Consumer Search with Observational Learning*

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Abstract

This paper studies social learning in a search environment. We model consumers who observe other consumers' purchasing decisions before embarking on their own search for the best-fitting product. This form of social learning has two distinct effects on consumer search and firm pricing. First, consumers *emulate* others in the sense that they always make the first visit to the firm where their predecessor has purchased. Second, consumers *free-ride* on their predecessor's information and search less intensively than in the standard search model. Emulation encourages price competition because firms fight for consumer visits, while free-riding has the opposite effect due to reduced search. Both effects increase in the search cost, but emulation is shown to dominate for most commonly used distributions of consumer preferences, therefore prices (eventually) fall as search cost increases, and may even go down to the marginal cost. We show that the results derived in a static duopoly model remain valid with a large number of firms and can be extended to a dynamic framework.

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1 Introduction

Observational learning has been the object of study of an increasingly large literature in economics since the seminal contributions of Banerjee (1992) and Bikhchandani et al. (1992). In the classical model, a sequence of individuals faces a simple decision problem under uncertainty and where each individual observes the history of decisions of her predecessors. As argued by Banerjee (1992), this simple environment closely resembles the problem faced by consumers in many markets, where previous customers' choices may be informative about the relative value of different products. Search markets, where consumers have to actively engage in costly activities to gather information about different options available to them, seem to be prominent examples of such environments. Intuitively, new consumers may *free ride* on the search effort of others and follow their advice. Importantly, this would substantially change the elasticity of demand and, hence, equilibrium prices. Thus, observational learning in search markets may have important implications on search behavior and ultimately on prices.

To the best of our knowledge, no paper has studied this important issue. We attempt to bridge this gap by analyzing a simple duopoly model of search with heterogenous products in the spirit of Wolinsky (1986). In the model, a large number of consumers derive utility from a given good, that comes in two varieties sold by two different firms. Consumers are initially uninformed about their valuation for each variety or the price charged by each firm, but learn them after engaging in costly sequential search. These valuations are randomly drawn from some variety-specific distribution. Firms simultaneously choose prices so as to maximize expected profits taking the behavior of consumers as given. To this fairly standard setup borrowed from Wolinsky (1986) and Anderson and Renault (1999) (henceforth *ARW*) we add observational learning by informing individual consumers of the purchasing decision of their predecessor whose valuation is positively correlated with consumer's own valuation. Importantly, consumers do not observe whether their predecessors searched actively nor the price or utility derived. Thus, this is a model of (limited) observational learning.

Strikingly, this change in the information structure of consumers radically changes equilibrium outcomes. In the *ARW* model, consumers' first visit is directed to each firm with equal probability (which is indeed optimal since the resulting equilibrium prices are equal). In our framework, because utilities are positively correlated, consumers decide to search first the variety purchased by their predecessor. As consequence, the proportion of first visits to a firm coincides (in the long-run) with her market share, and since consumers are more likely to buy the variety they sample first, this leads to a *social multiplier* of demand. To understand the effect of this change, we first assume that the correlation between consumers' valuation is positive but negligibly small, so that in a symmetric equilibrium consumers learn nothing from their predecessors but still follow them for their first visit (which is still optimal). We show that in the resulting equilibrium, prices are lower than in the *ARW* model, and for all commonly used distributions prices decrease in search cost once search cost is sufficiently large. The difference between our and Wolinsky-Anderson-Renault model stems from the behavior of consumers who always buy the variety they sample first (because their valuations for both products exceed their reservation utility). In the *ARW* model, the firm who is visited by these consumers first is effectively a monopolist and, thus, as the proportion of such consumers increase, as it does when search cost rises, prices tend to increase. In our model, however, these consumers are *allocated* according to the share of searchers who visit both stores. Since firms cannot price-discriminate between the two groups, as the proportion of consumers who stop at the first store increases, firms engage in increasingly fiercer competition for searchers who determine market shares, leading to Bertrand-like competition and eventually to prices that can be as low as the marginal cost.

We then introduce differences in the distributions of valuations of each variety. More precisely, we assume that the valuations that consumers derive from the variety offered by each firm are drawn from one of two distributions, and one of those distributions (High) dominates the other (Low) in the First Order Stochastic sense. Assuming that neither firms nor consumers observe which distribution realized, we can focus on the effect of learning in consumer behavior and prices. Note that a previous purchase of a given variety leads to an upwards update of its distribution of valuations and a downwards update in the rival's distribution. Hence, consumers are now willing to accept a lower surplus from their first visit, thus reducing search effort. We term this the *free-riding* effect of observational learning.

The effects on equilibrium prices are, however, not straightforward. On the one hand, as explained above, less search leads to higher competition because the social multiplier becomes more important. On the other hand, a price deviation triggers a change of beliefs that may overcome this effect. If a firm deviates to a higher price, the proportion of sales in each demand state changes, thus changing the beliefs that incoming consumers have. Since the elasticity of demand is lower for the firm with a higher distribution of valuations, consumers become more pessimistic about its rival's distribution the higher is the price of the store they visit. Hence, the surplus (net of the price) they demand for buying right-away decreases in the price, reducing the elasticity and increasing prices. We show that for some distributions, such as uniform and normal, the second effect is stronger for relatively small search costs while eventually the second effect dominates and prices may be lower than in the model without learning.

We are able to show that, just as in the model with emulation but no learning, in the model with learning, under a regularity condition, prices eventually decline in search cost and fall all the way down to marginal cost. The issue hinges on whether the likelihood ratio is sufficiently informative in the extreme case where almost no consumers search except those with the lowest valuations for one of the products. We are able to show that distributions with finite lower bound always satisfy the condition, and so ore prediction on the relationship between search cost and equilibrium prices is rather robust.

In the reminder of the paper we consider extensions to oligopoly and dynamic pricing. First we study the effects of competition on equilibrium prices in the model without learning. We show that prices decrease in search cost and converge to marginal costs as the proportion of movers vanish, for any number of firms. We further show that prices decrease in the number of firms for any search cost. Of special interest is the limit price when the number of firms grows large. In this case, whether there is correlation across individuals or not, there is nothing to be learnt about rival firms' qualities from the realization of sampled varieties. Armstrong and Chen (2009) shows that, as long as it is exogenous, the order of visits does not change prices in the ARW model since the elasticity of demand is the same for all firms. In our model, however, prices are lower than in the ARW model and have a U-shape relationship with search costs. If search costs are very low and there are many firms, Bertrand competition for buyers obtains. On the other hand, if search costs are very high, we know that firms fiercely compete for first visits and prices also converge to marginal costs. For intermediate search costs, however, firms charge positive prices.

Regarding dynamic pricing we show that if firms can adjust prices after the introductory period, and that in subsequent periods consumers make first visits proportional to first-period market shares, prices are further reduced in the introductory period with observational learning, and then go up to the levels of the ARW model. Thus observational learning mechanism is complementary to the dynamic market share buildup mechanism and together they lead to a very low introductory prices and then price hikes.

Literature Review

Our model of observational learning is inspired by recent evidence in economics and marketing. Mobius et al. (2005) present results of a field experiment designed to understand individual demand for different products for which individuals receive information. They show that those subjects who are connected through social links to others who are informed about a particular good, value the goods more. Importantly, they find that this effect is stronger for gadgets than for services, which are more likely to be subject to direct observation. In another field experiment, Cai et al. (2009) show that restaurant-goers are more likely to order those goods that are presented to them as more popular. Finally, Moretti (2011) analyzes the movie market where an unexpected increase in the first-week's box office has a persistent a significant effect in future attendance.

Two recent papers have analyzed consumer search with observational learning, but both assume that prices are fixed exogenously. Kircher and Postlewaite (2008) study consumers who differ in their willingness to search and firms differ in quality. Although prices are fixed, firms may decide to offer a valuable service to any consumer who visit their store. They show that equilibria may arise where high-quality firms offer service to those consumers who search more actively and those who search less actively follow their *advice*. Hendricks et al. (2012) presents a model of observational learning with multiple types in the spirit of Smith and Sørensen (2000) where each consumer has to decide between acquiring a costly signal about quality of a single good, buying it right-away or not buying. The model is cast in a more traditional herding framework where consumer receive a signal before deciding whether to engage in costly search. They focus on the long-run dynamics of sales for high and low quality products and the possibility of bad herds arising. Our model lacks pre-search information, therefore we cannot study bad herds, but instead focus on pricing.

In the consumer search literature with price competition, closest papers to ours are Armstrong et al. (2009) and Armstrong and Zhou (2011), who present a model of prominence in consumer search where one firm is sampled first by all customers. In Armstrong et al. (2009) a given firm is made prominent exogenously. In the resulting equilibrium, the prominent firm charges a lower price than her rivals because her share of returning customers (who are typically less responsive to prices) is lower. One may view our framework as that of endogenous prominence, where share of first visits depends on price. Armstrong et al. (2009) show that as the number of firms grows, (exogenous) prominence becomes irrelevant while in our model, competition becomes even fiercer and prices decrease in the number of firms. Armstrong and Zhou (2011) study several different models that endogenize firm's prominence. One such model is based on observable price competition where consumers rationally search the lowest-pricing firm first. Because demand is discontinuous in prices, the resulting equilibrium involves mixed strategies and has a property that higher search cost leads to (stochastically) lower prices. One may view observability of prices as an extreme example of observational learning where consumers observe market shares, and thus prices. We show that imprecise information about prices results in a pure strategy equilibrium featuring inverse relationship between search cost and prices. Another closely related model can be found in Haan and Moraga-Gonzalez (2011), where prominence depends on advertising efforts and firm profits may decrease in search cost (although prices increase and consumer surplus decreases).

Our paper is also related to the literature that studies effects of social learning on monopoly pricing. Campbell (2013) and Chuhay (2010) analyze the impact of word-of-mouth communication on the monopoly price and product design. Perhaps closest to our work are two contributions by Bose et al. (2006) and Bose et al. (2008) who study dynamic interaction between a monopolist and a sequence of consumers with common valuation who observe each other purchasing decisions. While most of this literature has studies monopoly, an exception is Kovac and Schmidt (2014) who study a dynamic market where two firms offer a homogenous product and consumers learn prices from others. Since our focus is on competition and we abstract from dynamic issues[1], we view our work as complementary to theirs.

Finally, a number of papers have studied the relation between current market shares and future demand. Becker (1991) directly introduces aggregate demand into the individual utility function. Caminal and Vives (1996) studies a dynamic signaling game where new cohorts of consumers observe past market shares of an experience good, but not prices and try to infer quality from this information. Firms use prices to manipulate market shares to attract consumers. While their setting is different in that consumers do not search, we also find that firms use prices to attract consumers, but here consumers free-ride on their predecessors' efforts and due to search cost, herds form, while in Caminal and Vives these effects are mute.

The next section introduces the general model with observational learning with correlated preferences and endogenous prices. Section 3 solve the model in the special case of no correlation between preferences of consumers. Section 4 studies the model with correlated preferences and establishes comparative statics results including with respect to the number of firms. Section 5 introduces an extension to dynamic pricing and Section 6 concludes.

2 Model

Consider a market populated by a large number of consumers $i = 1, 2, ..N$, interested in purchasing a single unit of a differentiated good that comes in two varieties, each sold by one firm, 1 and 2. Consumers are initially uncertain about their valuation of each firm's product, but may acquire this information through sequential search. We assume that the first visit is free but the second has a cost of c in utility units and all consumers can recall their previously sampled varieties at no additional cost. Following Anderson and Renault (1999) we make the simplifying assumption that all consumers will buy one of the two varieties.[2]

We describe now the demographics of the model. In every period, a cohort of consumers arrive to the market. Every consumer makes all his decisions in that one period. Let $1 \leq t_i \leq T$ be the period in which consumer i arrives. The utility that every consumer i derives from variety j depends on his type $k \in \{1, 2, .., K\}$, and in each cohort there is a single individual of each type, so that there are $N = K \cdot T$ consumers. Consumers do not know t_i and hold a common prior $\nu(t)$ over it. We introduce observational learning by informing every consumer arriving in a cohort $t > 1$ of the purchasing decision of the previous consumer of his type. Importantly, they do not observe the information that has

[1] See Section 4

[2] This amounts to assuming that their outside option is sufficiently bad.

led to the purchase or whether their predecessor sampled more than one variety.

We introduce correlation between preferences of individuals of a given type in the following fashion. All consumers of a type draw their valuations for each firm's product from one of two potential distributions, a High distribution (denoted by $G_H(u)$) and a Low distribution ($G_L(u)$). Both distributions are equally likely to realize for each type, and these realizations are independent across types and firms. We assume that both distributions have the same (possibly infinite) support $[\underline{u}, \bar{u}]$ and that G_H First Order Stochastically Dominates (FOSD) G_L. Let g_H and g_L be the corresponding densities. As is usual in the economics literature, we assume that the Monotone Likelihood Ratio property holds so that $\frac{g_H(x)}{g_L(x)}$ is an increasing function of x. Let $\lambda(u) = \frac{g_H(u)}{g_H(u)+g_L(u)}$ be the conditional probability of H given u. Let $G(u) = \frac{1}{2}G_H(u) + \frac{1}{2}G_L(u)$ be the unconditional distribution of valuations of a random consumer. We shall assume that $G(u)$ is log-concave.[3] Finally let $(S_1, S_2)_k \in \{H, L\}^2$ be the realized state for a given type. To summarize, all consumers of the same type draw their utility for a firm from the same distribution. E.g. consumers of type k may draw their utility from distribution H for firm 1 and L for firm 2. Even though consumers cannot know the utility of the consumer they observed, the fact that that consumer buys from a firm is informative about which state $(S_1, S_2)_k$ has realized.

In order to disentangle changes in information from changes in the distribution of valuations, it is convenient to define two auxiliary distributions $G_1(u)$ and $G_2(u)$ such that $G_H(u) = (1 - r)G(u) + rG_1(u)$ while $G_L(u) = (1 - r)G(u) + rG_L(u)$ and such that $G(u) = \frac{1}{2}G_1(u) + \frac{1}{2}G_1(u)$, where $r \in [0, 1]$ measures the degree of correlation across consumers of a given type. This specification guarantees that for any r the unconditional distribution (without knowing state S) for a firm is $G(u)$, while r controls how close G_L is to G_1 and G_H is to G_2. If $r = 0$ valuations are independent, and drawn from $G(u)$, while for $r > 0$ valuations of any two members of the same type are positively correlated. Conveniently, for every r the unconditional distribution of valuations remains constant but the amount of information contained in the purchasing decision of a predecessor may vary significantly (as allowed by the disparity between G_1 and G_2). Thus r proxies correlation of valuations among members of a type.

On the supply side, there are two firms supplying any quantity of their variety at a constant unit cost normalized to zero. Importantly, firms commit to serve all incoming customers at a single price p_j set simultaneously before the state of the market is realized in order to maximize expected profits. We shall devote most of our attention to a symmetric, pure-strategy equilibrium where both firms charge the same price p^*.

[3] Log-concavity of G_H and G_L does not guarantee Log-concavity of G.

2.1 Consumer Behavior

Consumers in the first cohort, lacking any information to discriminate firms, make their first visit randomly and buy there if and only if $u_i - p_i > \hat{w} - p^*$, where $\hat{w}$ solves

$$\int_{\hat{w}}^{\bar{u}} (u - \hat{w}) g(u) du = c, \tag{1}$$

where $g(u) = \frac{g_H(u)+g_L(u)}{2}$ and is the density of $G(u)$. $\hat{w}$ is the familiar reservation utility from Wolinsky (1986). It is found by equalizing expected benefit from search (with free recall) to the search cost. Unsurprisingly, the first consumer of any type is no different in our model as compared to the ARW model.

As is well known, in ARW there is no search for $c > \int_{\underline{u}}^{\bar{u}} ug(u)du$, adn in the version of the model where the outside option is infinitely bad, prices are infinitely large. To avoid this, henceforth we assume

Assumption 1. *The search cost satisfies* $c < \int_{\underline{u}}^{\bar{u}} ug(u)du \equiv \bar{c}$.

All remaining consumers observe a predecessor buying from firm j and expect the same price p^* in both stores before embarking on their first search. Since valuations are positively correlated, the expected surplus from firm visiting firm$j$ is (weakly) larger than that of firm $-j$ and so the consumer should visit that store. Upon visit consumer learns his utility realization for good j, u_{ij} and the price that the firm j charges. Consumer will search if and only if $u_{ij} - p_j < w(p_j) - p^*$, where $w(p_j)$ solves

$$\int_{w-p_j+p^*}^{\bar{u}} (u - w + p_j - p^*)(q(w; p_j) g_H(u) + (1 - q(w; p_j)) g_L(u)) du = c \tag{2}$$

where $q(u; p)$ is the (posterior) probability that at the other store the valuations are drawn from the high distribution. This probability will, in general, depend on the valuation drawn from firm j (because preferences are correlated) and the price of firm j because it affects the probability consumers buy at that firm, and thus conditional distribution of the other store. In principle, $q(u; p)$ is a very complicated object, since each consumer may infer from u not only how likely it is that a given distribution realized but also his cohort (which is potentially informative about the informational content of the purchasing decision of the predecessor). In particular, the probability that a consumer arriving in cohort t buys from firm 1 if the state is $(SS') \in \{H, L\}^2$ is

$$\begin{aligned} x^t_{SS'}(p_1, p_2) &= x^{t-1}_{SS'}(p_1, p_2)(1 - G^S(w(p_1))) + (1 - x^{t-1}_{SS'}(p_1, p_2)) G_{S'}(w(p_2))(1 - G_S(w(p_1))) \\ &+ \int_{\underline{u}}^{w(p_1)-p_1+p_2} G_{S'}(u - p_1 + p_2) g_S(u) du \end{aligned}$$

which increases in $x^{t-1}_{SS'}$, leading to a link between past and current market shares. It is straightforward to see that this mapping has a fixed point where market shares are stationary ($x_{SS'}(p_1, p_2)$). In Appendix 1 we show that, provided that the number of consumers of each type is sufficiently large, consumers' optimal search strategy is arbitrarily close to the one computed for a *stationary market share distribution*. In this case, dropping the time subscript, we can write a firm's market share in state SS' when it charges p while the other firm charges the equilibrium price as

$$\begin{aligned} x_{SS'}(p,p^*) &= x_{SS'}(p,p^*)(1-G^S(w(p))) + (1-x_{SS'}(p,p^*))G_{S'}(w(p))(1-G_S(w(p))) \\ &+ \int_{\underline{u}}^{w(p)-p+p^*} G_{S'}(u-p+p^*)g_S(u)du. \end{aligned}$$

We can solve for $x_{SS'}(p, p^*)$ from the above

$$x_{SS'}(p,p^*) = \frac{\int_{-\infty}^{-p^*+p+w^*} g_S(u)G_{S'}(p^*-p+u)\,du + (1-G_S(-p^*+p+w^*))G_{S'}(w^*)}{\int_{-\infty}^{-p^*+p+w^*} g_S(u)G_{S'}(p^*-p+u)\,du - \int_{-\infty}^{w(p)} g_S(u)G_{S'}(p^*-p+u)\,du + (1-G_S(-p^*+p+w^*))G_{S'}(w^*) + G_S(w(p))} \quad (3)$$

For stationary market shares, $q(u;p)$ satisfies

$$q^*(u;p) = \frac{x_{HH}(p)\lambda(u) + x_{LH}(p)(1-\lambda(u))}{(x_{HH}(p)+x_{HL}(p))\lambda(u) + (x_{LH}(p)+x_{LL}(p))(1-\lambda(u))} \quad (4)$$

The above formula uses market shares in every state and weights the conditional probability of H given u by these market shares. As expected, if in all states both firms have equal market share ($x_{SS'} = 1/2$), which given our assumption about G_H and G_L imply $G_L = G_H$, the first visit is not informative and $q^*(u;p) = 1/2$. Notice that $q^*(u;p)$ is increasing in u because λ is an increasing function and the market shares satisfy $x_{HH} \geq x_{LH}$ and $x_{HL} \geq x_{LL}$. Intuitively, a higher u leads the consumer to update upwards the probability he attaches to the current firm having a high distribution ($S = H$). Importantly, this also increases the belief he holds about the other firm's distribution, since it reduces the informativeness of the predecessor's purchase at this firm regarding the other firm's distribution.

This completes the characterization of the consumer search rule. In particular, $w(p)$ is implicitly defined in (2) where $q(u;p)$ is given by (4) where $x_{SS'}$ is defined in (3).

In the reminder of the paper we will use stationary market share distributions. One may think of a reduced-form model where consumers observed a predecessor, proportion of first visits matches firm's market share, and search and firm strategies are consistent with the above.

2.2 Equilibrium Conditions

In a symmetric equilibrium, prices are equal and, thus, do not affect search behavior. The consumers' search rule can be rewritten as

$$\int_{\hat{w}}^{\bar{u}} (u - \hat{w})(q(\hat{w})g_H(u) + (1 - q(\hat{w}))g_L(u))du = c \tag{5}$$

where $q(\hat{w}) = q(\hat{w}; p^*)$ is the equilibrium probability that the rival firm has the High distribution given the reservation utility. Notice that $q(u) \leq q(\bar{u}) \leq \frac{1}{2}$ since observing a previous consumer buying in a store is bad news about the prospects in the rival store. This effect is only fully mitigated if the consumer learns that the current firm offers high valuations, in which case it learns nothing about his prospects in the rival firm. Hence, we have the following trivial observation.

Proposition 1. *Fix $c > 0$, in a unique symmetric equilibrium, $\hat{w} < w^*$, and, hence, consumers free-ride on others' effort.*

Firms' profits depend on the search rule used by consumers both on and off-the-equilibrium path. In particular, let $w'(p)$ be the implicit derivative of the search cutoff with respect to p. In the ARW model model without learning, $w'(p) = 1$ so that an increase in price is compensated with a higher required utility. This is no longer the case with observational learning. Consumers visiting a firm with a different price adjust their beliefs about the distribution of valuations in the rival firm, while keeping their beliefs about its price constant.[4] In particular, the higher the price, the less likely it is that a consumer ends up in that firm if its rival has a High realization of valuations. Thus, in general, $w'(p) < 1$. Given this search behavior, consumers are split into four groups in the (u_1, u_2) space (plot). As is standard in Wolinsky-type models, those whose valuation for both varieties is lower than the reservation utility $\hat{w}$ search independently of the variety they sample first and then buy from the highest-surplus offering firm. Those whose utility profile satisfies $u_j < \hat{w} < u_k$ only search if they visit firm j and always buy from k. Finally, those whose valuations are higher than their reservation utility buy from the firm they visit first. This group has low-demand elasticity and is typically assigned randomly across firms. In our model, these consumers are assigned to each firm with probabilities equal to market shares and, thus, become quite elastic. We term this the *emulation* effect of observational learning.

Expected demand for a firm that charges p while its competitor charges p^* and consumers use reservation utility function $w(p)$ is the average of demands over all possible states. Because states between firms are independent, and equally likely, the demand is given by:

$$x(p, p^*) = \frac{1}{4} \sum_{SS'} x_{SS'}(p, p^*). \tag{6}$$

Here $x(p, p^*)$ implicitly depends on $w(p)$ and thus on consumer search rule.

[4] The assumption of passive beliefs is common in the literature. For a discussion see Janssen and Shelegia (2014)

In equilibrium, symmetric equilibrium price satisfies the following condition

$$\sum_{SS'} x_{SS'}(p^*, p^*) + p^* \sum_{SS'} \frac{\partial x_{SS'}(p, p^*)}{\partial p} = 0$$

and further $\sum_{SS'} x_{SS'}(p^*, p^*) = \frac{1}{2}$ by symmetry. So the equilibrium pricing rule simplifies to

$$\frac{1}{2} + p^* \sum_{SS'} \frac{\partial x_{SS'}(p_1, p^*)}{\partial p_1} = 0 \tag{7}$$

Equations for $x_{SS'}$ are non-trivial and depend on a complex way on the search rule consumers use. Because of this, while we derive some results for any G and $r > 0$, before proceeding to analyzing the general case we turn to the special case where $r = 0$.

3 Emulating Consumers

To better understand these two effects, we start by isolating emulation from free-riding. We do so by assuming that $r \to 0$ so that valuations across consumers are nearly independent, while consumers still follow others' purchasing decisions for their first visits. This is the limit case of the general model when distributions become increasingly similar, and since the model is ex-ante symmetric, making the first visit to the firm where the predecessor has purchased remains optimal (including when $r = 0$). More generally, one can view this model as a behavioral modification of Anderson and Renault (1999) where consumers' first visits are allocated according to market shares rather than randomly. While the modification is rather small, it has dramatic effect on prices.

Because the visit of the predecessor contains no information, the reservation utility is computed as in the *ARW* model and is given by (1). As for the firms, let x denote market share of firm 1. Because first visits also follow purchases, the share of firm 1's first visits is also x. Thus its demand can be written as

$$\begin{aligned} x(p_1, p^*) &= x(p_1, p^*)(1 - G(\hat{w} + p_1 - p^*)) + (1 - x(p_1, p^*))G(\hat{w})(1 - G(\hat{w} + p_1 - p^*)) \\ &+ \int_{\underline{u}}^{\hat{w}+p_1-p^*} G(u - p_1 + p^*)g(u)du \end{aligned}$$

From the above, it is straightforward to solve for $x(p_1, p^*)$ and find firm 1's pricing equation.

Proposition 2. *Suppose $r = 0$. In a symmetric equilibrium with emulating consumers, the reservation utility is computed as in (5) and the price is*

$$p^* = \frac{(2 - G(\hat{w}))G(\hat{w})}{2\int_{\underline{u}}^{\hat{w}} g^2(u)du + (1 - G(\hat{w}))g(\hat{w})} \tag{8}$$

while in the ARW model

$$\hat{p} = \frac{1}{2\int_{\underline{u}}^{\hat{w}} g^2(u)du + (1 - G(\hat{w}))g(\hat{w})} \tag{9}$$

Thus, $p^* \leq \bar{p}$, with a strict inequality for $w^* < \bar{u}$.

The intuition for this result is rather simple. In the *ARW* model, price competition is restricted to the small subset of consumers whose valuations for both firms are close and lower than the reservation utility. In the model with observational learning, those consumers whose valuation for both varieties is larger than the reservation utility become sensitive to price changes because they follow the lead of those with relatively low valuations. This is the social multiplier effect on demand. As the proportion of consumers who do not search beyond the first firm grows, price competition for searching consumers who bring all others to the store intensifies leading to lower prices. Recall that $\bar{c} = \int_{\underline{u}}^{\bar{u}} ug(u)du$ is the average utility at a given store. Using the argument above, we can state:

Proposition 3. *For any log-concave G with bounded support, for any* $\lim_{c\to\bar{c}} \hat{p} = 0$*. Also, if G has unbounded support but* $\lim_{u\to\underline{u}} G(u)/g(u) = 0$*, then* $\lim_{c\to\infty} \hat{p} = 0$*.*

The first part of the proposition has been explained. The second part puts a restriction on the ratio of the cumulative to the density at the lower bound that is satisfied by all distributions whose support is bounded below as well as for most log-concave distributions whose support is unbounded (e.g. Normal). This condition is not satisfied for the Logistic distribution, but it can also be readily verified that prices decrease in search costs for this distribution too.

4 Equilibrium with Learning

The effects of learning on prices is ex-ante ambiguous because learning adds two competing effects. First, as highlighted by Proposition 1, for a given c and a symmetric price p^*, $\hat{w}$ is lower the bigger the difference between distributions. Consumers are then willing to accept lower surplus the higher the price ($w'(p) < 1$), which pushes prices upwards. On the other hand, in the presence of emulation, lower $\hat{w}$ may lead to lower prices through fiercer competition for increasingly rare searchers. This effects can be seen in pictures...

Notice that, eventually, free-riding joins forces with emulation and prices decrease faster and converge to marginal cost for lower search costs than without learning. In particular, the maximum search costs at which a pure-strategy equilibrium exists equals the expected valuation of G_L. This is because, in the example, the likelihood ratio grows unboundedly at the lower bound. Hence, a consumer who arrives at a store offering sufficiently low utility is almost sure that the current firm's distribution of valuations is

Low, which implies that with arbitrarily high probability the rival offers Low valuations too, for otherwise the probability that my predecessor ended up here is negligible. To see this, notice that

$$\lim_{c \to \bar{c}} x_{SS'}(p, p^*) = \frac{G_{S'}(\hat{w})}{G_{S'}(\hat{w}) + G_S(w(p))} \tag{10}$$

If $\lambda(\underline{u}) = 1$, $x_{L,H}(p^*, p^*) = 0$ and thus, $q(\underline{u}) = 0$. In such a case, search is unattractive and the consumer stays. Define $\bar{c}_L = \int_{\underline{u}}^{\bar{u}} u g_L(u) du$. Since prices approach marginal costs as the share of movers vanishes, we have the following counterpart of Proposition 3:

Proposition 4. *Suppose G has a finite support. As $c \to \bar{c}_L$, we have $\hat{p} \to 0$.*

Prices converge to zero as the search cost converges to the expected valuation of G_L. The following Corollary suggests that learning itself may decrease prices by letting the ex-ante distribution of valuations constant. Let $\hat{p}(r)$ be the price if the distribution is governed by $r \in [0, 1]$ and recall that higher r correspond to higher correlation and, therefore, higher informativeness of the purchasing decision of a predecessor. We have the following Corollary

Corollary 1. *Take a distribution G. For $c \in (\bar{c}_L, \bar{c})$, $\hat{p}(1) = 0 < \hat{p}(0)$.*

That is, as the informativeness of the purchasing decision increases, the equilibrium price elasticity increases and, thus, prices decrease. This is because those consumers whose valuation for the variety they sample first is low become increasingly pessimistic about their prospects in the other store the higher is the correlation across consumers, and, therefore, the less inclined they are to search.

The following figures illustrate our results for a uniform distribution G on $[0, 1]$. In this example G_1 is a triangular distribution on $[0, 1]$ with mode 0 while G_2 is a triangular distribution on $[0, 1]$ with model 1. As required, the mixture of the two is uniform on $[0, 1]$, G_H FOSD G_L and the likelihood ration is monotone for any $r > 0$.

Figures 1 and 2 show prices and reservation utilities in various models.

As expected, prices are always lower in the model with pure emulation (black) than in the *ARW* model. As predicted by theory, in *ARW* model price is increasing in s and reaches 1 at $c = 1/2$. This is where even a consumer who draws 0 at the first store refuses to search further (after this, prices are infinite becuase outside option is $-\infty$). In our model with learning ($r = 1$), as predicted by Proposition 4, because $\underline{u} = 0 > -\infty$, price is zero at $c = 1/3$, which is the average of G_1. In our model without learning ($r = 0$), price converges to zero at $c = 1/2$, as implied by Corollary 2. Intuition for both is simple. When $c = 1/3$ in the model with learning, a consumer who is sure that utilities from the other store are drawn from G_L will not search. But this is what happens in equilibrium when a consumer draws utility close to $\underline{u}$ - she reasons that because distribution in the current firm is almost surely G_L, the fact that she came here indicates that in the other

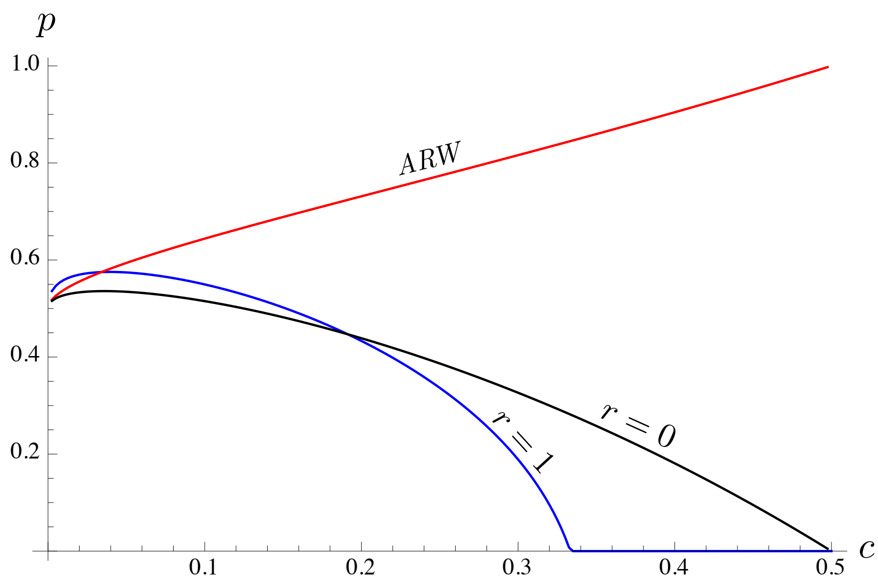

Figure 1: Prices in our model with $r = 1$ (blue), $r = 0$ (black) and ARW model (red) for triangular distributions.

store the distribution is also G_L, or otherwise almost all queues would end up at the other store. Thus even though u close to $\underline{u}$ is bad news about the current store, it is also bad news about the next store.

One interesting aspect revealed by Figure 1 is that prices in the model with information may be lower than without information. This seems to contradict with Proposition 1 (and also Figure 2) that states that with information ($r > 0$) there is free-riding, thus prices should be higher. This puzzle is resolved by noting that even though it is true that reservation utility is lower in the model with information, which on its own would result in higher prices, lower w also results in more elastic demand for firms (becuase very few consumers actually search), leading to lower prices.

Figure 3 shows $w'(p)$ as a function of c. For $c \to 0$, the derivative of reservation utility approaches 1. This is becuase when almost all consumers search, price contains almost no information and so reservation utility matches it one for one. For all $c > 0$, $w'(p)$ is less than 1, reflecting the informational content of price deviations.

Finally, Figure 4 illustrates equilibrium demands of a given type for various states as functions of c. In symmetric states (LL and SS), demand for both firms is equal to 0.5 and is independent of search cost. Matters are more interesting in asymmetric states. As the search cost increases, demand for a firm with low distribution facing a firm with high distribution shrinks (the opposite is true for high distribution vs low distribution). Naturally, when search cost is small some consumers buy at the store where utility is drawn from G_L, and so the same proportion of consumers visits this store first. As $c \to 1/3$, search vanishes and almost all consumers are steered first to the firm with high distribution, and because $\hat{w}$ approaches 0, almost all stay there. Thus, a "herd" forms where all consumers make the first visit to the store with high distribution, and

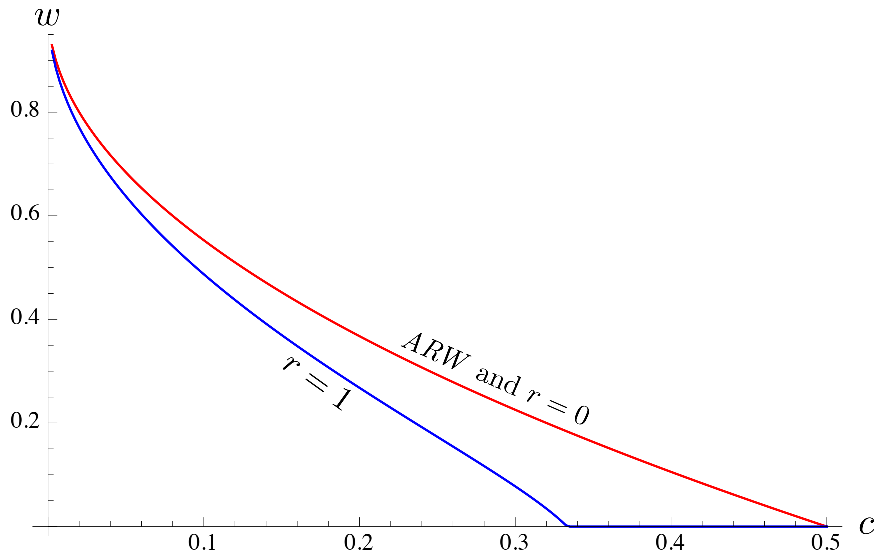

Figure 2: Reservation utilities in our model with $r = 1$ (blue) and our model with $r = 0$ and also ARW model (red) for triangular distributions.

then correctly avoid searching further.[5] The interesting aspect of such optimal collective behavior is that it coincides with zero prices. Note though that prices are zero not because of optimal collective behavior in equilibrium, but rather for the opposite reason out of equilibrium. Namely, if a firm were to deviate and charge a higher price, even though for a quarter of types the state is HL, almost no consumers of these types would visit it. This means that upon price deviations "bad herds" form where small price differences result in quarter of consumer incorrectly visiting a firm with worse distribution and slightly better price.

4.1 The Effects of Competition

We have assumed throughout that there are only two firms in the market. This assumption ensures that both decisions and information sets are binary and renders the analysis of consumer learning and behavior tractable. As the number of firms increases, the number of states increases exponentially and so do the complexity of the information structure. Thus, to analyze the effects of competition we go back to the simple model of consumers' emulation where the learning channel is muted (i.e. $r = 0$). Notice, however, that as the number of firms becomes larger, and learning becomes more complex, the amount of information about other firms' distribution contained in the purchasing decision of a predecessor also decays exponentially. Intuitively, as the number of firms grows large, the market share of a given firm is, approximately, independent of the realization of the state in any other firm (mean-field approximation). Thus, while a purchasing decision by a

[5]The word "herd" here is used in contrast to the economics literature. In our model consumers do not possess pre-search information, and have to obtain it actively, thus herds in traditional sense cannot arise.

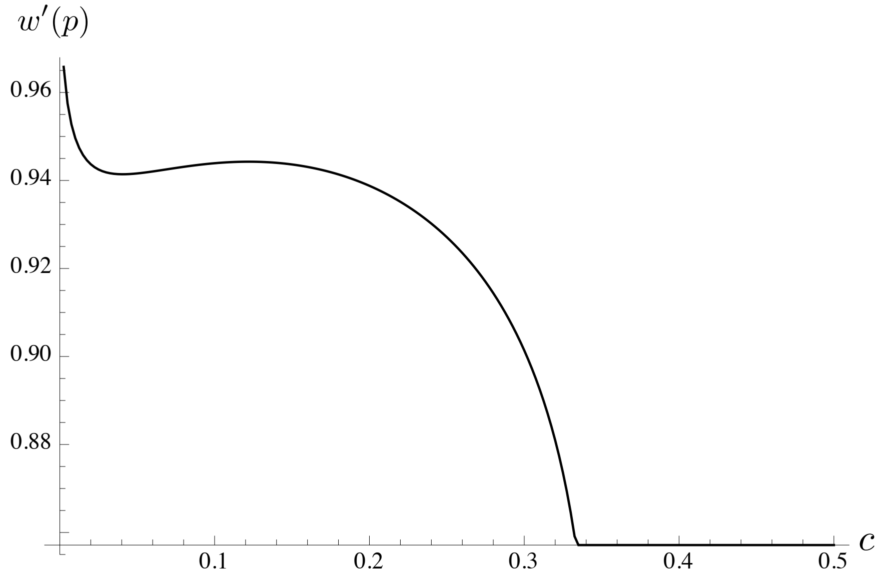

Figure 3: Derivative of reservation utility with respect to p in our model for $r = 1$ and triangular distributions.

predecessor remains informative about the distribution of valuations for the variety sold in the current firm, it is irrelevant in determining the distribution of valuations for any other variety.

Hence, assume that $r = 0$ and let n be the number of firms in the market. The stationary market share of a given firm choosing a price p if all its rivals choose p^* is

$$\begin{aligned} x(p,p^*) &= x(p,p^*)(1-G(\hat{w}-p^*+p)) + \frac{1-x(p,p^*)}{n-1}\sum_{j=1}^{n-1} G(\hat{w})^j(1-G(\hat{w}-p^*+p)) \\ &+ \int_{\underline{u}}^{\hat{w}-p^*+p} G(u-p+p^*)^{n-1} g(u-p+p^*)du. \end{aligned}$$

Notice that market shares only determine the number of first visits to each firm, but not the subsequent searches. Let p_n^* be the price in a symmetric equilibrium with n firms. For an arbitrary distribution G, we have

$$p_n^* = \frac{\frac{nG(\hat{w})-G(\hat{w})^n}{n-1}}{g(\hat{w})\left(\sum_{i=0}^{n-2} G(\hat{w})^i + (n-1)G(\hat{w})^{n-1}\right) - (n-1)n\int_{\underline{u}}^{\hat{w}} G(u)^{n-2} g(u)^2\, du}. \tag{11}$$

Recall that $\hat{w}$ is still defined by (5). This can be contrasted with the equilibrium price in the *ARW* model given by

$$\hat{p}_n = \frac{1}{g(\hat{w})\left(\sum_{i=0}^{n-2} G(\hat{w})^i - (n-1)G(\hat{w})^{n-1}\right) + (n-1)n\int_{\underline{u}}^{\hat{w}} G(u)^{n-2} g(u)^2\, du}. \tag{12}$$

Proposition 5. *Assume $r = 0$. For every n and bounded G, the price with emulating*

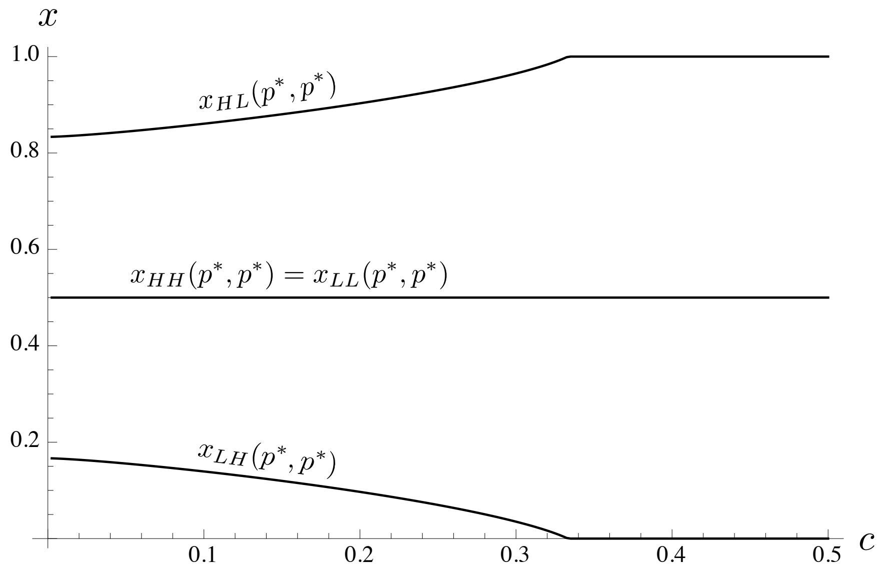

Figure 4: Equilibrium demand of a given type for various states for triangular distributions.

consumers is lower than the price in the ARW model and goes to zero with c. Further, $\hat{p}_n$ *but also* $p_n^*/\hat{p}_n$ *decreases in* n.

As with the *ARW* model prices go to zero with n in our model. Important distinction is that as the number of firms grows, the relative gap between prices in our and *ARW* model also grows, thus competition compounds the emulation effect.

The comparison becomes more clear when we take the limit as the number of firms grows. In this case, thee share of returning customers vanishes and the price converges to

$$p_\infty^* = \frac{(1 - G(\hat{w}))G(\hat{w})}{g(\hat{w})} \tag{13}$$

so that $\lim_{c\to 0} p_\infty^* = \lim_{c\to \bar{c}} p_\infty^* = 0$. On the other hand, as shown in Anderson and Renault (1999), the price in the *ARW* model (eq. 12) with an infinite number of firms coincides the classical Perloff-Salop formula

$$\hat{p}_\infty = \frac{1 - G(\hat{w})}{g(\hat{w})}. \tag{14}$$

Armstrong et al. (2009) shows that this price is independent of the way consumers decide their search paths, as long as this order is exogenous. This shows that the effect of prominence is qualitatively different from the effect of emulation, particularly when the number of firms is large.

The next figure illustrates equilibrium prices for different n. As shown in Proposition 5, prices decrease in n.

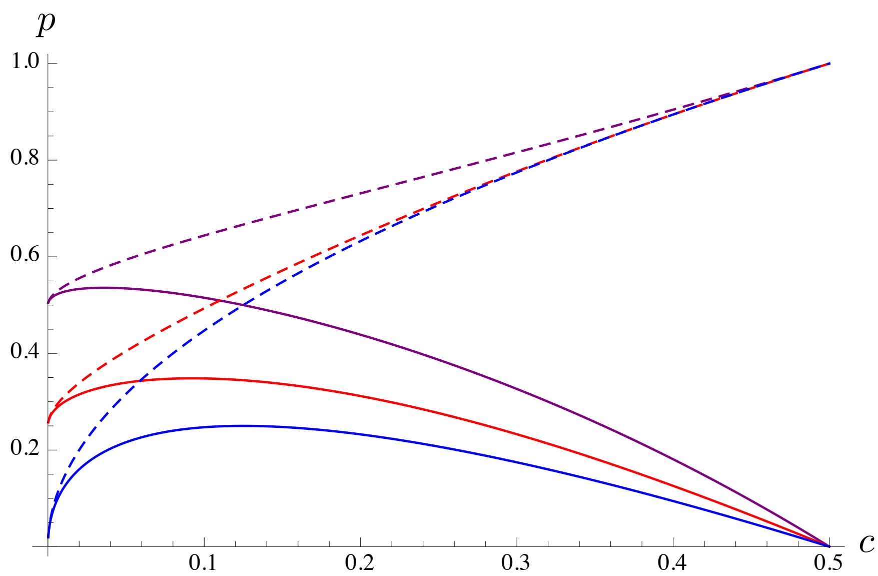

Figure 5: Equilibrium prices as function of search cost for $n = 2$ (purple), $n = 4$ (red) and $n = \infty$ (blue) for our model (solid) and ARW (dashed).

5 Dynamics and Lock-in Effects

In this Section we explore the connection between our model and the Switching Cost literature and, in so doing, introduce dynamic considerations. We consider the following extension of the Benchmark model. The economy has two dates $\tau = 1, 2$. In $\tau = 1$ the market structure is as presented above. Firms set prices for the first date and consumers arrive in different cohorts and observe their predecessors' purchasing decision. In $\tau = 2$, firms observe their sales and simultaneously choose prices for the second date. In order to introduce a simple dynamic link we assume that consumers visit first the store they purchased from in the first period but their utilities are drawn anew. As in the standard switching cost model, the market in the first date is more competitive than in the second. In this case, the difference in competition comes from the observational learning that does not occur in the second Again, for simplicity, we assume that $r = 0$, so that no learning about demand occurs in equilibrium. Further, we follow Armstrong et al. (2009) and assume that valuations are uniformly distributed in $[0, 1]$.

In the second periods, firms choose prices to maximize expected profits given their number of first visit. In particular, given a share of first-date market, her second-date market share is

$$x_2(p, p_2(x_1)) = x_1(1 - (w - p + p_2(x_1))) +$$
$$(1 - x_1)(w - p_2(x_1) + p_1(x_1))(1 - (w - p + p_2(x_1))) + \int_{\underline{u}}^{w-p+p_2(x_1)} (u - w + p - p_2(x_1))du$$

Let $\Delta^* = p_1(x_1) - p_2(x_1)$ be the expected difference in prices in the second period given some market shares and let $\Delta = p - p_2(x_1)$ be the realized difference. We have

$$x_2(\Delta) = x_1(1-(w-\Delta)) + (1-x_1)(w-\Delta^*)(1-(w-\Delta)) + \int_{\underline{u}}^{w-\Delta} (u-w+\Delta)du$$

From Haan and Moraga-Gonzalez (2011) we know that $\Delta(x_1)$ is increasing in x_1 and $\Delta^*(\frac{1}{2}) = 0$. If a symmetric equilibrium exists $\Delta^* = 0$ and $p_2(x_1^*) = p_2(\frac{1}{2}) = \hat{p}$. Now, let $\pi(x_1)$ be the expected profit in the second date given market share x_1. $\pi(x_1)$ is clearly increasing in x_1. Firm 1 solves then

$$\max_{p_1} p_1 x_1(p_1, p^*) + \pi(x_1(p_1, p^*)) \tag{15}$$

Clearly $p_1^* \leq p^*$ and in any symmetric $p_2(x_1^*) > p_1^*$ so that prices increase over time.

6 Nature of observational learning

In our model consumers observe the purchasing decision of a single predecessor. While this assumption is rather crude, it is perhaps more plausible than assuming that consumers observe the whole sequence of purchasing decisions or the true market shares. In any case, this assumption greatly simplifies the analysis since the number of possible information sets a given consumer may end up in grows exponentially in the number of predecessors. More importantly, in our model all consumers have the same reservation utility strategy. This would not be the case if they observe a larger set of consumers. On the other hand, if consumers observed market shares without noise, the equilibrium would converge to the mixed-strategy pricing equilibrium presented in Armstrong and Zhou (2011) for the case in which consumers observe prices. The reason for this is that even a small deviation in price would result in *all* consumers first visiting the deviating firm, thus profit would jump up discretely. This is the familiar Bertrand pressure on prices. But prices cannot be zero becuase for relatively small search cost a firm can still earn profit even if it is visited last. Therefore, there has to be a mixed strategy equilibrium.[6]

This shows that when consumers posses a lot of information about their predecessors' purchases, equilibrium results in mixed strategies. We analyze the other extreme where consumers have very limited information. What happens between these two extremes is extremely hard to analyze, but we believe that for relatively limited observational learning equilibrium is qualitatively similar to ours, until when a lot of learning results in mixed strategies.

[6] This is in contrast to our results (Propositions 3 and 4) that say that prices go down to zero.There search costs are so high that almost no one searches beyond the first firm, thus not matching the other firm's low price results in zero profits.

7 Discussion

Our aim in this paper was to introduce simple observational learning into a standard consumer search model with horizontally differentiated products. We did so and showed that consumer search models change qualitatively with such learning.

To achieve this goal, we have made several important assumptions that can be relaxed and or modified in future work. First, we followed Anderson and Renault (1999) in assuming that all consumers buy. This assumption greatly increases the tractability of the search model and simplifies learning across individuals. In its defense, it should be noted that an extensive margin of demand should push prices downwards. Since our main results concern surprisingly low prices, an active extensive margin would reinforce these results. Moreover, it allows us to concentrate on the *business-stealing* effect of information, since the amount of purchases is kept constant.

Second, we have introduced correlation across consumers' valuations in the simplest feasible way. If we went one step further in the direction of simplicity, and assumed that *all* consumers of the same type have the same utility, the equilibrium search rule would not have standard reservation utility property. This is because, in a putative cutoff-strategy equilibrium, the purchasing decision of a predecessor is more informative for utilities just below the cutoff and so search is less valuable there. Therefore, consumers would want to stop for utilities below the cutoff. As a result, the equilibrium would have to feature an interval of utilities where probability of searching further transitions smoothly from one to zero. Although very interesting, this search rule renders the full model less tractable. Moving in the direction of more general correlation of utilities within types is also hard to model since consumers' learning about the distribution in other stores is highly non-linear and their beliefs are formed by mixtures of truncated distributions. Hence, our model is a simple, yet appealing, compromise. We believe that pushing in either direction would be important for our further understanding of observational learning in search markets.

Finally, we have for the most part abstracted away from dynamic pricing considerations. It is highly intuitive that firms would take advantage of consumers' learning and change their prices accordingly. While we have addressed this issue in a somewhat rudimentary fashion by introducing a period where learning has been completed, price changes during the learning process may be interesting, but are fairly complicated to handle.

A Appendix 1

The following two lemmata prove that the stationary equilibrium we study can be reached as the limit of the dynamic economy of the model as N grows.

Lemma 1. *Suppose there is a unique cutoff $\hat{w}(p_i)$. Then, for every $\epsilon > 0$, there exists a $T* < \infty$ such that for all states and prices $\|w(p_i) - \hat{w}(p_i)\| < \epsilon$.*

Proof. The idea for the argument follows the ideas of Lemma 2 in Thomas and Cripps (2014), although our model is much simpler. Let $M_1(p_1, p^*)$ and $M_2(p_1, p^*)$ represent the probabilities that a consumer following $\hat{w}$ buys from firm 1 if he visits this firm first and second respectively. These probabilities are independent of t. The evolution of market shares can be readily computed as

$$|x^t - x^{t-1}| = |x^{t-1} - x^{t-2}|(M_1(p_1, p_2) - M_2(p_1, p_2)) \tag{16}$$

Notice that that $M_1 - M_2 \leq \max_{j,S}(1 - G_S(w(p_j))$. If $\min_{j,S} G_S(w(p_j)) = 0$, then market share for firm 1 is either 0 or 1 (one of the firms is an absorbing state). In such a case, the result holds trivially. Otherwise,[7]

$$|x^t - x^{t-1}| \leq \{\max_{j,S}(1 - G_S(w(p_j))\}^t |x^1 - x^0| \tag{17}$$

or

$$|x^t - x^{t-1}| \leq \{\max_{S}(1 - G_S(w(p_j))\}^t |x^W - \frac{1}{2}| \tag{18}$$

where x^W are the shares of the "wolinsky consumer" (i.e. the first consumer). Clearly, this Fixed Point converges to the "stationary market shares" by Blackwell's Sufficiency Conditions and the parameter of convergence is $\{\max_{j,S}(1 - G_S(w(p_j))\}$ which is strictly less than one. Since λ and x are continuous functions, for every $\epsilon > 0$, there exists $T_1 < \infty$ such that $T_1 = \ln(\frac{\epsilon}{\max_{j,S}(1-G_S(w(0))})$ and, therefore $\|q_{T^*}(u;p) - q^*(u;p)\| < \frac{\epsilon}{2}$. Let $T^* = LT_1$, for L large enough we have that

$$\sup \| \sum_{t=1}^{T^*} \frac{1}{T^*} q_t(u,p) \| < \frac{(L-1)\epsilon + 2}{2L} < \epsilon \tag{19}$$

Because (2) defines a contraction mapping between q and $\hat{w}(p)$, the result follows. □

Lemma 2. *If $\lambda'(u) < \frac{1}{\int u(g_H(u) - g_L(u))du}$, there exists a unique $w(p_i)$ satisfying equation (2).*

Proof. Taking derivatives in (2) we get

$$q'(u) \int_{w-p+p^*}^{\infty} (u - w + p - p^*)(g_H(u) - g_L(u)) - (1 - q(u)G_H(w) - (1 - q(u)G_L(w))) > 0 \tag{20}$$

[7] In equilibrium, $G_S(w(p^*)) > 0$ if $c < \bar{c}_L = \int u dG_L(u)$.

where $q'(u) < \lambda'(u)$. Rewriting we get

$$\lambda'(u) < \frac{1 - (q(u)G_H(w) + (1 - q(u))G_L(w))}{\int_{w-p+p^*}^{\infty}(u - w + p - p^*)} \tag{21}$$

RHS is a decreasing function of w, so the supremum is attained at $w = \underline{u}$. This condition is satisfied for all triangular distributions and for normal distributions with small enough different in means. TBC □

Proof of Proposition 2

Proof. First, in the case where consumers follow market shares, we need to compute demand as a fixed point of the market flow equation. In particular, given a conjecture price p^* for the opponent firm and a reservation utility $w(p) = \hat{w} + p - p^*$, the market share of a firm charging p if the rival sticks to equilibrium price is

$$x = 1 - \frac{G(w(p)) - \int_{\underline{u}}^{\hat{w}} G(u - p + p^*)dG(u)}{G(w(p)) + G(\hat{w})(1 - G(w(p))) - \int_{\underline{u}}^{\hat{w}} G(u - p + p^*)dG(u) + \int_{\underline{u}}^{w(p^*)} G(u - p + p^*)dG(u)}$$

taking derivatives with respect to p, equating $p = p^*$ and, therefore, $\hat{w} = w(p)$ and substituting $w'(p) = 1$

$$x'(p) = \frac{2\int_{\underline{u}}^{\hat{w}} g(u)^2 du + (1 - G(\hat{w}))G'(\hat{w})}{G(\hat{w})(2 - G(\hat{w}))} \tag{22}$$

Since, in a symmetric equilibrium, $x = \frac{1}{2}$, the pricing equation

$$x'(p)p + x(p) = 0 \tag{23}$$

is solved by

$$p^* = \frac{G(\hat{w})(2 - G(\hat{w}))}{2\int_{\underline{u}}^{\hat{w}} g(u)^2 du + (1 - G(\hat{w}))G'(\hat{w})} \tag{24}$$

It is easy to see that the Second Order Condition is satisfied for log-concave distributions (show). The price for the *ARW* model was derived in Anderson and Renault (1999). Simple inspection shows that the price is higher in the *ARW* model, for any $\hat{w}$. □

Proof of Proposition 3

Proof. Form (2), it is obvious that w is continuous and decreasing in c. Hence, it suffices to show that $p^*(\underline{u}) < p^*(\bar{u})$. To see this, notice that

$$p^*(\bar{u}) = \bar{p}(\underline{u}) = \frac{1}{2\int_{\underline{u}}^{\bar{u}} g^2(u)du} \tag{25}$$

That is, the price is inversely proportional to the "mean density". Clearly, for all continuous distributions, $p^*(\bar{u}) > 0$. On the other hand $p^*(\underline{u})$ satisfies

$$p^*(\underline{u}) = \frac{2G(\underline{u})}{g(\underline{u})} \tag{26}$$

which is zero for all distributions with finite lower bound. □

Proof of Corollary 1

Proof. $G_L(u)$ is stochastically increasing in r, meaning that $\bar{c}_L$ is decreasing in r. Since $\hat{p}$ is strictly positive for $c < \bar{c}_L$ and $p(\bar{c}_L; r) = 0$, the result follows. □

Proof of Proposition 5

Proof. As shown earlier for $n = 2$, price is lower in our model with emulation then in the ARW model. This is also true for any n because $\frac{nG(\hat{w})-G(\hat{w})^n}{(n-1)} < 1$. To see this notice that $\frac{nG(\hat{w})-G(\hat{w})^n}{(n-1)} < 1$ is equivalent to

$$n > \frac{1 - G^n(\hat{w})}{1 - G(\hat{w})} \tag{27}$$

$$= \sum_{j=0}^{n-1} G^j(\hat{w}) \tag{28}$$

but $\sum_{j=0}^{n-1} G^j(\hat{w}) < \sum_{j=0}^{n-1} 1 = n$. To see the second part notice that

$$\frac{nG(\hat{w}) - G^n(\hat{w})}{n-1} \leq \frac{(n-1)G(\hat{w}) - G^{(n-1)}}{n-2} \tag{29}$$

since

$$G(\hat{w})\frac{(n-1)^2 - n(n-2)}{(n-1)(n-2)} \geq G^{(n-1)}(\hat{w})\frac{(n-1) - G(n-2)}{(n-1)(n-2)} \tag{30}$$

which can be rewritten as

$$n \geq G(\hat{w})^{n-2}(n(1 - G(\hat{w})) - (1 - 2G(\hat{w}))) \tag{31}$$

for $n \geq 2$. It is easy to see that a sufficient condition is

$$nG(\hat{w}) \geq 2G(\hat{w}) - 1 \tag{32}$$

which holds trivially for $n \geq 2$. Finally for any number of firms, if $\underline{u} > -\infty$, prices must decrease in search cost. To see this notice that the price decreases in n and $\hat{w}$ is independent of n so that $p^*_\infty \leq p^*_n \leq p^*$. But since $p^*_\infty > 0$ for $c \in (0, \bar{c}_L)$ and $\lim_{c \to \bar{c}_L} p^* = 0$, $p^*_n > p^*_{n+1}$ for every n, $0 < c < \bar{c}_l$. □

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