Search with Learning

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Abstract

This paper provides a method to estimate search costs in a differentiated product environment in which consumers are uncertain about the utility distribution. Consumers learn about the utility distribution by Bayesian updating their Dirichlet process prior beliefs. The model provides expressions for bounds on the search costs that can rationalize observed search and purchasing behavior. Using individual-specific data on web browsing and purchasing behavior for MP3 players sold online we show how to use these bounds to estimate search costs as well as the parameters of the utility distribution. Our estimates indicate that search costs are sizable. We show that wrongfully assuming consumers are not learning while searching can lead to severely biased search cost and elasticity estimates.

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1 Introduction

Over the past fifty years, a large literature on search has tried to explain markets that are characterized by imperfectly competitive behavior. In a typical search model, consumers make a tradeoff between the gains from searching and the cost of searching to determine whether to continue searching or, alternatively, how many times to search. The gains from search are typically derived using the assumption that consumers "know" the distribution of prices or wages (Stigler, 1961; McCall, 1970; Mortensen, 1970). Several papers diverge from this view and have analyzed optimal search behavior when consumers are not only uncertain about actual draws but also about the distribution (Rothschild, 1974; Rosenfield and Shapiro, 1981; Chou and Talmain, 1993; Dana, 1994; Bikhchandani and Sharma, 1996). This is especially important since search behavior has been shown to be sensitive to the assumed distribution (e.g., Gastwirth, 1976).

In this paper we develop a method to estimate consumer search costs for differentiated products when consumers have only partial information about the distribution from which is being sampled. In the next section we present a model, based on the works of Rothschild (1974), Rosenfield and Shapiro (1981), and Bikhchandani and Sharma (1996), which relaxes the assumption that consumers "know" the distribution of offerings while deciding on their search strategy, and allows for learning of the utility distribution. More specifically, consumers learn about the utility distribution by Bayesian updating their Dirichlet process priors while sampling information about products and retailers. We use information on a consumer's sequence of searches to derive expressions for bounds on the search cost that rationalizes the consumer's observed search behavior. The intuition behind this approach is straightforward: if a consumer stops searching this means she found an alternative that has a higher utility than her reservation utility. Since reservation utility is a function of both search cost and the expected gains from searching, this inequality can be inverted to obtain a lower bound on the consumer's search cost. Similarly, if a consumer continues searching after having sampled an alternative, this means her search cost should have been lower than the expected gains from search, which can be used to obtain an upper bound on her search cost.

Our model applies to settings in which purchase decisions as well as search histories are observed. If products are homogeneous and consumers are only searching for the lowest price, the search cost bounds can be obtained directly from the observed gains from search. In the case of differentiated products, the bounds are conditional on the parameters of the utility function. We show how to map a differentiated product utility framework into the learning model and derive an estimation strategy for the parameters of both the utility and search cost distribution. As such, these bounds allow us to estimate the relationship between observed consumer characteristics and search costs using simulated maximum likelihood.

An important difference between the learning model and a model in which there is no learning is that in the learning model, reservation utilities are decreasing in the number of alternatives sampled, while constant in the no-learning model. We show that if initial priors are rational, i.e., the base distribution of the Dirichlet process equals the true utility distribution, the decreasing reservation utility property will result in less overall search activity than in the no-learning model. The decreasing reservation utility property may also trigger recall: a sampled alternative that was not good enough initially might pass the bar after a few searches, once more alternatives have been sampled and the reservation utility has gone down. This is important for explaining actual search data: the search patterns in the data reveal that in close to a third of transactions consumers recall a previously visited firm. As argued in De los Santos, Hortaçsu, and Wildenbeest (2012), this violates optimal behavior in the standard sequential search model.

In Section 3, we test the identification properties of our model using Monte Carlo experiments. The simulations suggest that our estimation method can recover the parameters of the utility function and the distribution of search costs. We demonstrate that wrongfully assuming that there is no learning leads to biased search cost and elasticity estimates. As predicted by the model, if consumers have rational initial priors the bias will be towards higher search costs. We show that if consumers have uniform initial priors, the direction of the bias depends on how optimistic the initial prior is in comparison to the true utility distribution. In particular, if consumers have relatively optimistic priors, there will be a downward bias in search costs. In this section we also study how measurement errors in the choice sets and prices affect the estimation. Finally, we show that in the special case of homogenous products the weight on the initial prior can be successfully recovered.

In Section 4, we present an application of the model using data on the web browsing and purchasing behavior of a large panel of consumers. We focus on purchases of MP3 players. Our data not only allow us to observe online transactions for all consumers in the panel, but also which online stores have been visited shortly before a transaction. We let utility be a function of retailer and product characteristics as well as an idiosyncratic unobserved component. Our estimates for the product differentiation model indicate that median search costs are $24.36. We find some evidence that search costs are negatively related to the speed of a household's Internet connection: a broadband connection makes search costs lower. Our model gives a better fit to the data than a standard sequential search model in which prices are known to the consumer. Moreover, we find search costs to be uniformly lower in the learning model.

Our paper relates to several recent papers that estimate search costs. The vast majority of these paper assumes consumers know the distribution of prices (Hortaçsu and Syverson, 2004; Hong and Shum, 2006; Moraga-González and Wildenbeest, 2008; Wildenbeest, 2011; De los Santos, Hortaçsu, and Wildenbeest, 2012; Koulayev, 2014). An exception is Koulayev (2013), who estimates a model of search with Dirichlet priors using aggregate data on prices and market shares. Unlike our dataset, Koulayev's data does not contain information on search sequences—to be able to estimate the model he derives closed-form ex-ante buying probabilities. Although less general than the Dirichlet *process* priors we use in our paper, Dirichlet priors also imply search decisions at any given time can be characterized by identity of the best alternative observed so far and the number of searches to date, which greatly simplifies integrating out the unobserved search histories. Another related paper is Häubl, Dellaert, and Donkers (2010), who estimate the parameters of a product differentiation and learning model in an experimental setting. Their experimental data allows them to observe the identity of the alternative with the highest utility for each subject at any point in the search sequence, which greatly simplifies the estimation of especially the parameters of the utility function. Our paper is also related to a literature that estimates Bayesian learning models (Erdem and Keane, 1996; Ackerberg, 2003; Crawford and Shum, 2005; Chernew, Gowrisankaran, and Scanlon, 2008). An important difference with our paper is that while we explicitly model consumers' joint search and learning decisions, in the literature on the estimation of Bayesian learning models, learning takes a more passive form.

We believe that our study makes several contributions relative to existing papers. First of all, we provide a methodology to estimate search costs in an environment in which learning is important, using individual-specific search data. The use of Dirichlet process priors allows us to build our model around an otherwise standard discrete choice product differentiation model of demand, and is therefore sufficiently flexible to allow for both horizontal and vertical differentiation. We show that modeling learning is important: ignoring learning may lead to biased search cost and elasticity estimates. Finally, our paper makes progress on the identification of the precision of the prior in the special case of homogenous products, and as such gives a better idea about the importance of learning while searching.

2 Model

In this section we present a model in which consumers learn about the utility distribution while searching. We use our framework, which is based on the learning models of Rothschild (1974), Rosenfield and Shapiro (1981), and Bikhchandani and Sharma (1996), to show how to estimate search costs in a learning context when the sequence of retailers visited prior to a purchase is observed. In particular, we use a consumer's search history to obtain search cost bounds that rationalize the consumer's observed behavior in a learning environment. We show how to map a standard model of product differentiation into the learning model and discuss how to estimate the model using a simulated maximum likelihood procedure.

2.1 Consumer Learning

Consumers learn about different options available by Bayesian updating their priors on an unknown utility distribution while sampling. Consumers are searching sequentially, which means that after each observation they have to determine whether to continue searching or not, making a tradeoff between the additional cost of another search and the potential gains of observing a better offer. We assume consumers can recall previous offers at no cost. The cost of each search, including the first, is specific to consumer i, and denoted by c_i. Consumers have imperfect information about the utility distribution; each new search provides the consumer with additional information, which is used to update priors on the utility distribution.

Suppose first that there are N options available in the market. Let $u = \{u_1, u_2, \ldots, u_N\}$ denote the utility values of the alternatives, where the subscript indicates the rank of the firm in terms of utility. The probability of sampling each utility is given by a vector $\rho = (\rho_1, \rho_2, \ldots, \rho_N)$, where $\sum_n \rho_n = 1$. The utility values are known to consumers, while the probabilities of sampling each utility value are not. Instead, consumers consider the probabilities to be random variables that are distributed according to a Dirichlet distribution of order N with density

$$f(\rho_1, \ldots, \rho_N) = \frac{\Gamma\left(\sum_{n=1}^{N} \alpha_n\right)}{\prod_{n=1}^{N} \Gamma(\alpha_n)} \prod_{n=1}^{N} \rho_n^{\alpha_n - 1},$$

where Γ is the gamma function and $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_N)$ are concentration parameters. The prior expected value of each probability ρ_n is given by

$$E[\rho_n] = \frac{\alpha_n}{W},$$

where $W = \sum_n \alpha_n$ can be interpreted as the weight put on the initial prior. As consumers start searching and sampling utilities, the prior is updated. Since the Dirichlet distribution is the conjugate prior of the multinomial distribution, the posterior distribution will be Dirichlet as well. Specifically, the posterior expected value of ρ_n after the consumer has sampled an alternative is

$$E[\rho_n] = \begin{cases} \dfrac{\alpha_n}{W+1} & \text{if } n \text{ is not sampled;} \\ \\ \dfrac{\alpha_n + 1}{W+1} & \text{if } n \text{ is sampled.} \end{cases}$$

A simple example illustrates the updating process. Suppose there are three options and consumers have an uninformative prior, i.e., $\alpha = (1, 1, 1)$ so the prior expected values of the probability of sampling each option are given by $E[\rho] = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. If a consumer starts searching and samples option 2, we add 1 to α_2 to get a Dirichlet posterior distribution with concentration parameters $\alpha = (1, 2, 1)$ and posterior expected values $E[\rho] = \left(\frac{1}{4}, \frac{2}{4}, \frac{1}{4}\right)$.

When deciding whether to continue searching, consumers make a trade-off between the cost of an additional search and the expected gains from search, where the latter is a function of the expected probability of finding a better alternative. A useful feature of this particular learning environment is that the posterior probability of sampling an alternative with a higher utility than the best alternative observed so far only depends on the weight put on the initial prior, W, and how many alternatives have been sampled to date. More specifically, suppose alternative k is the alternative with the highest utility observed after t searches. Since none of the higher-utility alternatives could have been sampled at time t, the posterior on the expected probability of sampling each of the alternatives that offers a higher utility than u_k is given by $\alpha_m/(W + t)$, with $m = k + 1, \ldots, N$.

In the setting we are studying, a continuous distribution of utilities is more applicable. Bikhchandani and Sharma (1996) discuss the case of a Dirichlet process (see also Häubl, Dellaert, and Donkers, 2010), which generalizes the multinomial Dirichlet case discussed above to a continuous distribution and can be thought of a distribution over probability distributions. More formally, following Ferguson's (1973) definition, probability distribution D is distributed according to a Dirichlet process with base distribution H and concentration parameter W, i.e., $D \sim DP(W, H)$, if for any finite measurable partition $(T_1, T_2, \ldots, T_N)$ of a measurable space Θ,

$$(D(T_1), \ldots, D(T_N)) \sim \text{Dir}\left(WH(T_1), \ldots, WH(T_N)\right),$$

where $\text{Dir}\left(WH(T_1), \ldots, WH(T_N)\right)$ is a Dirichlet distribution with parameters $WH(T_1), \ldots, WH(T_N)$.

Since a Dirichlet process is the conjugate prior for any arbitrary distribution, the posterior distribution will be a Dirichlet process as well. In particular, if $D \sim DP(W, H)$, then the posterior distribution of D given t utility draws $u_1, u_2, \ldots, u_t \sim D$, is a Dirichlet process with concentration parameter $W + t$ and posterior base distribution $\frac{WH + \sum_{i=1}^{t} \delta_{u_i}}{W+t}$, where $\delta_{u_i} = 1$ if $u_i \in T$ and $\delta_{u_i} = 0$ if $u_i \notin T$. The posterior base distribution is also the predictive distribution of u_{t+1} and can be written as

$$u_{t+1} | u_1, u_2, \ldots, u_t \sim \frac{W}{W+t} H + \frac{t}{W+t} \hat{H}_{u_1,\ldots,u_t}, \tag{1}$$

where $\hat{H}_{u_1,\ldots,u_t}$ is the empirical distribution of observed utilities, i.e., $\hat{H}_{u_1,\ldots,u_t} \equiv \frac{1}{t} \sum_{i=1}^{t} \mathbb{1}_{[u_i,\infty)} = \frac{1}{t} \sum_{i=1}^{t} \delta_{u_i}$ (see also Bikhchandani and Sharma, 1996). This means the updated distribution is just the weighted average of the base distribution H and the empirical distribution of observed utilities $\hat{H}$. The concentration parameter, which can be interpreted as the weight put on the initial prior, determines how quickly searchers update: the smaller W, the faster weight is shifted from the base distribution to the empirical distribution of observed utilities.

Let $\hat{u}_{it}$ denote consumer i's highest observed utility after having observed t utility draws. By definition the gains from search at the maximum observed utility at time t are zero for the empirical distribution of observed utilities. This means that if we calculate the gains from search at time t using the updated distribution in equation (1), only the first part of this distribution is relevant (since this reflects the option value of searching), i.e., the gains from search are only a function of the base distribution H with updated weight $W/(W + t)$. As a result, the gains from search at utility $\hat{u}_{it}$ can be given by

$$G(\hat{u}_{it}) = \frac{W}{W+t} \int_{\hat{u}_{it}}^{\infty} (u - \hat{u}_{it}) \cdot h(u) \; du, \tag{2}$$

where $h(u)$ is the density of the base distribution of the Dirichlet process. Intuitively, the gain is equal to the expected utility when searching minus the offer at hand, considering that a consumer will keep the current offer in the event that a utility lower than $\hat{u}_{it}$ is sampled. The term $W/(W+t)$ reflects consumers' updating process: less weight is put on offers that exceed $\hat{u}_{it}$ every time a utility is drawn that is lower than $\hat{u}_{it}$. In the case of t equal to zero this expression describes the gains from search for the standard (non-learning) sequential search model.

2.2 Product Differentiation Model

Denote consumer i's indirect utility for product j, sold by retailer k, as

$$u_{ijk} = \alpha p_j + X_j \beta + X_k \gamma + \varepsilon_{ijk}, \tag{3}$$

where X_j are product characteristics, X_k are firm characteristics, and ε_{ijk} is a utility shock from a type I extreme value distribution. Let there be K retailers and J products. The overall indirect utility distribution of these alternatives in the market follows a mixture distribution of $J \times K$ type I extreme value distributions with location parameter $\delta_{jk} = \alpha p_j + X_j\beta + X_k\gamma$ and scale parameter 1, i.e., the density of the utility distribution is given by

$$f(u) = \frac{1}{K}\sum_{k=1}^{K}\frac{1}{J}\sum_{j=1}^{J}\exp(-(u-\delta_{jk}+\exp(-(u-\delta_{jk})))).$$

We assume consumers do not know ε_{ijk} before searching. Moreover, we simply matters by assuming consumers know the empirical distribution of the mean utilities δ_{jk}. This means consumers know the available variety of mean utilities, but do not know which retailer is offering which mean utility until they start sampling retailers.

We assume that by visiting a retailer a consumer observes prices and characteristics of all products sold by retailer k. Since this means a consumer samples J times from the utility distribution when visiting a firm k, the gains from search in equation (2) change to

$$G(\hat{u}_{it}) = \frac{W}{W+t\cdot J}\sum_{k=1}^{K}\frac{1}{K}\int_{\hat{u}_{it}}^{\overline{u}}(u-\hat{u}_{it})\cdot\frac{d}{du}\left(\prod_{J}H_{jk}(u-\delta_{jk})\right)du,$$

where $H_{jk}(u)$ is the CDF of the initial prior of product j's utility sold by retailer k. The integral reflects the expected maximum utility of J draws from a firm's utility distribution, taking into account that each firm is selling products with different mean utilities δ_{jk}; to get the overall gains from search from randomly sampling the retailers, we can take the average over the gains from search from visiting each individual firm.

2.3 Priors

So far we have left the density of the initial prior, which corresponds to the density $h(u)$ of the base distribution of the Dirichlet process, unspecified. In what follows we consider two cases: rational priors, i.e., consumers' initial priors correspond to the overall utility distribution such that $h(u) = f(u)$, and uniform priors. If we assume rational initial priors, the type I extreme value assumption allows us to simplify the gains from search equation to

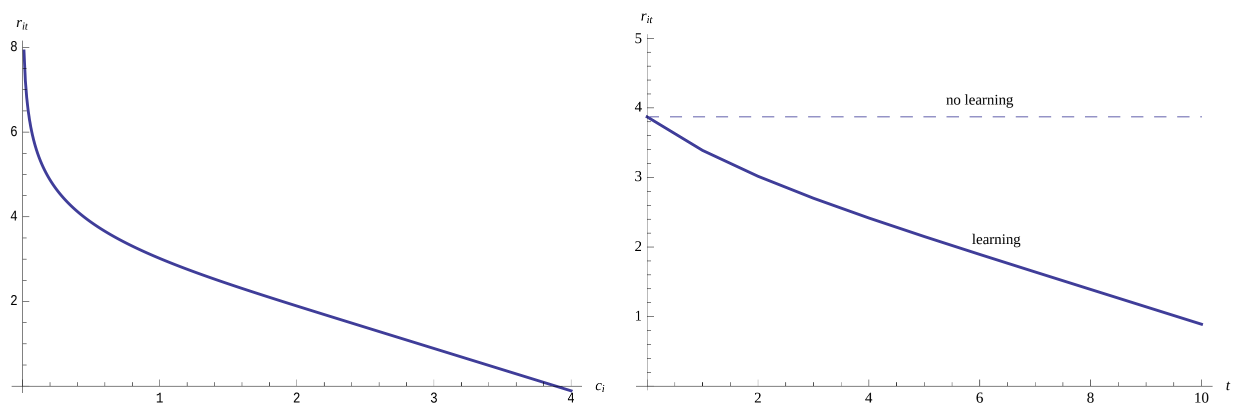
$$G(\hat{u}_{it}) = \frac{W}{W+t\cdot J}\sum_{k=1}^{K}\frac{1}{K}\left(\gamma+\log\left[\sum_{J}\exp\left(\delta_{jk}\right)\right]-\hat{u}_{it}+\int_{\sum_{J}\exp[\delta_{jk}-\hat{u}_{it}]}^{\infty}e^{-x}/x\ dx\right), \tag{4}$$

where γ is the Euler constant. In this expression, which is derived in the Appendix, the term between brackets can be broken down into three parts. The first part, given by $\gamma+\log\left[\sum_{J}\exp\left(\delta_{jk}\right)\right]$, is the expected maximum utility when taking J draws from $h(u)$. The second part consists of the reference utility $\hat{u}_{it}$, which needs to be subtracted since we are calculating the gains from search in comparison to this reference utility. The last part is the exponential integral function evaluated at $\sum_J \exp[\delta_{jk} - \hat{u}_{it}]$ and captures the option value of sticking to $\hat{u}_{it}$ in case the maximum utility out of J draws is less than $\hat{u}_{it}$.

Alternatively, if we assume consumers have uniform priors with support $[\underline{u}, \overline{u}]$, the gains from search equation is

$$G(\hat{u}_{it}) = \frac{W}{W + t \cdot J} \sum_{k=1}^{K} \frac{1}{K} \left(\frac{1}{J+1} \left(\underline{u} + \overline{u}J - (J+1)\hat{u}_{it} + (\hat{u}_{it} - \underline{u}) \left(\frac{\hat{u}_{it} - \underline{u}}{\overline{u} - \underline{u}} \right)^J \right) \right). \tag{5}$$

A consumer with search cost c_i will continue searching as long as the gains from an additional search more than offset the cost of searching once more, i.e., $G(\hat{u}_{it}) \geq c_i$. Once this is no longer the case, the consumer will stop and buy from the store selling at the highest utility observed so far. Let the reservation utility r_{it} be the utility at which the consumer is indifferent between searching and not searching, i.e., r_{it} solves $G(r_{it}) = c_i$. In general, reservation utilities are decreasing in search costs. Figure 1(a) shows this for the gains from search equation (4), assuming $W = 2$, $\delta_1 = 2$ and $\delta_2 = 3$.



(a) Reservation utility r_{it} for $t = 0$

(b) Resevation utility r_{it} for $c = 0.5$

Figure 1: Reservation Utility for $W = 2$, $\delta_1 = 2$, and $\delta_2 = 3$

Rosenfield and Shapiro (1981) have shown that in the related setting of a multinomial distribution with a Dirichlet prior, consumers' optimal search policy is myopic and can be characterized by a reservation utility that is non-increasing in the number of alternatives sampled. Figure 1(b) gives an example of such non-increasing reservation utilities for our model using the same parameter values as used in Figure 1(a). Also plotted is the reservation utility for the no-learning model (dashed line), using the same parameter values. Since the initial prior (i.e., the prior at $t = 0$) is the same in both models, the decreasing reservation utility for the learning model means consumers are more likely to accept a sampled alternative over time in the learning model than in the model without learning. The decreasing reservation utility is also the reason that consumers may recall in the learning model: a sampled alternative may not have passed the bar initially, but may do so after more alternatives have been sampled, resulting in the consumer recalling a previously sampled alternative. Since the reservation utility is constant over time in the no-learning model, consumers will not recall in that model. In Section 4 we show that there are nontrivial amounts of recall in the data we use in our application, which highlights the importance of modeling learning.

It is important to keep in mind that whether consumers will search more or less in the learning model than in the no-learning model depends on what is assumed on the initial priors. In the example in Figure 1(b) the initial priors are the same in both models, and as a result consumers will always be searching less in the learning model. However, it is easy to see that if consumer have very optimistic priors, consumers may in fact search more. In Section 3 we study these differences in more detail in a Monte Carlo study, including any biases that may arise when wrongfully assuming there is no updating, and while we confirm that search costs are overestimated in the learning model when consumers have rational priors, if consumers have overly optimistic priors, the bias may go in the opposite direction.

In the next subsection we show how to estimate consumers' search costs when we observe the sequence of retailers visited by a consumer prior to a transaction. More specifically, we explain how we can use this search history to obtain bounds on the consumer's search cost while simultaneously estimating the parameters of the utility function.

2.4 Estimation Strategy

A consumer will keep searching until the gains from search no longer exceed the cost of searching, which means bounds on the consumer's search cost can be obtained from observed search patterns. More specifically, depending on what is assumed on the priors, equations (4) and (5) give expressions for the gains from search as a function of the highest utility offer observed so far, $\hat{u}_{it}$. To obtain the lower bound of a consumer's search cost, consider a transaction as the outcome of the search process: buying a product corresponds to a decision not to continue searching, hence, search cost for the consumers exceeded the gains from search, i.e., $c_i > G(\hat{u}_{it})$. The lower bound on the search cost for such a consumer, denoted by $\underline{c}_i$, is therefore

$$\underline{c}_i = G(\hat{u}_{it}). \tag{6}$$

The upper bound for the search cost is not identified for those consumers that only sample one firm. For those consumers who sampled multiple firms, the search cost upper bound, denoted by $\overline{c}_i$, corresponds to the gains from search at the best offer observed before the last search. Since the consumer found it optimal to sample more than once, we know that the gains from search in the last period, $t-1$, were higher than her search costs, hence $c_i \leq G(\hat{u}_{it-1})$. This means the search cost upper bound for sampling more than once is given by

$$\overline{c}_i = G(\hat{u}_{it-1}). \tag{7}$$

For each consumer in our sample, we use equation (6) to obtain a lower bound on the consumer's search cost, conditional on utility parameters. In addition we can use equation (7) to obtain an upper bound for those consumers that searched more than once. Since in the product differentiation model the gains from search will depend on actual utility values that have been observed while sampling, we have to simulate consumers by drawing from the distribution of ε_{ijk} conditional on the parameters of the utility function in equation (3). This gives us specific utility draws for each alternative, allowing us to calculate the gains from search at the highest observed draw using equation (4) or equation (5), depending on what we assume on the distribution of the initial prior. The search cost bounds for a specific observation are then obtained by taking the average over the simulated bounds of the simulated consumers.

In order to derive the probability that a consumer's search cost is within the observed search cost bounds, we assume search costs follow a log-normal distribution, and relate a consumer's search cost c_i to both demographic-related covariates as well as a stochastic term in the following way:

$$\ln c_i = \beta X_i + \eta_i, \tag{8}$$

where X_i is a vector of consumer demographics and η_i is a standard normal distributed error term. The probability that consumer i's search cost is within the relevant search cost bounds is then given by

$$P\big(\underline{c}_i < c_i \leq \overline{c}_i\big) = \Phi\big(\ln \overline{c}_i - \beta X_i\big) - \Phi\big(\ln \underline{c}_i - \beta X_i\big). \tag{9}$$

where $\Phi(\cdot)$ is the standard normal CDF. Note that if a consumer searches once, the search cost upper bound is not identified and we set $\Phi\big(\ln \overline{c}_i - \beta X_i\big) = 1$.

In addition to conditions on the search cost bounds, the chosen product should also be the preferred product among the set of products sold by the seller. Denote $\mathcal{J}_k$ the set of products sold by retailer k. For products sold by the same retailer the stochastic utility term is i.i.d., which means the probability that the purchased product, denoted by subscript ℓ, has the highest utility of all products sold by the seller is given by the familiar multinomial logit probability, i.e.,

$$P\big(u_{i\ell} > u_{ij}\ \forall\ j \in \mathcal{J}_k \backslash \{\ell\}\big) = \frac{\exp[\delta_\ell]}{\sum_{j \in \mathcal{J}_k} \exp[\delta_j]}. \tag{10}$$

Taking both the stopping decision and the product choice decision into account, the total likelihood is

$$\prod_i P\big(\underline{c}_i < c_i \leq \bar{c}_i\big) \cdot P\big(u_{i\ell} > u_{ij}\ \forall\ j \in \mathcal{J}_k \backslash \{\ell\}\big). \tag{11}$$

We estimate the parameters of the search cost distribution and utility function using maximum likelihood, where the log-likelihood function is constructed by taking the log of equation (11).

A special case of the model is when there is no vertical or horizontal product differentiation, so the utility function simplifies to $u_{ijf} = u_j = -p_j$. In this case, since prices are observed by the econometrician, we can calculate the search cost bounds used in equation (9) directly.[1]

3 Monte Carlo Experiments

In this section we investigate the performance of our estimation procedure using a number of Monte Carlo experiments. Our first objective is to confirm that our estimation procedure is able to recover the unknown parameters of the search cost distribution and the utility function. Next, we study to what extent estimates will be biased if we do not take learning into account when estimating data generated by a learning model. Finally, we look at possible measurement error in the composition of choice sets, as well as measurement error in prices.

The setup of the experiments is as follows. We randomly generate 1,000 observations, where each observation corresponds to one household. An observation includes data on the search sequence prior to a transaction, as well as the product bought, its price, and the identity of the retailer. We simulate 5 different retailers, each selling 3 different products.

We assume consumers' search costs are drawn from a log-normal search cost distribution with the standard deviation of the associated normal distribution set to 1. We let the mean of the associated normal distribution depend on a constant as well as an indicator for whether the household has a broadband connection (randomly drawn from a Bernoulli distribution with $p = 0.3$) using parameter values as shown in the search cost panel of the first column of Table 1.

[1] Note that in this case the utility distribution will correspond to the empirical distribution of prices. Moreover, in the homogenous good case we only use the stopping decision in the likelihood function.

Household i's utility for product j sold at retailer k is given by equation (3). The stochastic term in the utility specification is randomly drawn from a standard type I extreme value distribution and each household has a different draw for each product-retailer combination. Prices are randomly drawn from a uniform distribution with product-specific parameters.[2] In the first column of Table 1 we provide the utility parameter values used for generating the data.

Table 1: Monte Carlo Simulations

	(1) True	(2) Learning		(3) No Learning		(4) Noise Choice Set		(5) Noise Prices	
Variable	Coeff.	Coeff.	Std. Dev.	Coeff.	Std. Dev.	Coeff.	Std. Dev.	Coeff.	Std. Dev.
Search Cost									
Constant	-1.000	-0.930	(0.053)	-0.281	(0.046)	-1.101	(0.050)	-0.940	(0.055)
Broadband	-0.500	-0.439	(0.085)	-0.282	(0.081)	-0.368	(0.085)	-0.426	(0.092)
Utility									
Firm 1	-2.000	-2.051	(0.188)	-1.739	(0.197)	-1.785	(0.187)	-2.031	(0.183)
Firm 2	-1.500	-1.551	(0.163)	-1.362	(0.164)	-1.341	(0.162)	-1.517	(0.194)
Firm 3	-1.000	-1.049	(0.153)	-0.953	(0.152)	-0.901	(0.156)	-1.006	(0.151)
Firm 4	-0.500	-0.523	(0.128)	-0.480	(0.130)	-0.444	(0.117)	-0.488	(0.133)
Product 2	-1.000	-0.991	(0.103)	-0.952	(0.126)	-0.974	(0.104)	-0.799	(0.106)
Product 3	1.000	1.015	(0.107)	0.987	(0.121)	0.999	(0.109)	0.809	(0.097)
Price	-2.000	-2.014	(0.191)	-1.935	(0.222)	-1.974	(0.191)	-1.452	(0.154)

Notes: Number of observations is 1,000. Weight on the initial prior $W = 15$.

Table 1 presents the mean of the parameter estimates across 100 replications, as well as the standard deviation for four different experiments. We assume that initial priors are rational and correspond to the true joint utility distribution, which means that consumers use equation (4) to calculate the gains from search. We set the weight on the initial prior equal to the total number of product-retailer combinations, so $W = 15$. Simulations of the main learning specification, given in the second column, show that the estimates are relatively close to the true parameter values and have low standard deviations. This can also be seen in Figure 2, which plots the actual search cost CDF (black dashed curve) as well as the estimated CDF (solid curve) and corresponding 90 percent confidence interval for the main specification.

The third column of Table 1 gives parameter estimates for the standard sequential search model, in which consumers they do not update their initial priors on the utility distribution. Although estimation of the price parameter and product dummies appear unaffected by this, the search cost parameters are biased upward and both parameters are no longer within two standard deviations of their true values. Figure 3 plots the true search cost CDF (black dashed curve) as well as the

[2] Prices for product 1 are uniform $U(100, 175)$, prices for product 2 are uniform $U(75, 125)$, and prices for product 3 are uniform $U(125, 225)$.

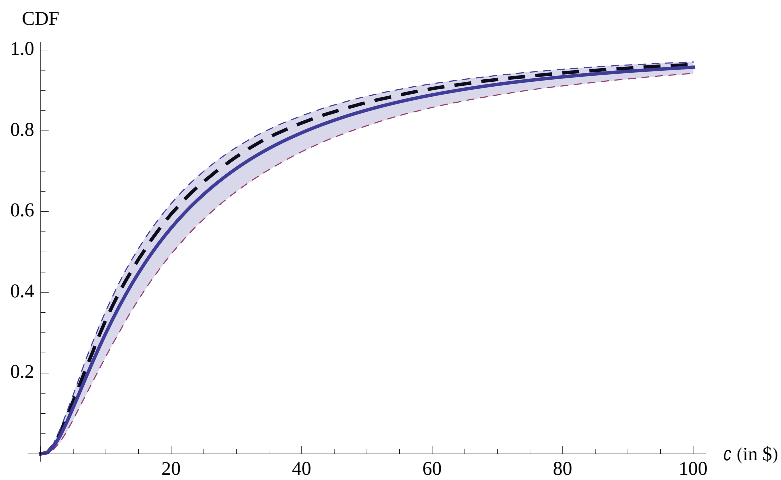


Figure 2: Estimated Search Costs Learning Specification

estimated CDF (solid curve) and corresponding 90 percent confidence interval for the no-learning model, and shows that search cost estimates will be severely biased when the data is generated from a learning model and learning is unaccounted for in the estimation. As we argued in Section 2.3, reservation utilities in the learning model are decreasing in the number of alternatives sampled. Since we are assuming a rational initial prior, the initial prior will be the same in both learning and no-learning models, which means consumers will be searching less in the learning model (which is used to generate the data) than they would in the no-learning model. This means that the no-learning model can only rationalize the observed search patterns in the Monte Carlo experiment by having higher search costs. Notice that Koulayev (2013) finds the direction of the bias to be similar for his learning model.

The bias in search cost estimates as a result of wrongfully estimating a model with no learning is also reflected in the elasticity estimates. Table 2 gives the true own-price elasticities for each retailer as well as the own-price elasticities based on the estimates from the no-learning model.[3] The table shows that elasticity estimates are biased towards zero for all retailers, and that the bias is most severe for the firms with the lowest market shares.

Notice that if the initial prior is not rational, the direction of the search cost distribution bias depends on the shape of the initial prior. Figure 4 gives the estimated search cost distribution if we assume the data is generated from a learning model in which initial priors are uniform with

[3] The elasticities are calculated by simulation. For the learning model, we simulate the percentage change in demand as a result of a 10 percent increase in price for 100,000 consumers, using the true modeling parameters. We use a similar approach for the no-learning model, using the estimated parameters in column (3) of Table 1.

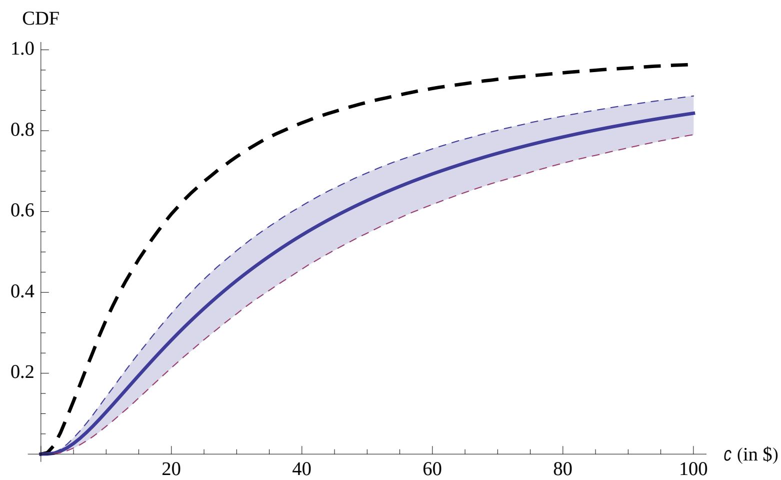


Figure 3: Estimated Search Costs No-Learning Specification

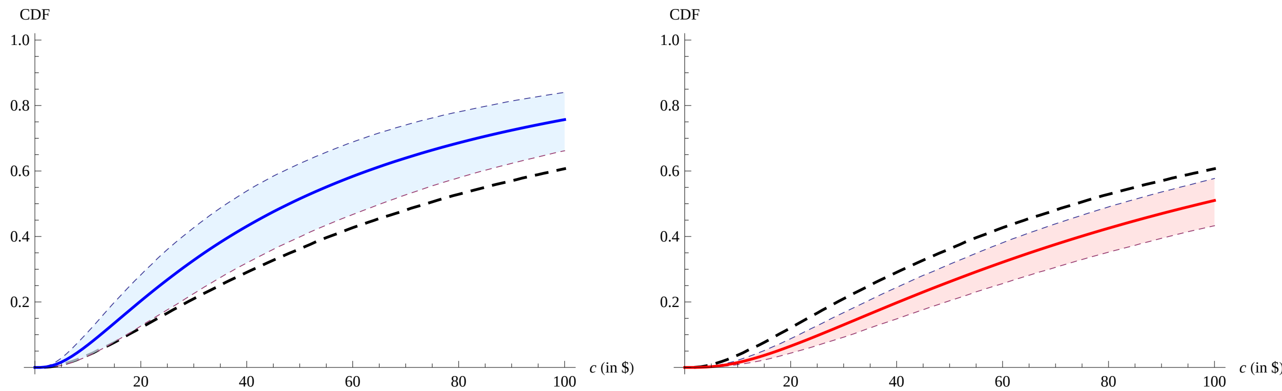
Table 2: Own-Price Elasticity Estimates

	Learning	No Learning
Firm 1	-1.735	-1.385
Firm 2	-1.631	-1.343
Firm 3	-1.474	-1.241
Firm 4	-1.271	-1.104
Firm 5	-1.022	-0.932

Notes: Firms are ordered by increasing market shares.

utility bounds $[1, 5]$ (blue curve in Figure 4(a)) or with utility bounds $[-2, 2]$ (red curve in Figure 4(b)), along with the 90 percent confidence intervals and the true search cost CDF (dashed black curve).[4] This initial prior distribution with bounds $[1, 5]$ puts more weight at higher utility values and is therefore more optimistic than the actual utility distribution. The gains from search are higher, and as a results consumers are searching on average three times as much in the simulated data as they would when having rational priors. The non-learning model can only rationalize this surge in search activity by having lower search costs, explaining the direction of the bias in Figure 4(a). The initial prior distribution with bounds $[-2, 2]$ is closer to the true utility distribution, and hence the bias is going in the other direction.

[4]The parameters of the search cost distribution are zero for the constant and -1 for the broadband dummy. The price coefficient is set to -1, whereas all other utility parameters are zero.



(a) Uniform with bounds $[1, 5]$

(b) Uniform with bounds $[2, 2]$

Figure 4: Estimated Search Costs No Learning (Uniform Priors)

Robustness of Estimates

In our application as well as in most online settings, visits to retailers are observed at the domain level, which makes it difficult to infer as to whether a particular product was sampled by the consumer during her visit. The common strategy is to treat prior visits to retailers—during a set search window—as being related to a subsequent purchase. This means we may wrongfully attribute a visit to a retailer as a consumer sampling a particular product sold by this retailer. On the other hand, consumers may get some price and product information from non-retailer websites such as price comparison sites, which means we may fail to account for some retailers in consumers' choice sets. To see to what extent our estimates are affected by the potential error that could arise as a result of these assumptions, we add noise to the sets of searched retailers in the data used for estimating the learning model. We let 50 percent of the observations be affected by this noise—to half of these observations we add a retailer at a random position in the search sequence, while for the other half we take out a randomly selected retailer from the sequence.[5] The estimation results are shown in column (4) of Table 1. Although the mean parameter estimates are somewhat affected by the noise, standard deviations do not change that much in comparison to those in column (2) and all parameter estimates are within two standard deviations of the true parameter values.

Another potential concern is that in the actual application prices may be measured with error. Prices are typically obtained from transactions, and hence we infer prices at the other retailers from transaction prices of other consumers. Since prices may change over time, this means there is potential for measurement error in prices. To study how this will affect the estimation of the

[5]We only delete a randomly selected retailer if the consumer has searched more than one firm and if this selected retailer is not the seller.

learning model, we add noise to the simulated prices by allowing for a multiplicative noise term that is drawn from a log-normal distribution with an associated mean-zero normal distribution with 0.1 standard deviation.[6] The estimation results shown in column (5) of Table 1 indicate that the noise does not affect the estimation of search cost parameters. However, the price noise does affect some of the utility parameters: even though the firm dummies appear unaffected, there is a bias in the estimation of the product dummies and the price coefficient. The direction of the bias is as expected: the effect of prices on search and purchase decisions is less pronounced when prices are measured with error, which leads to a price coefficient that is biased towards zero.

Estimation of the Weight on the Initial Prior

So far we have been taking the weight consumers put on their initial priors as fixed to the number of product-retailer combinations, using $W = 15$. Although this allows us to identify the parameters of both the search cost distribution and the utility function, an important question is to what extent consumers are behaving as if they know the distribution of utilities. Intuitively, the importance of the initial prior may be identified from recall patterns in the search data: as discussed in Section 2.3, recall is a result of a declining reservation utility, so the rate of "decline" should reflect how the prior departs from the sampled utiity distribution.

To investigate to what extent the weight initial prior can be estimated in practice, we parameterize the weight on the initial prior as $W = \omega JK$, where JK corresponds to the number of product-retailer combinations and ω is a parameter to be estimated. Inspection of the likelihood function reveals that even though this parameter should be identified due the parametric form of the gains from search equations, identification is likely to be weak.[7]

Due to these concerns we shift focus to a homogenous product version of the learning model. In this special case of the more general product differentiation model the utility function simplifies to $u_j = -p_j$. In the homogenous product model there is no stochastic utility term, which means we can no longer use equation (4) to calculate the gains from search. Instead, we use equation (5), which means we assume consumers' initial prior is uniform, with the lower and upper bound equal to respectively the lowest and highest price in the market. Since prices are observed, this means we can directly obtain bounds on the consumer's search costs using equations (6) and (7).[8]

Table 3 gives results for two Monte Carlo simulations with this constrained model: in column

[6] The noise term has a mean of 1.005 and a variance of 0.010, which implies that 95 percent of draws is between 0.8 and 1.2.

[7] More specifically, it will be difficult to separately identify ω from the search cost constant. To see this, when taking logs of equation (2) ω appears twice: by itself and as $\omega JK + t$. Notice that only the second term will allow us

Table 3: Monte Carlo Simulations of the Homogenous Product Model

	(1)	(1)		(2)	
	True	W fixed		W estimated	
Variable	Coeff.	Coeff.	Std. Dev.	Coeff.	Std. Dev.
Search Cost					
Constant	-2.000	-1.992	(0.059)	-2.022	(0.077)
Broadband	-1.000	-0.997	(0.105)	-1.004	(0.108)
Weight on prior	1.000	1.000		0.912	(0.175)
Price	-1.000	-1.000		-1.000	

Notes: Number of observations is 1,000.

(1) W is set to the total number of product-retailer combination, i.e., $W = JK$, while in column (2) we parameterize W as $W = \omega JK$ and estimate the parameter ω, The search cost parameters are very similar across the two specifications. Moreover, the estimated weight on the prior in column (2) is close to the true value, suggesting that the weight on the prior can be recovered in at least the homogenous goods version of the search and learning model.

4 Application

4.1 Data

The dataset was constructed from the comScore Web-Behavior Panel which includes detailed online browsing and transaction data from 91,689 users in 2007. The users in the sample are randomly chosen from a universe of 1.5 million global users. ComScore is a leading provider of information on consumers' online behavior and supplies Fortune 500 companies and large news organizations with market research on e-commerce sales trends, website traffic, and online advertising campaigns. Each user's online activity is channeled through comScore proxy servers that record all Internet traffic, including information on visits to a website or domain (browsing), as well as secure online transactions.[9] The data include date, time, and duration of visits, as well as price, quantity, and description of each product purchased during a session.

The dataset contains 731 transactions that constitute sales in 2007 of 10 MP3 players. Table 4 summarizes the transactions of the products in the sample. The iPod Nano 4Gb is with 192 transactions the most popular product. Prices range from a $28 Sandisk Sansa Shaker MP3 player

to identify ω (the first term will be collinear with the search cost constant).

[8]For estimation of the homogenous product model we do not take into account that the product bought should have higher utility than all other products sold by the same retailer, so the likelihood function is $\prod_i P\big(\underline{c}_i < c_i \leq \overline{c}_i\big)$.

[9]The servers only capture web-browsing on one computer per household, hence it might not capture browsing and transactions at work.

to up to \$349 for the iPod 80Gb, which is the most expensive item in our sample. The concentration ratio of the largest retailer in terms of sales (CR1) ranges from 33 to 74 percent of the market.

Table 4: Transaction Characteristics by Product

Product	Obs.	Number of firms	CR1	Transaction price Mean	Std. Dev.	Min	Max
iPod Nano 4Gb	192	9	0.71	170.05	25.43	129.00	249.99
iPod Shuffle 1Gb	147	10	0.60	78.28	2.69	58.95	129.00
iPod Nano 8Gb	110	9	0.74	203.08	18.06	179.94	249.00
iPod 80Gb	65	8	0.68	282.23	48.66	229.00	349.99
iPod 30Gb	60	6	0.65	242.74	13.62	184.99	299.99
iPod Nano 2Gb	47	5	0.45	141.98	13.78	99.99	199.99
Zune 30Gb	46	8	0.42	174.94	52.80	89.99	250.00
iPod Touch 8Gb	43	5	0.73	296.64	5.93	276.99	299.99
Sandisk Sansa Shaker MP3	13	4	0.37	30.87	3.47	28.28	39.99
Sandisk Sansa E250 MP3	8	5	0.33	73.83	23.78	38.99	139.98
Total	731	8	0.74	174.93	73.94	137.68	233.29

In order to identify a user's visit to a website as search behavior related to a particular transaction, we link the browsing history up to seven days before a transaction. Whereas there is no evidence to guide the definition of a search time span in relation to a transaction, one week is long enough to capture all search behavior related to a transaction; any longer intervals are likely to also capture unrelated website visits. A search history could be less than seven days if another transaction has occurred within seven days. The browsing activity of users consists of 9,742 visits to the retailers in the sample. Although some user search activity may not be linked to the transaction that immediately follows, but to a subsequent one, there is no clear way to link this intervening search to a later transaction. Another limitation of the comScore data is that although we observe consumer visits to different stores, we only observe the price of the transaction. To recover missing prices for all visited retailers we use the most recent transaction prices at those retailers with missing values. The average difference between a transaction and the transaction from which the price was imputed is 16 days. For a popular product like the iPod Nano 4Gb, the average difference is as low as less than one day, for a less popular product like the Sandisk Sansa E250 the average difference is 31 days.

Table 5 presents descriptive statistics of the search patterns for a particular transaction per product. Overall, consumers visit on average 2.82 firms before buying an MP3 player.[10] Price dispersion of the searched firms is small for most products, ranging from 1 to 11 percent. The iPod

[10] De los Santos, Hortaçsu, and Wildenbeest (2012) report that there was only limited use of price comparison sites for 2004 ComScore (book price) data.

80Gb is the product with the most dispersed prices, as measured by the coefficient of variation. The average price difference between the transaction price and the lowest price across all transactions and stores is $37.25. Most consumers will not visit all stores and encounter the full range of prices. The difference between the transaction price and the lowest price of the stores visited during the customer's search is on average $30.05.

Table 5: Descriptive Characteristics of Retailers Searched by Product

Product	Mean number of firms searched	Prices of searched firms		
		Mean	Std. Dev.	CV
iPod Nano 4Gb	2.79	161.81	11.29	0.07
iPod Shuffle 1Gb	2.62	75.86	0.62	0.01
iPod Nano 8Gb	2.88	194.29	3.36	0.02
iPod 80Gb	2.91	262.30	28.21	0.11
iPod 30Gb	2.60	244.78	2.68	0.01
iPod Nano 2Gb	2.47	134.96	6.49	0.05
Zune 30Gb	3.46	168.25	13.84	0.08
iPod Touch 8Gb	3.23	294.82	3.39	0.01
Sandisk Sansa Shaker MP3	4.15	31.60	0.49	0.02
Sandisk Sansa E250 MP3	1.75	86.57	3.18	0.04
Total	2.82	168.52	68.13	0.40

As shown in Table 6, when counting all electronic store visits in the seven days prior to a purchase as searches, close to fifty percent of consumers recalled a previously visited firm conditional on having searched at least twice. We have conditioned our recall statistic on "awareness," by not including recalls for consumers that visited all stores they are aware of (as measured by having previously visited the store)—as noted by De los Santos, Hortaçsu, and Wildenbeest (2012), the standard sequential search model predicts a consumer always buys from the last store she visited, unless she has visited all stores she is aware of. Since almost seventy percent of consumers searched at least twice, this means that in almost a third of transactions consumers recalled a previously visited firm. Although some of these recalls could be an artifact of the relatively large search window of seven days, as shown in Table 6, most recalls remain if the search window is made smaller. As argued in the Section 2, these non-trivial amounts of recall violate optimal behavior in the standard sequential search model, but can be rationalized by a learning model and provide an additional motivation for estimating a learning model.

Notice that recall may also be rationalized by a non-sequential search model. De los Santos, Hortaçsu, and Wildenbeest (2012) look at several testable predictions of the sequential search model versus the non-sequential search model and argue that a robust prediction of sequential search (with and without learning) is that the decision to continue searching depends upon the

Table 6: Consumer Recall Patterns by Search Window

Product	Two or more stores visited (as percentage of total)			Recalled (as percentage of those having visited two or more stores)		
	One-day	Three-day	Seven-day	One-day	Three-day	Seven-day
iPod Nano 4Gb	42.7	56.3	69.8	35.4	38.9	44.8
iPod Shuffle 1Gb	42.2	51.0	61.2	14.5	34.7	42.2
iPod Nano 8Gb	49.1	60.0	74.5	24.1	48.5	46.3
iPod 80Gb	43.1	55.4	70.8	39.3	44.4	45.7
iPod 30Gb	50.0	56.7	63.3	33.3	35.3	50.0
iPod Nano 2Gb	40.4	53.2	70.2	10.5	20.0	24.2
Zune 30Gb	56.5	63.0	76.1	42.3	48.3	51.4
iPod Touch 8Gb	39.5	60.5	72.1	64.7	57.7	74.2
Sandisk Sansa Shaker MP3	61.5	84.6	84.6	37.5	54.5	54.5
Sandisk Sansa E250 MP3	12.5	25.0	25.0	100.0	100.0	100.0
Total	44.7	56.4	68.7	30.6	41.3	46.4

outcome of the previous search, while it does not with non-sequential search. Although this is relatively straightforward to test for in a homogenous product setting, testing between our learning model and a non-sequential search model is more complicated in a differentiated product setting, especially since we allow for multi-product search.[11]

Firms and Consumers

Table 7 presents market shares, the number of consumer search visits, and general information for the main websites in the sample. The products included in the sample were bought from 18 different retailers. Given that there are few transactions for MP3 players for several retailers we show the largest retailers in terms of MP3 players and group smaller retailers into two categories.[12] The market for these goods is less concentrated than other online markets. Given the inclusion of popular Apple products in the sample, Apple has the largest number of transactions, followed by Amazon. The presence of other large retailers in our sample, like for instance Walmart, BestBuy, and Circuit City, means Amazon is less dominant in the market for electronics than it is for books: even if we exclude Apple products from the sample, Amazon's market share for the remaining products is still significantly lower than Amazon's share of the book market (which is 66 percent according to De los Santos, Hortaçsu, and Wildenbeest, 2012).

Table 8 presents household characteristics in the sample. The overall comScore sample is rep-

[11] De los Santos, Hortaçsu, and Wildenbeest (2012) develop a test for sequential search versus non-sequential search that accounts for differentiation across stores. In our learning model consumers are searching for a right product match in addition, which means their test cannot be directly applied.

[12] Other electronic stores include bhphotovideo.com, compusa.com, dell.com, macmall.com, newegg.com, and tigerdirect.com. Other Retailers include buy.com, costco.com, officedepot.com, sears.com, and staples.com

Table 7: Firms' Market Shares and Shares of Search Visits

Firms	Market Share %	Search visits %
apple.com	59.10	13.56
amazon.com	16.69	19.91
circuitcity.com	9.71	9.80
target.com	3.15	9.10
bestbuy.com	2.33	9.96
walmart.com	2.33	12.55
overstock.com	1.50	4.54
Other Electronic Stores	3.69	9.77
Other Retailers	1.50	10.80
Observations	731	9,742

resentative of online buyers in the U.S. in terms of age, education, income, household composition, and other observable characteristics.[13] As this data is composed of electronics' consumers, the sample has a larger fraction of consumers with high income. Consumers with income above $75,000 represent 41 percent of the sample compared to 14 percent of the U.S. population that has bought a product online.

4.2 Results

We estimate the model using transactions for MP3 players as well as the corresponding search history before a transaction. As a benchmark case, we first assume there is no differentiation at the product level as well as at the retailer level, i.e., $u_j = -p_j$, and assume consumers have uninformative initial priors. Since prices are observed, we can directly obtain bounds on the consumer's search costs using the gains from search equation (5). We follow Koulayev (2013) in setting the weight on the initial prior W equal to the total number of product-retailer combinations, which is in our application is 62, although we will relax this assumption momentarily. This means retailers do not necessarily sell all products—as shown in Table 4 only the iPod Shuffle is sold by all retailers.[14] To make prices comparable across products we de-mean prices by subtracting average

[13]De los Santos (2010) shows that the comScore sample is representative of the U.S. by comparing it with users that have bought a product online from the sample with the Internet and Computer Use Supplement of the Current Population Survey (CPS) and the Forrester Technographics Survey. The main differences between the CPS and ComScore samples are that in the ComScore sample Internet users are older, have higher income, and are more likely to have some college but no degree. The racial composition is similar across samples—online users are predominantly white. However, compared with CPS, comScore over samples Hispanics and Forrester over samples whites. The geographic distribution of users is similar to CPS population estimates at the regional and state levels.

[14]In the application we use the actual number of products sold for a given retailer to calculate the gains from search in equations (4) and (5).

Table 8: Descriptive Statistics of Consumer Characteristics

	Mean	Std. Dev.
Broadband connection	0.95	0.21
Household size	3.31	1.35
Children present	0.75	0.43
Age		
18-20	0.00	0.06
21-24	0.02	0.15
25-29	0.04	0.20
30-34	0.09	0.29
35-39	0.13	0.34
40-44	0.18	0.39
45-49	0.18	0.39
50-54	0.14	0.35
55-59	0.07	0.25
60-64	0.06	0.23
65 and over	0.08	0.28
Household income		
Less than $15,000	0.09	0.29
$15,000 - $25,000	0.05	0.22
$25,000 - $35,000	0.08	0.28
$35,000 - $50,000	0.13	0.33
$50,000 - $75,000	0.23	0.42
$75,000 - $100,000	0.16	0.37
More than $100,000	0.25	0.44
Race		
White	0.94	0.24
Black	0.05	0.21
Hispanic	0.01	0.10
Other	0.00	0.04

Notes: Number of consumers is 637.

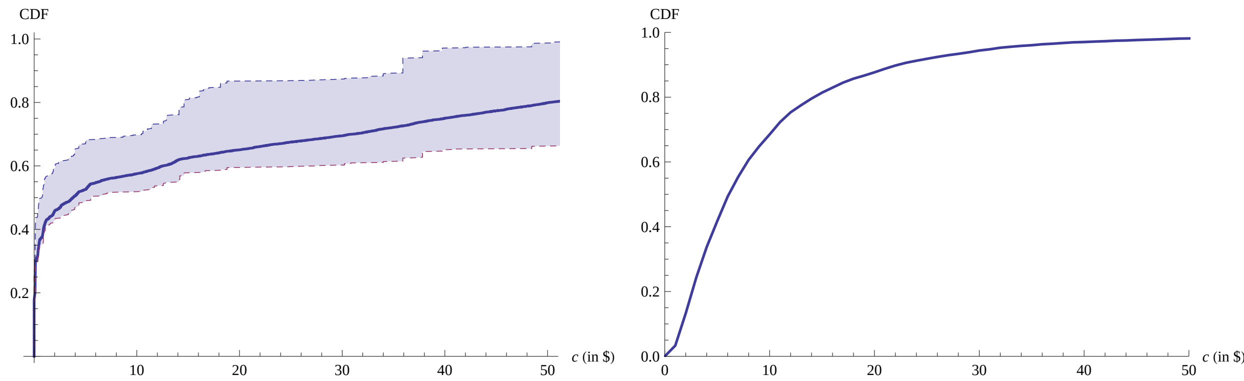
product prices.

The search cost bounds give us an idea about the range of search costs for the individuals in our sample. The dashed curves in Figure 5(a) represent the empirical CDF of our estimate of the search cost lower bound $\underline{c}_i$ and the upper bound $\overline{c}_i$.[15] The solid line is the empirical CDF of ten (uniform) draws from the search cost range for each observation and can be interpreted as an average search cost CDF. The median of this distribution is $3.86, whereas the medians of the lower and upper bound are $0.75 and $5.56, respectively.

Next, we estimate how search costs relate to household demographics using the maximum likelihood procedure described above. The parameters of the search cost specification in equation (8) are chosen in such a way that the probability that a consumer's search cost is within corresponding search cost bounds is maximized.[16] Figure 5(b) gives the estimated search cost CDF for a

[15] Since we can only calculate an upper bound if the consumer searches more than once, we set the upper bound of those searching once to the maximum upper bound of consumers searching at least twice ($90.74).

[16] Here we use the same likelihood function as in the Monte Carlo experiments for the homogenous good case, i.e., we do not factor in the probability that the product bought has the highest utility among all products sold by the same retailer. See also Footnote 8.



(a) Empirical CDF Lower and Upper Bound (b) Estimed Search Costs

Figure 5: Search Costs Homogeneous Model

specification that lets search cost be a function of a constant, household size, and indicators for 60 years and older, a household income of at least \$75,000, and a broadband connection.[17] We also let the weight on the initial prior be a function of the number of product-retailer combinations, i.e., $W = \omega JK$, where $JK = 62$ and ω is a parameter to be estimated. The estimated parameter for weight on the initial prior is 0.071 (standard error 0.017), which means W is estimated to be slightly over 4. Median search costs are \$6.11.

The homogenous product model assumes price is the only factor that is important when buying an MP3 player. Although this may be true for some buyers, this is unlikely to be the case in general. In fact, the average difference between the transaction price and the lowest price among the retailers visited is slightly over \$30, which is difficult to explain in the homogenous product model. This also suggests other attributes than price affect consumer choices, such as product attributes and retailer characteristics (ease of payment, reliability, speed of shipment, etc.). The product differentiation model discussed in Section 2.2 can accommodate most of these factors, and we will focus on this model for the remainder of this section.

Table 9 presents the results for various specifications of the product differentiation model. Due to the concerns about identification we discussed in the Monte Carlo section, we set the weight on the initial prior equal to the total number of product-retailer combinations, which means $W = 62$. Unobserved product and retailer characteristics may be correlated with prices, which, if not corrected for, may lead to biased estimates. We deal with this in two ways; firstly, all

[17] The estimated search cost parameters (standard errors within parenthesis) are:

$$\ln c_i = \underset{(0.288)}{-3.091} + \underset{(0.130)}{0.045} \cdot \text{Age } 60^{+} - \underset{(0.094)}{0.034} \cdot \text{Income} > 75\text{k} + \underset{(0.210)}{0.415} \cdot \text{Broadband} - \underset{(0.034)}{0.027} \cdot \text{Household Size} + \eta_i.$$

Table 9: Estimates Differentiated Product Model

	(1) No Control Function		(2) Control Function		(3) SNP	
Variable	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.
Search Cost						
Constant	-1.039	(0.176)***	-1.060	(0.196)***	-0.970	(0.162)***
Broadband	-0.204	(0.159)	-0.202	(0.174)	-0.229	(0.132)*
Age 60+	-0.061	(0.107)	-0.059	(0.103)	-0.060	(0.098)
Income >75k	-0.096	(0.065)	-0.077	(0.071)	-0.066	(0.063)
Household size	-0.017	(0.028)	-0.018	(0.029)	-0.020	(0.024)
Retailer Fixed Effects						
Apple	1.136	(0.199)***	1.271	(0.179)***	1.260	(0.129)***
Circuit City	-0.200	(0.242)	-0.109	(0.205)	-0.088	(0.194)
Overstock	-1.036	(0.585)**	-1.328	(0.572)**	-1.326	(0.431)***
Target	-1.051	(0.322)***	-0.977	(0.265)***	-1.030	(0.235)***
Walmart	-1.051	(0.431)**	-0.810	(0.344)**	-0.769	(0.280)***
Other electronics	0.035	(0.346)	-0.251	(0.349)	-0.250	(0.298)
Other general merchandise	-0.914	(0.565)	-1.168	(0.610)*	-1.128	(0.465)**
Product Attributes						
Zune	0.336	(0.027)**	0.257	(0.162)	0.243	(0.170)
Sansa	-1.482	(0.000)***	-1.973	(0.213)***	-1.960	(0.250)***
Storage/weight	0.019	(0.137)***	0.077	(0.018)***	0.075	(0.018)***
Price	-0.454	(0.065)***	-1.065	(0.110)***	-1.043	(0.105)***
Control Function						
Unobserved attributes			1.005	(0.168)***	1.009	(0.160)***
Log-likelihood	-2,638.22		-2,613.31		-2,538.98	

Notes: *** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level. The number of observations is 731. The number of simulated consumers is 50 per observation. Bootstrapped standard errors within parentheses.

our specifications include retailer and brand fixed effects, which will accommodate most of the correlation between prices and unobserved characteristics that is retailer and brand specific; and secondly, in columns (2) and (3) we use a control function approach (Petrin and Train, 2010) to correct for any remaining endogeneity bias. The approach consists of two stages. In the first stage we regress prices on all exogenous variables as well as instruments.[18] In a second stage we estimate our model adding the residuals of the first-stage regression as an additional control. A comparison of the log-likelihood values of the specification in column (1), which does not use the control function, and column (2) indicates that accounting for price endogeneity using the control function approach results in a slightly better fit. Most parameter estimates increase in magnitude, including the price coefficient, and the parameter that reflects unobserved heterogeneity is significantly different from zero. The Sansa brand dummy is negative and highly significant, whereas the Zune dummy is not

[18] We use two instruments for the control function, which are inspired by the instruments used in Berry, Levinsohn, and Pakes (1995). As each observation reflects the sale of a particular MP3 player at a particular retailer, one instrument is the number of retailers that sell the MP3 player and the second is the total number of other MP3 players sold at all other retailers.

significantly different from zero. Storage per weight has a positive marginal utility. All estimated search cost parameters have a negative sign, although only the constant is significantly different from zero at the 1 percent level. Figure 6 plots the estimated search cost CDF using the estimates in column (2) of Table 9. Search costs are sizable: median search costs are \$24.36 and 25 percent of households have search costs that exceed \$48. Although these figures seem relatively large, by assumption consumers sample all MP3 players sold by a retailer (on average 7 different products) during one search, which means that search costs per sampled product are substantially lower.

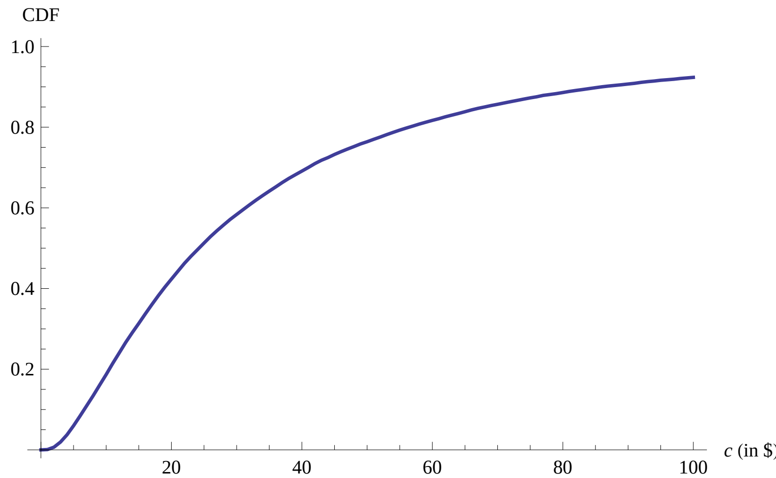


Figure 6: Estimated Search Cost Distribution

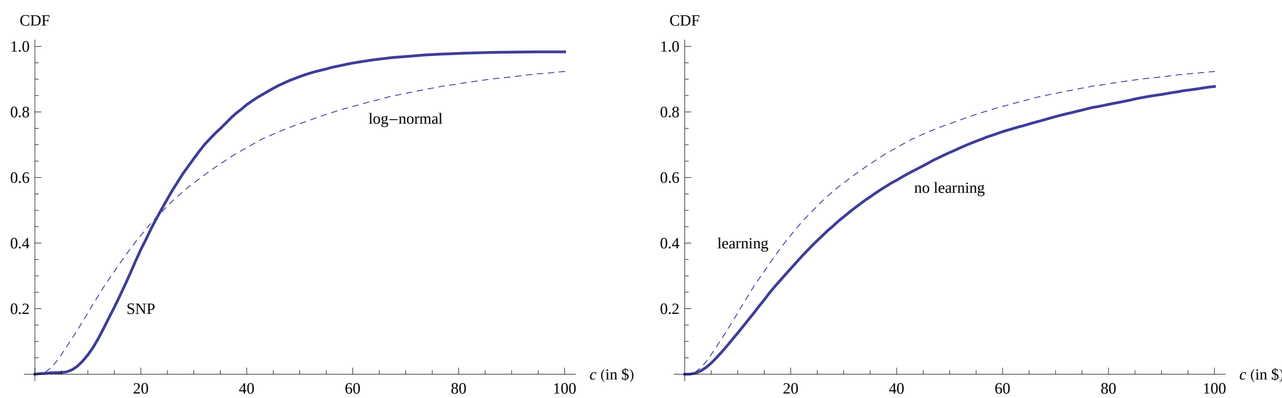
We use a log-normal distribution as the parametric form of the search cost distribution. To see how sensitive our estimates are to this assumption, we have also estimated the model using a semi-nonparametric (SNP) approach. In this approach the density is based on flexible Hermite polynomial functions that approximate arbitrarily closely a large class of sufficiently smooth density functions (Gallant and Nyckha, 1987). In most applications a normal density is used as a base function. In our context, where search costs are positive, a log-normal density is a more natural application. We use the following parametric form for the search cost density:

$$g(c_i;\beta X_i,\theta) = \frac{\left[\sum_{n=0}^{N} \theta_n w_n(c_i)\right]^2}{\sum_{n=0}^{N} \theta_n^2}, \ \theta \in \Theta_N \tag{12}$$

where $\Theta_N = \{\theta : \theta = (\theta_0, \theta_1, \ldots, \theta_N), \theta_0 = 1\}$, N is the number of polynomial terms and

$$\begin{aligned} w_0(c_i) &= (c\sigma\sqrt{2\pi})^{-1/2} \exp[-(\log c_i - \beta X_i)^2/4], \\ w_1(c_i) &= (c\sigma\sqrt{2\pi})^{-1/2} (\log c_i - \beta X_i) \exp[-(\log c_i - \beta X_i)^2/4], \\ w_n(c_i) &= \left[(\log c_i - \beta X_i) w_{n-1}(c_i) - \sqrt{n-1}\, w_{n-2}(c_i)\right] / \sqrt{n}, \text{ for } n \geq 2. \end{aligned}$$

This univariate SNP estimator is equivalent to that in Moraga-González, Sándor, and Wildenbeest (2013), except that we constrain the standard deviation parameter of the associated normal distribution to be 1. The SNP approach increases the vector of search cost parameters to be estimated by maximum likelihood to $\{\beta, \theta_1, \ldots, \theta_N\}$. The number of SNP parameters N can be made arbitrarily large as the number of observations increases to infinity; in our application we follow recommendations by Fenton and Gallant (1996) and set $N = 4$, which equals the closest integer to the fifth root of the total number of observations. A comparison of log-likelihood values between columns (2) and (3) suggests the fit has improved substantially by allowing for the more flexible SNP estimator, although most parameter estimates appear unaffected. However, the estimated search cost distribution may still be different since it now also depends on the additional SNP parameters we have estimated.[19] Indeed, a comparison of the two estimated search cost distributions in Figure 7(a) shows that search costs are higher at especially the lower end of the distribution for the SNP specification.



(a) SNP (b) No learning

Figure 7: Estimated Search Costs SNP and No-Learning Model

[19]The parameter estimates are (bootstrapped standard errors within parentheses) $\theta_1 = -0.117$ (0.018), $\theta_2 = -0.507$ (0.005), $\theta_3 = 0.090$ (0.013), $\theta_4 = 0.302$ (0.008).

Alternative Specifications

Table 10 gives parameter estimates for various alternative specifications. Our data only allows to observe visits to retailers at the domain level—for all estimates so far we have linked visits to a retailer for up to seven days prior to a MP3-player transaction as being related to the purchase of the MP3-player. To see how sensitive our estimates are to assumptions about the length of this window, we have also estimated the model assuming only visits to a retailer in the three days prior to a purchase are related. The results assuming this different window are shown in column (1) of Table 10. Except for the search cost constant, which is higher for the three-day window, all parameter estimates are very similar to the results when assuming a seven-day window, as reported in column (2) of Table 9. The difference in search cost estimates is intuitive: a lower number of retailers is visited when assuming a smaller window, which can only be rationalized by having higher search cost estimates.

Table 10: Alternative Specifications

	(1) Three-Day Window		(2) Uninformative Prior		(3) $W = 31$		(4) No Learning	
Variable	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.
Search Cost								
Constant	-0.701	(0.202)***	-0.796	(0.345)**	-1.266	(0.185)***	-0.742	(0.133)***
Broadband	-0.221	(0.173)	-0.338	(0.303)	-0.217	(0.177)	-0.210	(0.115)*
Age 60+	-0.194	(0.122)	-0.126	(0.256)	-0.062	(0.118)	-0.031	(0.103)
Income >75k	-0.083	(0.094)	-0.178	(0.188)	-0.095	(0.078)	-0.049	(0.068)
Household size	-0.027	(0.032)	-0.008	(0.074)	-0.017	(0.032)	-0.019	(0.025)
Retailer Fixed Effects								
Apple	1.478	(0.170)***	1.148	(0.120)***	1.454	(0.177)***	0.875	(0.127)***
Circuit City	-0.056	(0.214)	-0.404	(0.182)**	-0.105	(0.248)	-0.219	(0.151)
Overstock	-1.133	(0.521)**	-1.580	(0.259)***	-1.419	(0.428)***	-1.055	(0.261)***
Target	-0.870	(0.301)***	-0.950	(0.201)***	-1.251	(0.369)***	-0.788	(0.170)***
Walmart	-1.745	(0.557)***	-0.638	(0.232)***	-1.160	(0.417)***	-0.499	(0.186)***
Other electronics	-0.059	(0.388)	-0.556	(0.220)**	-0.282	(0.441)	-0.196	(0.256)
Other general merchandise	-1.439	(5.211)	-1.323	(0.331)***	-1.441	(0.862)*	-0.626	(0.201)***
Product Attributes								
Zune	0.252	(0.176)	0.330	(0.134)**	0.266	(0.163)	0.252	(0.166)
Sansa	-1.948	(0.246)***	-1.805	(0.241)***	-1.967	(0.261)***	-1.710	(0.255)***
Storage/weight	0.079	(0.018)***	0.086	(0.019)***	0.079	(0.015)***	0.085	(0.019)***
Price	-1.063	(0.118)***	-1.109	(0.135)***	-1.055	(0.117)***	-1.130	(0.140)***
Control Function								
Unobserved attributes	1.023	(0.165)***	1.091	(0.203)***	1.035	(0.173)***	1.054	(0.212)***
Log-likelihood	-2,392.52		-2,565.68		-2,566.44		-2,736.33	

Notes: *** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level. The number of observations is 731. The number of simulated consumers is 50 per observation. Bootstrapped standard errors within parentheses. All specifications have retailer specific dummies.

The second specification in Table 10 we use a uniform distribution instead of type I extreme value distribution as the initial prior. Here we assume the lower and upper bound of the uniform are known to the consumer and correspond to the lowest and highest utility in the market. Estimated search costs are higher than when assuming consumers have rational initial priors. Since a uniform distribution puts equal weight to all utility values in excess of the highest utility observed so far, the gains from search are uniformly higher, which means search costs need to be higher in order to rationalize observed search patterns.

In the third specification in Table 10 we set $W = 31$, which means the weight on the initial prior is half of what we assume in our main specification. The estimated search cost constant is slightly lower; the rest of the estimated parameters do not change much in comparison to those reported in column (2) of Table 9. Although not reported, estimates assuming $W = 93$ are very similar to the main specification as well.

In our final specification in Table 10, we fit a standard sequential search model to the data by assuming there is no Bayesian updating. This is equivalent to setting $t = 0$ in equation (2) and means consumers know the utility distribution with certainty. As shown in the last column of Table 10 the model does worse in terms of fitting the data. Part of this is explained by the model being unable to account for recall patterns. The estimates are also quite different: according to the learning model median search costs are \$24.36, while in the model with no updating median search costs are with \$31.73 more than 30 percent higher. The difference in search cost estimates can also be seen in Figure 7(b), which plots the search cost CDFs for both specifications.

5 Conclusions

In this paper we have presented a methodology to estimate a consumer search model with learning. The distribution of utilities is assumed to be unknown to consumers and is learned in the search process by Bayesian updating Dirichlet process priors. We have shown how to use information on the sequence of searches as well as prices to derive expressions for the bounds on a consumer's search costs. We relate search costs to the characteristics of households in our sample and find search costs to be sizable: our estimates indicate that search costs are on average slightly over $24. Estimated search costs are uniformly lower than in a sequential search model with a known distribution of utilities. Moreover, the learning model gives a better fit to the data than the model in which there is no updating.

The Monte Carlo experiments have shown that in the case of homogenous products, our estimation method allows us to recover how much weight consumers put on their initial priors. Nevertheless, it is difficult to say anything definitive on the identification of the prior in the more realistic product differentiation model. We have shown that to estimate our model, the number of searches as well as information from the last search is needed: the search cost bounds in our model are derived from the gains from search when actually purchasing the product and the period before that. We are hopeful that using the entire sequence of searches will help in identifying the importance of learning in a product differentiation setting, and leave this for future research.

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Appendix

Additional Details on the Derivation of Equation (4)

In general, the expected maximum utility out of J draws from distribution $H(u)$ is given by

$$\begin{aligned}
E\left[\max_{j\in J}\{u_{ij}\}\right] &= \int_{-\infty}^{\infty} u\frac{d}{du}\left(\prod_{j\in J} H(u-\delta_j)\right)du; \\
&= \int u\sum_{j\in J}\left(h(u-\delta_j)\prod_{r\in J\setminus j} H(u-\delta_r)\right)du; \\
&= \int u\sum_{j\in J}\left(\exp[-((u-\delta_j)+\exp[-(u-\delta_j)])]\prod_{r\in J\setminus j}\exp\left[-\exp\left[-(u-\delta_r)\right]\right]\right)du; \\
&= \gamma+\log\left(\sum_{j\in J}\exp[\delta_j]\right).
\end{aligned}$$

Note that $E\left[\max_{j\in J}\{u_{ij}\}\right]$ is relevant when searching non-sequentially, for instance when the decision is whether to search J or $J+1$ times. In our setting the decision is whether to sample J firms given that the utility at hand is $\hat{u}_{it}$. Just focusing on this, and ignoring the $W/(W+t)$ term, this means the gains from search equation is

$$G(\hat{u}_{it}) = \int_{\hat{u}_{it}}^{\infty}(u-\hat{u}_{it})\frac{d}{du}\left(\prod_{j\in J} H(u-\delta_j)\right)du.$$

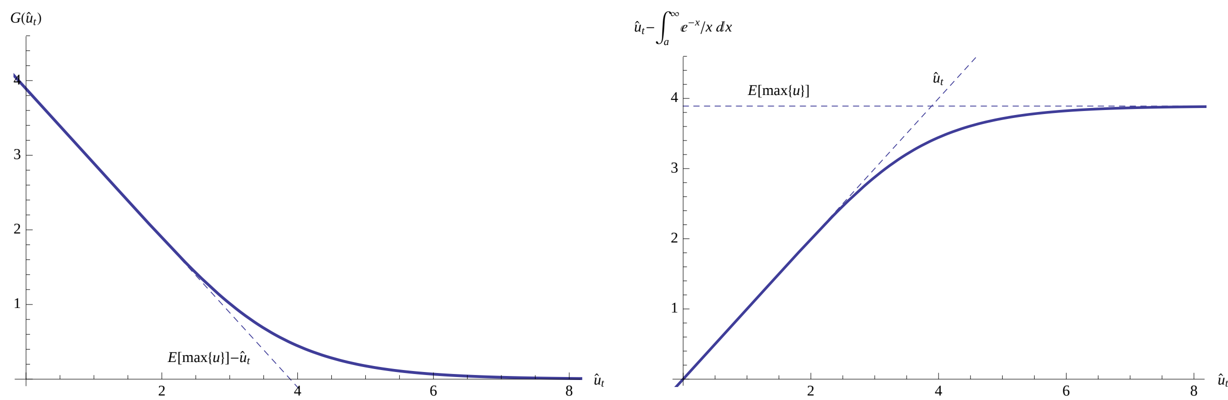
We can rewrite this integral as to sum of three separate integrals:

$$G(\hat{u}_{it}) = \int_{-\infty}^{\infty} u\frac{d}{du}\left(\prod_{j\in J} H(u-\delta_j)\right)du - \int_{-\infty}^{\infty}\hat{u}_{it}\frac{d}{du}\left(\prod_{j\in J} H(u-\delta_j)\right)du - \int_{-\infty}^{\hat{u}_{it}}(u-\hat{u}_{it})\frac{d}{du}\left(\prod_{j\in J} H(u-\delta_j)\right)du.$$

The first integral is the expected maximum utility out of J draws from $H(u)$, which, using the results from above, has the closed-form solution $\gamma+\log\left(\sum_{j\in J}\exp[\delta_j]\right)$. The second integral is just the reference utility $\hat{u}_{it}$. Finally, the last integral reflects a situation in which the maximum utility of J draws is less than $\hat{u}_{it}$, so the consumer will find it optimal to stick to $\hat{u}_{it}$. This, integral does not have a closed-form solution, but can be described by the exponential integral function evaluated at $a=\sum_{j\in J}\exp\left[\delta_j-\hat{u}_{it}\right]$, i.e., the integral equals $-\int_a^{\infty} e^{-x}/x\;dx$. Therefore, the gains from search equation simplifies to

$$G(\hat{u}_{it}) = \gamma+\log\left(\sum_{j\in J}\exp[\delta_j]\right) - \left(\hat{u}_{it} - \int_a^{\infty} e^{-x}/x\;dx\right). \tag{13}$$

As an example, Figure 8(a) gives $G(\hat{u}_{it})$ as a function of $\hat{u}_{it}$ for $J = 2$ with $\delta_1 = 2$ and $\delta_2 = 3$. For relatively low values of $\hat{u}_{it}$, the gains from search correspond to $E[\max\{u\}] - \hat{u}_{it}$ (the dashed line in the figure), since the maximum utility of (in this case) two draws from the utility distribution is almost always larger than $\hat{u}_{it}$. For higher values of $\hat{u}_{it}$, the probability of finding a higher utility than $\hat{u}_{it}$ becomes smaller and smaller, and in the limit the gains from search are zero. The exponential integral in equation (13) captures the option value of sticking to $\hat{u}_{it}$ in case the maximum utility drawn is less than $\hat{u}_{it}$. The difference between $\hat{u}_{it}$ and the exponential integral (the second term between brackets in equation (13)) is plotted in Figure 8(b). The graph shows that while for relatively small values of $\hat{u}_{it}$ the exponential integral is close to zero, for larger values of $\hat{u}_{it}$ that term is substantial, and $\hat{u}_{it} - \int_a^\infty e^{-x}/x\ dx$ converges to the expected maximum utility.



(a) Gains from search $G(\hat{u}_{it})$ (b) Reference utility minus exponential integral

Figure 8: Gains from Search for $\delta_1 = 2$ and $\delta_2 = 3$