obtain.

Specifically, consider the case where f is i.i.d. uniform, $N = 2$, and $u_k^S = \lambda_k a_k$. In this case, the sender is indifferent between two action profiles $a = (a_1, a_2)$ and $a' = (a_1', a_2')$ if and only if

$$\lambda_1 a_1 + \lambda_2 a_2 = \lambda_1 a_1' + \lambda_2 a_2'. \tag{9}$$

This is a restriction only on the slope of the line joining a and a'. Consequently, many types of equilibria are possible. We consider a simple one with two messages in the affine form, $m = \{\theta_2 \leqslant \alpha + \beta\theta_1\}$ and $m' = \{\theta_2 > \alpha + \beta\theta_1\}$, with $\beta \in [0, 1]$ and $\alpha = 1/2 - \beta/2 \geqslant 0$. In the space of a_1, a_2, such a line $\theta_2 = \alpha + \beta\theta_1$ passes through the point $(1/2, 1/2)$. Normalizing $\lambda_1 = 1 \geqslant \lambda_2 = \lambda > 0$, and substituting $a_k = E[\theta_k|m]$ and $a_k' = E[\theta_k|m']$ for $k = 1, 2$ into equation (9), one sees that such a comparative cheap talk equilibrium exists for all $\lambda \in (0, 1]$ with

$$\beta = \sqrt{3 + \frac{1}{\lambda^2}} - \frac{1}{\lambda}.$$

In the symmetric case where $\lambda = 1$, the line forms the 45° diagonal and the sender's statement is equally informative about both issues. As λ becomes smaller and issue 2 becomes relatively less important, the line becomes increasingly flat and provides less and less information about issue 1. However, even in the limit when the sender puts overwhelming weight on issue 1, the ranking still provides considerable information about issue 2 that is of relevance to the receiver.

[18] Distinct from the payoff effects due to concavity/convexity, in the auction example if there are buyer information rents then revealing information through such a ranking increases auction revenues via the linkage principle [7].

4.2. Recommendation games

We now turn to games where the receiver chooses a binary action in each dimension in any equilibrium, e.g. a legislator votes for or against a bill after listening to a lobbyist, a consumer buys a product or not after listening to a salesperson, or an employer hires a student or not after looking at recommendation letters. Such games have all the properties of continuous action games, and in addition, have some novel and empirically interesting implications of their own.

To make things concrete, assume that the sender is a professor who knows the quality of N students and the receiver is an employer who will hire a student k if his expected quality given all available information is above some threshold. To capture this let the sender's and receiver's payoff for each student be

$$u_k^S = \lambda_k(\theta_k - \tau_k^S)a_k, \tag{10}$$

$$u_k^R = (\theta_k - \tau_k^R)a_k, \tag{11}$$

where $\tau_k^i \in (0, 1)$ is a threshold quality for student k above which the student is worth employing according to i. With such preferences, the receiver's responses in any equilibrium will be binary—he will choose the action $a_k = 1$ (i.e., hire the student) if the expected quality of student k is above τ_k^R given all available information, and choose the action $a_k = 0$ otherwise. For simplicity assume that there is no limit to the number of students an employer can hire, but such limits would have no effect on the results.[19]

Before analyzing comparative cheap talk, first consider interval cheap talk on one dimension that is analogous to that examined in the quadratic example. If the professor expects the cheap talk to be influential, the professor will recommend a student if and only if $\theta_k \geqslant \tau_k^S$. And if the employer believes the recommendation, the employer will hire a recommended student if $E[\theta_k|\theta_k \geqslant \tau_k^S] > \tau_k^R$ and not hire an unrecommended student if $E[\theta_k|\theta_k < \tau_k^S] < \tau_k^R$. Therefore, it is an influential equilibrium for the professor to disclose whether student quality θ_k is in the interval $(\tau_k^S, 1]$ or not if $E[\theta_k|\theta_k < \tau_k^S] < \tau_k^R < E[\theta_k|\theta_k \geqslant \tau_k^S]$. For τ_k^S sufficiently close to τ_k^R this condition will be satisfied, but if τ_k^S and τ_k^R are too far apart then the conflict of interest is too great for interval cheap talk to be credible.

Since both u^R and u^S are continuous and supermodular, Theorem 1 applies in the symmetric model where $\lambda_k = \lambda$, $\tau_k^S = \tau^S$ and $\tau_k^R = \tau^R$ for all k and f is symmetric. For instance, consider the case where $\tau^S = 0$, $\tau^R = \frac{3}{5}$ and f is i.i.d. uniform. Under the uniformity assumption, the expected value of the jth worst student is $E[\theta_{j:N}] = j/(N + 1)$. Thus, when the professor ranks two students the employer will infer that the lower ranked student has an expected quality of $\frac{1}{3}$ while the higher ranked student has an expected of quality of $\frac{2}{3}$ so the employer will only hire the latter. Given this, the professor indeed recommends the better student since supermodularity implies that the better student provides her with a higher marginal benefit from being hired.

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[19] Similarly, there might be multiple employers, each of which can hire a limited number of students. As long as the professor's message is public, the results are unchanged. In such a case the more appropriate interpretation of the example is that of a college disclosing information about students via transcripts, rather than a professor providing possibly private recommendations.

Regarding ex ante sender payoffs, revealing a partial ranking may be preferable to providing either the complete ranking or no information at all. For instance, in the example above with $N = 3$, revealing no information implies that no student will be hired since $E[\theta_k] = \frac{1}{2} < \tau^R$. In contrast, disclosing the complete ranking results in estimated qualities of $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ for the three students, so that only the top student is hired. However, if the professor only identifies the top two students and does not differentiate between them, then the employer estimates a quality of $\frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{3}{4} = \frac{5}{8} > \tau^R$ for each of them and they are both hired. Such a partial ranking is thus ex ante payoff optimal for the professor. As N increases, Theorem 2 applies and the complete ranking identifies almost precisely the fractions of the students who are above and below τ^R, which is approximately the same outcome as under full information, and is the ideal case for the employer. However, the professor can ensure that a strictly larger fraction of students is hired by withholding some information through a partial ranking.[20]

The payoff gains from a partial ranking might explain why some colleges either explicitly withhold transcript information or inflate grades so much that the best students are hard to differentiate from merely good students. Note that as the distribution of students becomes more favorable, a higher proportion of students can be put in the top category and still make the threshold. Hence, "grade inflation" should be more severe when average student quality is increasing and grades should be more inflated in elite schools.

These results apply to the symmetric case, i.e. the professor does not have a "favorite" student and the employer has no reason to expect that any one student is better than another. With such asymmetries communication may not be credible when the number of students who make the threshold τ^R depends on the ranking. For instance, it is a more positive signal for an unfavored student to be ranked higher than for a favored student to be ranked higher. Therefore, regardless of which student is actually better, if the threshold τ^R is very high a professor might be tempted to rank the unfavored student higher so that at least one student gets a job. However, even though u_k^R fails the regularity conditions that guarantee an interior solution to the receiver's problem, the discreteness of the receiver's actions makes robustness arguments to small asymmetries even simpler than those employed in Theorem 3. As long as the asymmetries are relatively small, the number of students that make the threshold will not depend on the ranking. Since supermodularity still ensures that the professor wants the best students to make the threshold, comparative cheap talk is still an equilibrium.

5. Extensions: a discussion

5.1. Interdependent actions

We have assumed that the receiver action in each dimension is independent of his actions in other dimensions, but in some applications this might be inappropriate. For ex-

[20] Letting F be the distribution of each i.i.d. θ_k, in the limit the sender can put fraction $1 - F(\tau^*)$ of the issues in the top category where τ^* satisfies $E[\theta_k|\theta_k \geqslant \tau^*] = \tau^R$. In this example where F is uniform and $\tau^R = \frac{3}{5}$, $\tau^* = \frac{1}{5}$ so in the limit the proportion of students above the threshold is $1 - F(1/5) = \frac{4}{5}$. In contrast, under the complete ranking the proportion of students above the threshold is only $1 - F(\tau^R) = \frac{2}{5}$.

ample, in the recommendation game a buyer might be interested in purchasing only one product even if multiple products are above the threshold τ^R. Clearly, this does not reduce the salesperson's incentive to rank products truthfully and so it does not affect any of our results. Similarly, in the professor-employer example, a limit on the number of positions available does not change the results. In this example a more interesting case of interdependent actions is where θ represents different attributes of *one* student (e.g. research skills, teaching ability, etc.). In such a case, if the action profile a represents the details of the contract (teaching load, research support, salary, etc.) that will be offered to the student, then comparative statements like "the student is relatively better at research" might still be influential and enable the sender to trade-off teaching loads against research requirements. Aspects of this problem are addressed in Chakraborty and Harbaugh [9].

5.2. Sender actions

The model can be extended to include actions taken by the sender as well as the receiver. For instance in coordination games if both sides agree on the ranking of outcomes, cheap talk can sometimes resolve strategic uncertainty over which action each side intends to take [17,25,18]. Baliga and Morris [4] consider coordination games in which there is one-sided uncertainty about payoffs so that the ranking of outcomes is state-dependent as in our model, rather than state-independent as in models with only strategic uncertainty. For instance, two firms must each decide whether to invest in complementary research projects and one company has private information about the profitability of their own investment. They find the strong negative result that in a binary action game if the informed side always wants the uninformed side to take a particular action, e.g., wants the other firm to invest, then no cheap talk of any kind is possible. Note that this result applies to games in a single dimension. If the two firms are considering several different projects, it is straightforward to show that comparative cheap talk can be used to credibly rank the different projects and thereby increase investment efficiency.

5.3. Private receiver information

Often the receiver will also have some private information that is relevant to his decision. Such information creates asymmetries from the perspective of the receiver that can give the sender an incentive to lie about the ranking. For instance, if an employer privately interviews different job applicants and the threshold for employment is relatively low, a professor might try to artificially boost a weak job candidate at the expense of a stronger candidate who is likely to interview well enough to receive a job anyway. In the two types of games analyzed in Section 4 it can be shown that comparative cheap talk is robust to private information held by the decision maker in that, even if the complete ranking is not credible, there always exists a two-category partial ranking that is credible for a large enough number of issues. For an analysis of this equilibrium in valuation games see Chakraborty et al. [8].

6. Conclusion

We find that simple complementarity and symmetry conditions are sufficient for cheap talk across dimensions to be credible even when interests are too opposed to support cheap talk in a single dimension. We also find that the amount of information revealed by comparative cheap talk can be considerable. When we allow for asymmetries, comparative cheap talk continues to be credible if the asymmetries are not too large. For a sufficiently large number of issues, there are always some issues which are sufficiently close to being symmetric for comparative cheap talk to be credible. These results broaden our understanding of how mere cheap talk can be used by individuals and institutions to communicate information.

Appendix

Proof of Theorem 3. Fix some $p^* \in \mathbf{P}^*$ and, via Theorem 1, let the equilibrium actions be (α_1^*, α_2^*) when the announced message is $\theta_1 > \theta_2$ so that (α_2^*, α_1^*) is the action profile when the message is $\theta_1 \leqslant \theta_2$. Let $Z = \{a = (a_1, a_2) | a_1 \geqslant \frac{\alpha_1^* + \alpha_2^*}{2} \geqslant a_2\}$ and $Z' = \{a' = (a_2', a_1') | a_2' \geqslant \frac{\alpha_1^* + \alpha_2^*}{2} \geqslant a_1'\}$. Note that for each $a \in Z$, $a' \in Z'$, $a \neq a'$, and arbitrary $\mathbf{u}^S \in \mathbf{U}_S^2$, the sender prefers a to a' iff

$$u_1^S(\theta_1, a_1) - u_1^S(\theta_1, a_1') \geqslant u_2^S(\theta_2, a_2') - u_2^S(\theta_2, a_2) \tag{12}$$

which using strict supermodularity can be written as a slightly modified comparative cheap talk inequality

$$\Delta_1(\theta_1; a_1, a_1') \geqslant \Delta_2(\theta_2; a_2', a_2) \tag{13}$$

for Δ_i strictly increasing in its first argument. By the inverse function theorem, there exist increasing functions $l_1(\cdot)$ and $l_2(\cdot)$ such that (13) can also be written as $\theta_2 \leqslant l_1(\theta_1; a, a')$ and $\theta_1 \geqslant l_2(\theta_2; a, a')$.

Given $f \in D_2$ and $\mathbf{u}^R \in \mathbf{U}_R^2$, for $k = 1, 2$ let $\alpha_k : Z \times Z' \to \mathbb{R}^2$ and $\alpha_k' : Z \times Z' \to \mathbb{R}^2$ be the solution to the receiver's problem:

$$\alpha_k(a, a'; p) = \arg\max_{a_k} E_f[u_k^R(\theta_k, a_k) | \Delta_1(\theta_1, a_1, a_1'; p) \geqslant \Delta_2(\theta_2, a_2', a_2; p)], \tag{14}$$

$$\alpha_k'(a, a'; p) = \arg\max_{a_k} E_f[u_k^R(\theta_k, a_k) | \Delta_1(\theta_1, a_1, a_1'; p) \leqslant \Delta_2(\theta_2, a_2', a_2; p)]. \tag{15}$$

Define $\psi(a, a'; p) : Z \times Z' \times \mathbf{P} \to \mathbb{R}^4$ as the ordered tuple

$$\begin{aligned}
\psi^1(a, a'; p) &= \alpha_1(a, a'; p) - a_1,\\
\psi^2(a, a'; p) &= \alpha_2(a, a'; p) - a_2,\\
\psi^3(a, a'; p) &= \alpha_2'(a, a'; p) - a_2',\\
\psi^4(a, a'; p) &= \alpha_1'(a, a'; p) - a_1'.
\end{aligned}$$

An equilibrium is a pair a, a' such that $\psi(a, a'; p) = 0$. Whenever such an equilibrium exists it is influential, by construction. By Theorem 1, it exists for any $p^* \in \mathbf{P}^*$ with equilibrium actions $\alpha^* = (\alpha_1^*, \alpha_2^*)$ and $\alpha^{*\prime} = (\alpha_2^*, \alpha_1^*)$.

It is immediate that ψ is continuous in its arguments. Let $J = \partial_{a,a'}\psi(\alpha^*, \alpha^{*\prime}; p^*)$ be the Jacobian of ψ. We wish to apply the implicit function theorem at such p^* i.e., show that $\det J \neq 0$.[21] Define the function $\xi : [0, 1]^2 \to \mathbb{R}$

$$\xi(t_1, t_2) = -\frac{\int_0^1 \dfrac{u_a^{R*}(z, t_1) u_a^{S*}(z, t_2)}{u_\theta^{S*}(z, \alpha_1^*) - u_\theta^{S*}(z, \alpha_2^*)} f^*(z, z)\, dz}{\int_0^1 \int_{z_2}^1 u_{aa}^{R*}(z_1, t_1) f^*(z_1, z_2)\, dz_1\, dz_2}, \tag{16}$$

where subscripts denote partial derivatives. Keeping in mind the symmetries in the symmetric equilibrium, especially the fact that the receivers optimal actions α, α' depend on the initial actions a, a' only through their effect on the functions l_1 and l_2, it is not difficult to verify that ψ is continuously differentiable in a, a' in a neighborhood of the symmetric solution and that the following symmetries obtain:

$$\begin{aligned}
\frac{\partial \alpha_1^*}{\partial a_1} &= -\frac{\partial \alpha_1^*}{\partial a_2'} = \frac{\partial \alpha_2'^*}{\partial a_2'} = -\frac{\partial \alpha_2'^*}{\partial a_1} \equiv x_1 = \xi(\alpha_1^*, \alpha_1^*),\\
\frac{\partial \alpha_1^*}{\partial a_2} &= -\frac{\partial \alpha_1^*}{\partial a_1'} = \frac{\partial \alpha_2'^*}{\partial a_1'} = -\frac{\partial \alpha_2'^*}{\partial a_2} \equiv x_2 = \xi(\alpha_1^*, \alpha_2^*),\\
\frac{\partial \alpha_2^*}{\partial a_1} &= -\frac{\partial \alpha_2^*}{\partial a_2'} = \frac{\partial \alpha_1'^*}{\partial a_2'} = -\frac{\partial \alpha_2'^*}{\partial a_1} \equiv x_3 = \xi(\alpha_2^*, \alpha_1^*),\\
\frac{\partial \alpha_2^*}{\partial a_2} &= -\frac{\partial \alpha_2^*}{\partial a_1'} = \frac{\partial \alpha_1'^*}{\partial a_1'} = -\frac{\partial \alpha_2'^*}{\partial a_2} \equiv x_4 = \xi(\alpha_2^*, \alpha_2^*),
\end{aligned} \tag{17}$$

where superscripts * denote derivatives evaluated at symmetric point $(\alpha^*, \alpha^{*\prime}; p^*)$. Using the block symmetric form of J we see that

$$\det J = 1 - 4(x_1 x_4 - x_2 x_3) - 2(x_1 + x_4). \tag{18}$$

It is immediate that the set of primitives for which $\det J \neq 0$ is open, since its complement is closed. It remains to show that this set is dense in $\mathbf{P}^*$.

Suppose that $\det J = 0$ for some p^*. We show explicitly that one can perturb u^{S^*} slightly to make $\det J \neq 0$. Pick $\varepsilon > 0$ sufficiently small and perturb u^{S*} as follows

$$\widehat{u}^{S*}(z, a) = u^{S*}(z, a) + \tau^\varepsilon(z, a),$$

where τ^ε is $\mathcal{C}^2$ satisfying

$$\lim_{\varepsilon \to 0} \tau^\varepsilon(z, a_k) = 0 \quad \text{all } z, a_k$$

and

$$\begin{aligned}
&\tau^\varepsilon(z, \alpha_1^*) = 0,\\
&\tau^\varepsilon(z, \alpha_2^*) = -\varepsilon(u^S(z, \alpha_1^*)) - u^S(z, \alpha_2^*),\\
&\tau_a^\varepsilon(z, \alpha_2^*) = (2\varepsilon + \varepsilon^2) u_a^S(z, \alpha_2^*)
\end{aligned}$$

[21] The specific version of the implicit function theorem we use is that from Mas-Collell [24, Chapter 1, C.3.3].

for all z. For ε small, this can always be done preserving the strict supermodularity of $\widehat{u}^{S*}$. Letting $\widehat{}$ denote the case corresponding to $\widehat{u}^{S*}$ this perturbation has the effect that

$$\widehat{x}_i = \frac{x_i}{1+\varepsilon}, \quad i = 1, 3$$
$$\widehat{x}_i = x_i(1+\varepsilon), \quad i = 2, 4.$$

Using $\det J = 0$, we then have:

$$\begin{aligned}\det \widehat{J} &= 1 - 4(\widehat{x}_1\widehat{x}_4 - \widehat{x}_2\widehat{x}_3) - 2(\widehat{x}_1 + \widehat{x}_4)\\ &= 1 - 4(x_1x_4 - x_2x_3) - 2\left(\frac{x_1}{1+\varepsilon} + x_4(1+\varepsilon)\right)\\ &= -2\varepsilon\left(x_4 - \frac{x_1}{1+\varepsilon}\right)\end{aligned}$$

which can be made to be non-zero by perturbing ε, unless $x_1 = x_4 = 0$ in which case, since $\det J = 0$, we obtain that $x_2, x_3 \neq 0$, so that a perturbation very similar to the one above will suffice (e.g. by dropping the ε^2 term in the expression for τ_a^{ε}). $\square$

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