Search Costs and Investment in Quality*

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Abstract

In this paper we analyze how lower search costs affect firms' incentives to invest in quality. We identify two conflicting effects. On the one hand, lower search costs increase incentives to invest in quality by eroding the market share of low quality firms and increasing the market share of high quality firms. On the other hand, by intensifying price competition, lower search costs adversely affect high quality firms more than low quality firms. The net effect of a change in the search cost on quality is shown to depend on the initial quality distribution. There is a critical value such that, if the average quality in the market is initially above this value, lower search costs lead to lower quality, whereas if the initial quality is below this value, lower search cost leads to higher quality. We show that our results are consistent with a "superstar effect".

keywords: search, internet search, quality, investment, superstar effect

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1 Introduction

One striking development associated with the explosion of e-commerce is the increased transparency of sellers' quality. Sites like yelp.com, tripAdvisor.com and cnet.com in which consumers and professionals rate the quality of a wide variety of products and services, facilitate comparison of competing vendors by new consumers. Indeed, consumers seem to have become increasingly dependent on such sources. For example, according to a survey by Forrester Research,[1] some 86% of respondents use ratings and reviews for online purchases and 44% go online before buying products in-store. Chevalier and Mayzlin (2006) find that an improvement in a book's reviews on Amazon.com significantly increases relative sales at that site.

In this paper we explore the impact of such technological innovation on sellers' incentives to invest in quality. It seems natural to suppose that as it becomes less costly for consumers to search for quality products, the negative consequences for firms which fail to deliver superior quality become more severe. Indeed, Jarvis (2009) documents a striking example in support of this intuition. Frustrated over the poor performance and service of a dell laptop, he posted a blog with the title 'dell sucks'. The blog went viral, produced a 'blogstorm' of similar complaints, which eventually led to a total transformation of Dell's customer support division. Jarvis argues persuasively and at length about how unlikely such a development would have been in the pre internet era.

Such anecdotes suggest that lower search costs increase incentives to invest in quality by eroding the market share of low quality firms and increasing the market share of high quality firms. However, lower search costs may also intensify price competition for both low and high quality goods. If this effect reduces the profits of high quality firms more those of low quality firms, then reducing search costs might lower incentives to invest.

To explore this issue formally, we develop a consumer search model in which firms sell differentiated products (as in Wolinsky (1986) and Anderson and Renault (1999)) and invest in product quality. Consumers search for both (subjective) match value and (objective) high quality. We derive the symmetric equilibrium of the model and study how the distribution of realized quality changes with the search cost. Our main result is that the effect of a change in the search cost on the proportion of high quality firms depends on the initial quality

[1] ComputerWeekly.com, November 10, 2010.

distribution. Specifically, there is a critical value such that if the proportion of high quality firms is initially below this value, lower search costs increase this proportion, whereas if the initial quality is above this value, lower search cost decreases the proportion of high quality firms.

The explanation is as follows. In our model, high quality firms have a higher market share than lower quality firms. Therefore the downward pressure on prices from lower search costs reduces the profits of high quality firms more than those of low quality firms. This effect, the *revenue effect*, dampens the incentive to invest in quality. On the other hand, lower search costs increase the gap between the market shares of high and low quality firms. This effect, the *quantity effect*, increases the incentive to invest.

If the initial proportion of high quality firms is small, and the search cost is reduced, a relatively large number of consumers switch from low to high quality firms and is distributed over a relatively small number of high quality firms. Then the change in quantity per high quality firm is large, therefore the quantity effect on high quality firms is sufficiently large to offset the revenue effect and thus the proportion of high quality firm increases.

In contrast, if the initial percentage of high quality firms is large, and the search cost is reduced, the total number of consumers who switch from low quality to high quality firms is relatively small and this number is distributed over a large number of high quality firms. In that case, the increase in quantity per high quality firm is small, therefore the revenue effect dominates and therefore the proportion of high quality firms is reduced.

The latter case is consistent with the view that the internet leads to more competitive markets dominated by a small number of high demand firms – the "superstar effect" (Brynjolfsson, Hu and Smith (2010), Goldmanis, Hortacsu, Syverson and Emre (2010), Bar-Isaac, Caruana and Cuñat (2012)). In our model, lower search costs which makes it easier for consumers to find better firms and increase the amount sold by each high quality firm, also increases competition, reducing the return to investment and hence the number of high quality firms.

More broadly, our paper is a part of a recent search literature which analyzes the effect of changes in search costs on product design (for example Kuksov (2004), Larson (2011)). We differ from the bulk of this literature by focusing explicitly on the relationship between search costs and investment in quality.

Less directly, we are also related to a large theoretical and empirical literature on the effect of competition and consumer information on firms' investment and quality (e.g. Schmidt (1997), Aghion et. al. (2005), Hörner (2002), Kranton (2003), Bar-Isaac (2005) with respect to the effect of competition and Dranove, Kessler, McClellan and Satterthwaite (2003) and Moav and Neeman (2010) on the effect of consumers' information[2]).

2 Model

There is a continuum of firms and consumers. Firms' products are both horizontally and vertically differentiated. Specifically, there are two qualities, low and high, where high quality is denoted by $\overline{v}$ and low quality is denoted by $\underline{v} < \overline{v}$. Let the proportion of high quality firms be μ and the proportion of low quality firms be $1 - \mu$. The unit cost of production is c irrespective of quality.[3] However, firms' ability to produce high quality depends on a prior investment as we explain in detail below. For the moment we take μ to be exogenous and later endogenize it.

A consumer which buys a unit of quality $v \in \{\underline{v}, \overline{v}\}$, for a price p derives a utility:

$$v + \varepsilon - p$$

where ε is an idiosyncratic match value, distributed according to a CDF $G(\varepsilon)$, which is assumed to be uniform with support $[-k, k]$, where k measures the extent of consumer heterogeneity.

Consumers have unit demand. Consumers do not know firms' match values, product qualities or price ex ante but can learn them by searching sequentially, where each search costs the consumer $s > 0$.

Let $\overline{p}$ and $\underline{p}$ be the price of high and low quality firms respectively (assuming that firms with the same quality charge the same price). Under these assumptions, it is well known that consumers' optimal search strategy is characterized by a reservation value, u, a function of (s, μ), such that she stops searching and buys only if she finds a firm which provides her with utility greater or equal to u, and continues searching otherwise, where u is defined by:

[2] Relatedly, Ater and Orlov (2010) show that increased price information enabled by internet penetration, led to lower product quality, as measured by scheduled flight times and flight delays.

[3] It can be shown that none of the results below are changed if the unit production cost of high quality is $\overline{c}$ and the unit production cost of low quality is $\underline{c} < \overline{c}$, as long as $\overline{v} - \overline{c} > \underline{v} - \underline{c}$.

$$\mu \int_{\overline{\varepsilon}}^{k} (\overline{v} + \varepsilon - \overline{p} - u)\, g(\varepsilon)\, d\varepsilon + (1-\mu) \int_{\underline{\varepsilon}}^{k} (\underline{v} + \varepsilon - \underline{p} - u)\, g(\varepsilon)\, d\varepsilon = s \tag{1}$$

where $g(\varepsilon) = G'(\varepsilon)$. Here $\overline{\varepsilon}$ is a consumer's reservation match value (the minimum match value she requires to stop searching) for a high quality firm and $\underline{\varepsilon}$ is the reservation match values for a low quality firm. The LHS of the preceding equation is the expected utility, over and above u, that a consumer obtains from searching once more and the RHS is the cost of an additional search.

The above equation implies that

$$\begin{aligned} \overline{\varepsilon} &= \overline{p} + u - \overline{v} \\ \underline{\varepsilon} &= \underline{p} + u - \underline{v}. \end{aligned} \tag{2}$$

Let $\overline{Q}$ and $\underline{Q}$ and $\overline{\pi}$ and $\underline{\pi}$ be the expected quantities and profits of firms with qualities $\overline{v}$ and $\underline{v}$ respectively. Then

$$\begin{aligned} \overline{\pi} &= (\overline{p} - c)\overline{Q} \\ \underline{\pi} &= (\underline{p} - c)\, \underline{Q}. \end{aligned} \tag{3}$$

Since a firm of quality $\overline{v}$ ($\underline{v}$) only sells to consumers to whom it provides a match value greater or equal than $\overline{\varepsilon}$ ($\underline{\varepsilon}$),

$$\begin{aligned} \overline{Q} &= \left[1 - \overline{G}\right] M \\ \underline{Q} &= [1 - \underline{G}]\, M, \end{aligned} \tag{4}$$

where $\overline{G} = G(\overline{\varepsilon})$, $\underline{G} = G(\underline{\varepsilon})$ and where M is the proportion of consumers that sample a firm. Since the number of consumers who arrive at any firm equals the mean number of searches per consumer (which has a geometric distribution),

$$M = \frac{1}{\mu\left[1 - \overline{G}\right] + (1-\mu)\left[1 - \underline{G}\right]}. \tag{5}$$

We can now derive the equilibrium prices. A firm with quality v and price p sells only to consumers for whom $v + \varepsilon - p \geq u$, or $\varepsilon > u - v + p$, Thus, the firm solves:

$$\max_{p}\, (p - c)\left[1 - G(u - v + p)\right]$$

where u is defined by (1). The first-order condition gives:

$$p = c + \frac{1 - G(u - v + p)}{g(u - v + p)}. \tag{6}$$

Substituting $G(\varepsilon) = \frac{\varepsilon + k}{2k}$ and $g(\varepsilon) = \frac{1}{2k}$ and solving for p gives:

$$p = c + k - u + v - p.$$

In equilibrium

$$\begin{aligned} \overline{p} &= \frac{k - u + \overline{v} + c}{2}, \\ \underline{p} &= \frac{k - u + \underline{v} + c}{2}. \end{aligned} \tag{7}$$

Substituting in (2), gives

$$\begin{aligned} \overline{\varepsilon} &= \frac{k + u - \overline{v} + c}{2} \\ \underline{\varepsilon} &= \frac{k + u - \underline{v} + c}{2}. \end{aligned} \tag{8}$$

Note that $\overline{\varepsilon} < \underline{\varepsilon}$ implies $\underline{G} > \overline{G}$.

3 Investment in quality

Up to now we've assumed that the proportion of high quality firms μ is fixed. We now endogenize μ and assume that by investing $c(\widehat{\mu})$ a firm produces high quality products with probability $\widehat{\mu}$, where $c'(\widehat{\mu}) \geq 0$ is continuously differentiable, $c''(\widehat{\mu}) \geq 0$ and $c'(1) = \infty$.

Denote by μ the equilibrium probability of high quality. For each firm, the expected (gross) profit from investing $c(\widehat{\mu})$ given μ and the search cost s is:

$$R(\widehat{\mu}; \mu, s) = \widehat{\mu} \times \overline{\pi}(\mu, s) + (1 - \widehat{\mu}) \times \underline{\pi}(\mu, s) \tag{9}$$

The firm's choice of investment is a solution to

$$\max_{\widehat{\mu}} R(\widehat{\mu}; \mu, s) - c(\widehat{\mu})$$

The optimal investment, if positive, is given by equating the marginal return on investment, $\overline{\pi} - \underline{\pi}$, to the marginal cost:

$$\overline{\pi}(\mu, s) - \underline{\pi}(\mu, s) - c'(\widehat{\mu}) = 0.$$

The second-order condition is clearly satisfied since $c'' > 0$. A necessary condition for an interior solution ($\mu > 0$) is

Assumption 1 $c'(0) < \overline{\pi}(0, s) - \underline{\pi}(0, s)$.

In a symmetric equilibrium, all firms choose $\widehat{\mu} = \mu$ and thus μ is the proportion of high-quality firms. Thus the equilibrium μ is determined by:

$$\overline{\pi}(\mu, s) - \underline{\pi}(\mu, s) - c'(\mu) = 0. \tag{10}$$

Our main purpose is to determine how a change in the search cost affects investment in quality. If $d\mu/ds > 0$, increasing the search cost increases the average quality in the market. If $d\mu/ds < 0$, increasing the search cost reduces average quality in the market.

Differentiating equation (10), and by the Implicit Function Theorem we get:

$$\frac{d\mu}{ds} = -\frac{\partial\left(\overline{\pi} - \underline{\pi}\right)/\partial s}{\partial\left(\overline{\pi} - \underline{\pi}\right)/\partial\mu - c''(\mu)}. \tag{11}$$

The numerator is the change in the marginal return on investment with respect to the search cost, for a given μ, where

$$\frac{\partial\overline{\pi}}{\partial s} = \frac{\partial\overline{p}}{\partial s}\overline{Q} + \left[\overline{p} - c\right]\frac{\partial\overline{Q}}{\partial s}, \tag{12}$$

$$\frac{\partial\underline{\pi}}{\partial s} = \frac{\partial\underline{p}}{\partial s}\underline{Q} + \left[\underline{p} - c\right]\frac{\partial\underline{Q}}{\partial s}. \tag{13}$$

The first term on the RHS of (12) and (13) is the effect of the search cost on the firms' revenue, keeping quantity fixed (the revenue effect). The second term is the effect of the search cost on the quantity sold, keeping prices fixed (the quantity effect).

Consider first the revenue effect. By (7),

$$\frac{\partial\overline{p}}{\partial s} = \frac{\partial\underline{p}}{\partial s} = -\frac{1}{2}\frac{\partial u}{\partial s}$$

and by (1), $\partial u/\partial s < 0$ and thus $\partial\overline{p}/\partial s = \partial\underline{p}/\partial s > 0$. Intuitively, when the search cost increases, consumers settle for a lower match value which lowers their reservation utility. Firms' market power thus increases and equilibrium prices increase. By (4), $\overline{Q} > \underline{Q}$. Thus, since the price of both qualities increases by the same amount, and high quality firms sell a greater quantity, *the revenue effect is greater for high quality firms.*

Consider next the quantity effect. Differentiating (4),

$$\begin{aligned}\frac{\partial \overline{Q}}{\partial s} &= -g\left(\overline{\varepsilon}\right)\frac{\partial \overline{\varepsilon}}{\partial s}M + \left[1-\overline{G}\right]\frac{\partial M}{\partial s},\\ \frac{\partial \underline{Q}}{\partial s} &= -g\left(\underline{\varepsilon}\right)\frac{\partial \underline{\varepsilon}}{\partial s}M + \left[1-\underline{G}\right]\frac{\partial M}{\partial s},\end{aligned}$$

where

$$\begin{aligned}\frac{\partial M}{\partial s} &= \frac{1}{2k}\frac{\mu\frac{\partial \overline{\varepsilon}}{\partial s} + (1-\mu)\frac{\partial \underline{\varepsilon}}{\partial s}}{\left[\mu\left(1-\overline{G}\right) + (1-\mu)\left(1-\underline{G}\right)\right]^2}\\ &= \frac{1}{4k}\frac{\frac{\partial u}{\partial s}}{\left[\mu\left(1-\overline{G}\right) + (1-\mu)\left(1-\underline{G}\right)\right]^2}.\end{aligned}$$

After some simplifications, we obtain

$$\begin{aligned}\frac{\partial \overline{Q}}{\partial s} &= \frac{1}{4k}\frac{\partial u}{\partial s}\frac{(1-\mu)\left[\underline{G}-\overline{G}\right]}{\left[\mu\left(1-\overline{G}\right) + (1-\mu)\left(1-\underline{G}\right)\right]^2} < 0\\ \frac{\partial \underline{Q}}{\partial s} &= \frac{1}{4k}\frac{\partial u}{\partial s}\frac{-\mu\left[\underline{G}-\overline{G}\right]}{\left[\mu\left(1-\overline{G}\right) + (1-\mu)\left(1-\underline{G}\right)\right]^2} > 0.\end{aligned} \tag{14}$$

Thus, when the search cost increases, *the quantity effect is negative for high quality firms and positive for low quality firms* (and the reverse when the search cost decreases). Intuitively, when the search cost increases, consumers become less selective and settle for a lower match value. In particular this means that they are more likely to accept low quality and therefore the quantity sold by low quality firms increases and the quantity sold by high quality firms decreases.

Because the revenue effect is greater for the high quality firms whereas the quantity effect is greater for the low quality firms, the net effect of a change in the search cost on the (marginal) return on investment is ambiguous. The following lemma establishes that the sign of the net effect depends on whether or not the initial proportion of high quality firms in the market exceeds the proportion of low quality firms.

Lemma 1

1. $\frac{\partial\left(\overline{\pi}-\underline{\pi}\right)}{\partial s} > 0$ *if* $\mu > 0.5$ *and* $\frac{\partial\left(\overline{\pi}-\underline{\pi}\right)}{\partial s} < 0$ *if* $\mu < 0.5$.

2. $\overline{\pi}\left(0.5, s\right) - \underline{\pi}\left(0.5, s\right) \equiv \Delta$, *where* Δ *is constant with respect to* s.

Thus, when there are more high quality firms than low quality firms, the revenue effect dominates the quantity effect. Intuitively, when $\mu > 0.5$, then most consumers are initially matched with high quality firms. In that case when the search cost is reduced, only a relatively small number of consumers switch from low to high quality firms and this number is distributed over a large number of high quality firms. Thus, the quantity effect per high quality firm is relatively small. In contrast, when μ is small a relatively large number of consumers switch to high quality firms, and are distributed over a relatively small number of high quality firms. Then, the quantity effect is relatively large.

Recall that the parameter k measures the extent of the horizontal heterogeneity in consumer preferences. The following proposition, our main result, states that there is a unique investment equilibrium if consumer preferences are sufficiently heterogenous.

Proposition 1 *If k is sufficiently large, there is a unique equilibrium, characterized as follows:*

1. *If $\Delta \geq c'(0.5)$ then $\mu \geq 0.5$ and $d\mu/ds > 0$.*
2. *If $\Delta < c'(0.5)$ then $\mu < 0.5$ and $d\mu/ds < 0$.*

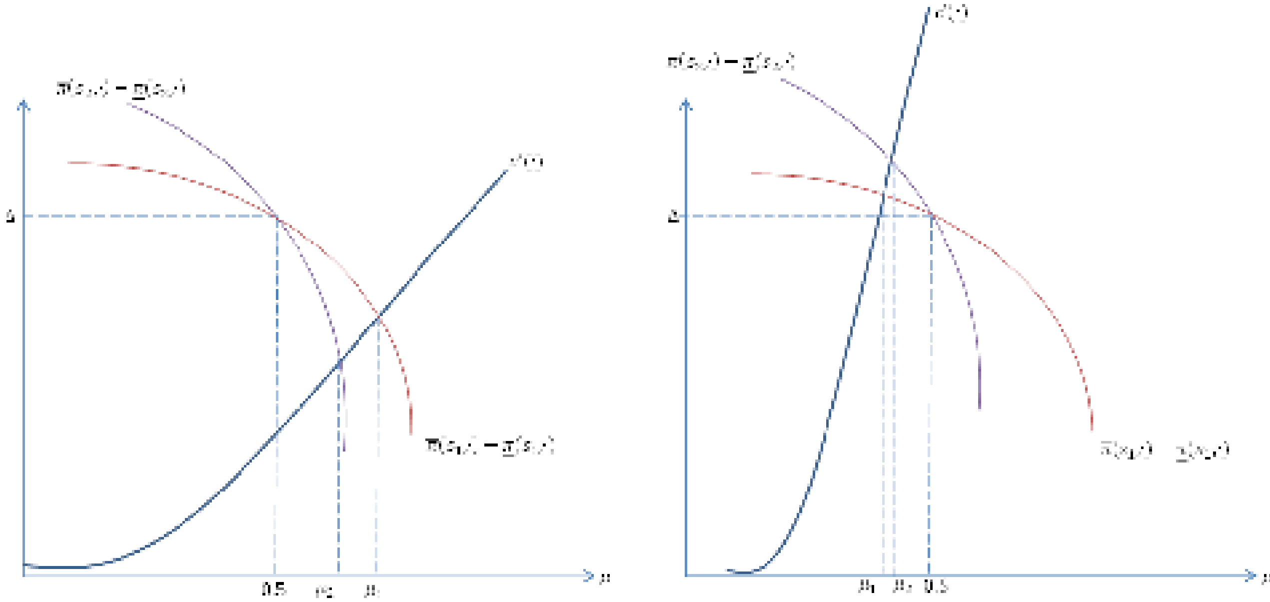
Proof. In the appendix (Lemma 2) we prove that $\partial\left(\overline{\pi} - \underline{\pi}\right)/\partial\mu < 0$ everywhere if k is sufficiently large. Recall that

$$\frac{d\mu}{ds} = -\frac{\partial\left(\overline{\pi} - \underline{\pi}\right)/\partial s}{\partial\left(\overline{\pi} - \underline{\pi}\right)/\partial\mu - c''(\mu)}$$

Thus it follows directly from Lemma 1 that $\frac{d\mu}{ds} > 0$ if and only if $\mu > 0.5$. Since $\overline{\pi} - \underline{\pi}$ and c' intersect only once, the intersection is to the right of $\mu = 0.5$ if and only if $\Delta > c'(0.5)$. ■

The results of Proposition 1 are illustrated in Figure 1. It follows from Lemma 1 that if the search cost decreases from s_1 to $s_2 < s_1$, the return on investment $\overline{\pi} - \underline{\pi}$ rotates around $\mu = 0.5$. The left panel shows case 1, where $\Delta \geq c'(0.5)$. In that, the average quality decreases from μ_1 to μ_2. The right panel shows case 2, where $\Delta < c'(0.5)$. Here, the average quality increases from μ_1 to μ_2

The effect of changing the search cost on investment depends on the initial quality, which ultimately depends on the curvature of the investment function. If the cost of investment increases sufficiently slowly, so that initially most firms have high quality (consistent with



(1) $\Delta > c'(0.5)$ (2) $\Delta < c'(0.5)$

Figure 1: Change in average quality when search cost decreases from s_1 to s_2

Case 1 of the proposition), then decreasing the search cost always leads to lower average quality. Conversely, if the cost of investment rises rapidly (consistent with Case 2 of the proposition), then decreasing the search cost always leads to higher average quality.[4]

A common view is that the cheaper access to information on the internet reduces the number of, and increase the market share of high-demand firms – a phenomena known as the "superstar effect" (see the references cited in the introduction).

[4] Although driven by a very different logic and in a very different setting, there is a similarity between our result and Aghion et al (2005). They find both theoretical and empirical support for an inverted-U shape relationship between competition – as measured by the ability of firms collude – and innovation, such that increased competition leads to more innovation if the initial level of competition is low and the reverse if the initial level is high (see also Schmidt (1997)).

In our framework, there are two channels which determine the intensity of competition. One is the magnitude of the search costs s, and the other is the proportion of high quality firms μ. Both a decrease in s and an increase in μ intensifies competition by motivating consumers to search more intensely. As is established by the proposition, when the initial μ is low (i.e. $\mu \leq 0.5$) corresponding to a low intensity of competition, further intensifying competition by lowering the search cost, leads to more investment in quality and the reverse if the initial μ is high.

In our setting, the high-quality firms are the high-demand firms. Thus, in our context, there is a superstar effect if a decrease in the search cost reduces the number of the high quality firms and increases the market share of the remaining ones.

The next proposition shows that our results are consistent with a superstar effect, whenever the initial number of high quality firms is greater than 0.5.

Proposition 2 (Superstar effect) *If* $\Delta \geq c'(0.5)$ *then if* s *decreases (i) the proportion of high quality firms* μ *decreases and (ii) sales per high quality firm* $\overline{Q}$ *increases.*

Proof.

(i) From Proposition 1.

*(ii)*The total change in sales per high quality firm resulting from a change in the search cost is

$$\frac{d\overline{Q}}{ds} = \frac{\partial\overline{Q}}{\partial s} + \frac{\partial\overline{Q}}{\partial\mu}\frac{d\mu}{ds}.$$

By Proposition 1, $\Delta \geq c'(0.5) \implies \frac{d\mu}{ds} > 0$ and $\mu > 0.5$. It is shown in the proof of step 2 of Lemma 2 (in the appendix) that $\frac{\partial\overline{Q}}{\partial\mu} < 0$ in this case. Hence the second term on the right-hand side of the above equation is negative. Finally $\frac{\partial\overline{Q}}{\partial s} < 0$ by (14) above. Hence $\frac{d\overline{Q}}{ds} < 0$. ■

The preceding result is complementary to a related result by Bar-Isaac et al. (2012). In their model quality is exogenous and firms choose between broad and niche horizontal product characteristics ("designs"). The higher quality firms choose the broad design (and have higher sales) and the lower quality firms choose niche designs. As search costs decrease, the quality threshold above which the broad design is chosen increases and the market shares of those firms increase. Thus, in their model, the superstar effect reflects a change in the firms' choice of the horizontal aspect. By contrast, in our setup, the horizontal product characteristic is exogenous and firms choose product quality. The superstar effect derives from a change in the choice of the vertical aspect.

4 Extensions and concluding remarks

We conclude with a brief discussion of some possible extensions.

One key assumption of the analysis is that all consumers are ex ante identical. One implication of this assumption is that the market is always fully covered. Suppose alternatively that consumers have heterogenous fixed costs of participating in the market (for example, a fixed cost of learning how to search online). Then, consumers with higher "entry costs" require higher expected consumer surplus from search (net of entry costs) to enter the market than those with lower costs, and thus, when search costs decrease, the proportion of active consumer will generally increase. Since high quality firms will capture a higher proportion of these new entrants and at a higher margin than the low quality firms, this additional effect further increases the difference between the profits of high and low quality firms. Thus, when the market is not fully covered, the pro-investment effect of lower search cost is enhanced.

Another interesting extension is to consider alternative settings in which, in contrast to the above analysis, low quality firms have larger market shares than high quality firms. This could be the case if, for example, consumers differ with respect to their willingness to pay for high quality, and a sufficiently large proportion of consumers have only low willingness to pay. In this case, our preceding analysis suggests that the (negative) revenue effect of lower search costs might be greater for low quality firms. In that case, both the revenue and quantity effects would work in the same pro-investment direction and thus lower search costs would always lead to higher quality.

5 Appendix

Proof of Lemma 1. Differentiating the profits in (3) with respect to s gives:

$$\begin{aligned} \frac{\partial \overline{\pi}}{\partial s} &= \frac{\partial \overline{p}}{\partial s} \cdot \overline{Q} + [\overline{p} - c] \frac{\partial \overline{Q}}{\partial s} \\ \frac{\partial \underline{\pi}}{\partial s} &= \frac{\partial \underline{p}}{\partial s} \cdot \underline{Q} + [\underline{p} - c] \frac{\partial \underline{Q}}{\partial s} \end{aligned}$$

By (4) and as $\partial\overline{\varepsilon}/\partial s = \partial\underline{\varepsilon}/\partial s = 1/2 \cdot \partial u/\partial s$,

$$\begin{aligned} \frac{\partial \overline{Q}}{\partial s} &= -Mg(\overline{\varepsilon}) \frac{\partial \overline{\varepsilon}}{\partial s} + \left(1 - \overline{G}\right) \frac{\partial M}{\partial s} = -\frac{M}{4k} \frac{\partial u}{\partial s} + \left(1 - \overline{G}\right) \frac{\partial M}{\partial s} \\ \frac{\partial \underline{Q}}{\partial s} &= -Mg(\underline{\varepsilon}) \frac{\partial \underline{\varepsilon}}{\partial s} + \left(1 - \underline{G}\right) \frac{\partial M}{\partial s} = -\frac{M}{4k} \frac{\partial u}{\partial s} + \left(1 - \underline{G}\right) \frac{\partial M}{\partial s} \end{aligned}$$

where

$$\begin{aligned} \frac{\partial M}{\partial s} &= \frac{\mu g(\overline{\varepsilon}) \frac{\partial \overline{\varepsilon}}{\partial s} + (1-\mu) g(\underline{\varepsilon}) \frac{\partial \underline{\varepsilon}}{\partial s}}{\left[\mu\left(1 - \overline{G}\right) + (1-\mu)\left(1 - \underline{G}\right)\right]^2} \\ &= \frac{\frac{1}{4k}\frac{\partial u}{\partial s}}{\left[\mu\left(1 - \overline{G}\right) + (1-\mu)\left(1 - \underline{G}\right)\right]^2} \end{aligned}$$

Thus,

$$\begin{aligned} \frac{\partial \overline{Q}}{\partial s} &= \frac{1}{4k}\frac{\partial u}{\partial s} \frac{(1-\mu)\left[\underline{G} - \overline{G}\right]}{\left[\mu\left(1 - \overline{G}\right) + (1-\mu)\left(1 - \underline{G}\right)\right]^2} < 0 \\ \frac{\partial \underline{Q}}{\partial s} &= \frac{1}{4k}\frac{\partial u}{\partial s} \frac{-\mu\left[\underline{G} - \overline{G}\right]}{\left[\mu\left(1 - \overline{G}\right) + (1-\mu)\left(1 - \underline{G}\right)\right]^2} > 0 \end{aligned}$$

By (7),

$$\frac{\partial \overline{p}}{\partial s} = \frac{\partial \underline{p}}{\partial s} = -\frac{1}{2}\frac{\partial u}{\partial s}$$

and thus

$$\frac{\partial \overline{\pi}}{\partial s} = -\frac{1}{2}\frac{\partial u}{\partial s}\left[\frac{1 - \overline{G}}{\mu\left(1 - \overline{G}\right) + (1-\mu)\left(1 - \underline{G}\right)} - (\overline{p} - c)\frac{1}{2k}\frac{(1-\mu)\left[\underline{G} - \overline{G}\right]}{\left[\mu\left(1 - \overline{G}\right) + (1-\mu)\left(1 - \underline{G}\right)\right]^2}\right]$$

$$= -\frac{1}{2}\frac{\partial u}{\partial s} \cdot \frac{\left(1 - \overline{G}\right)\left[\mu\left(1 - \overline{G}\right) + (1-\mu)\left(1 - \underline{G}\right)\right] - \frac{1}{2k}(\overline{p} - c)(1-\mu)\left[\underline{G} - \overline{G}\right]}{\left[\mu\left(1 - \overline{G}\right) + (1-\mu)\left(1 - \underline{G}\right)\right]^2}$$

and

$$\frac{\partial \underline{\pi}}{\partial s} = -\frac{1}{2}\frac{\partial u}{\partial s}\left[\frac{1 - \underline{G}}{\mu\left(1 - \overline{G}\right) + (1-\mu)\left(1 - \underline{G}\right)} + (\underline{p} - c)\frac{1}{2k}\frac{\mu\left[\underline{G} - \overline{G}\right]}{\left[\mu\left(1 - \overline{G}\right) + (1-\mu)\left(1 - \underline{G}\right)\right]^2}\right]$$

$$-\frac{1}{2}\frac{\partial u}{\partial s}\cdot\frac{(1-\underline{G})\left[\mu\left(1-\overline{G}\right)+(1-\mu)(1-\underline{G})\right]+\frac{1}{2k}\left(\underline{p}-c\right)\mu\left[\underline{G}-\overline{G}\right]}{\left[\mu\left(1-\overline{G}\right)+(1-\mu)(1-\underline{G})\right]^2}$$

Denoting $Z=(1-\underline{G})\left[\mu\left(1-\overline{G}\right)+(1-\mu)(1-\underline{G})\right]+\frac{1}{2k}\left(\underline{p}-c\right)\mu\left[\underline{G}-\overline{G}\right]$, we thus get:

$$\begin{aligned}
\frac{\frac{\partial\overline{\pi}}{\partial s}}{\frac{\partial\underline{\pi}}{\partial s}} &= \frac{\left(1-\overline{G}\right)\left[\mu\left(1-\overline{G}\right)+(1-\mu)(1-\underline{G})\right]-\frac{1}{2k}(\overline{p}-c)(1-\mu)\left[\underline{G}-\overline{G}\right]}{(1-\underline{G})\left[\mu\left(1-\overline{G}\right)+(1-\mu)(1-\underline{G})\right]+\frac{1}{2k}\left(\underline{p}-c\right)\mu\left[\underline{G}-\overline{G}\right]} \\
&= \frac{Z+\left[\underline{G}-\overline{G}\right]\left[\mu\left(1-\overline{G}\right)+(1-\mu)(1-\underline{G})\right]-\frac{1}{2k}\left(\underline{p}-c\right)\mu\left[\underline{G}-\overline{G}\right]-\frac{1}{2k}(\overline{p}-c)(1-\mu)\left[\underline{G}-\overline{G}\right]}{Z} \\
&= \frac{Z+\left[\underline{G}-\overline{G}\right]\left(\mu\left(1-\overline{G}\right)+(1-\mu)(1-\underline{G})-\mu\frac{1}{2k}\left(\underline{p}-c\right)-(1-\mu)\frac{1}{2k}\left(p\left(\overline{v}\right)-c\right)\right)}{Z}
\end{aligned}$$

Thus $\frac{\partial\overline{\pi}}{\partial s}>\frac{\partial\underline{\pi}}{\partial s}$ if and only if the second term in the numerator is positive, i.e. if:

$$\mu\left(1-\overline{G}\right)+(1-\mu)(1-\underline{G})-\mu\frac{1}{2k}\left(\underline{p}-c\right)-(1-\mu)\frac{1}{2k}(\overline{p}-c)>0.$$

Substituting for $\overline{p}$ and $\underline{p}$ in the previous expression gives:

$$\begin{aligned}
& \mu\left(1-\overline{G}\right)+(1-\mu)(1-\underline{G})-\mu(1-\underline{G})-(1-\mu)\left(1-\overline{G}\right) \\
=\ & \mu\left(\underline{G}-\overline{G}\right)-(1-\mu)\left(\underline{G}-\overline{G}\right) \\
=\ & (2\mu-1)\left(\underline{G}-\overline{G}\right)
\end{aligned}$$

Thus

$$\frac{\partial\overline{\pi}}{\partial s}>\frac{\partial\underline{\pi}}{\partial s}\text{ if and only if }\mu>0.5$$

■

The next lemma is used in the proof of Proposition 1 in the text.

Lemma 2 $\partial\left(\overline{\pi}-\underline{\pi}\right)/\partial\mu<0$ *for all* μ *if* k *is sufficiently high.*

Proof. Differentiating the profits in (3) with respect to μ and substituting from (6), gives:

$$\begin{aligned}
\frac{d\overline{\pi}}{d\mu} &= \frac{\partial\overline{p}}{\partial u}\cdot\frac{\partial u}{\partial\mu}\overline{Q}+(\overline{p}-c)\frac{\partial\overline{Q}}{\partial\mu} \\
&= -\frac{1}{2}\cdot\frac{\partial u}{\partial\mu}\left[1-\overline{G}\right]M+\frac{1-\overline{G}}{\overline{g}}\frac{\partial\overline{Q}}{\partial\mu} \\
&= \left[1-\overline{G}\right]\left\{-\frac{1}{2}\frac{\partial u}{\partial\mu}M+2k\frac{\partial\overline{Q}}{\partial\mu}\right\}
\end{aligned}$$

From (4)

$$\frac{\partial\overline{Q}}{\partial\mu}=-\frac{1}{4k}\frac{\partial u}{\partial\mu}M+\left[1-\overline{G}\right]\frac{\partial M}{\partial\mu},$$

where

$$\frac{\partial M}{\partial \mu}=-\frac{\underline{G}-\overline{G}-\frac{1}{4k}\frac{\partial u}{\partial \mu}}{\left(\mu\left(1-\overline{G}\right)+(1-\mu)\left(1-\underline{G}\right)\right)^2}=\left[\overline{G}-\underline{G}+\frac{1}{4k}\frac{\partial u}{\partial \mu}\right]\cdot M^2,$$

and thus

$$\frac{\partial \overline{Q}}{\partial \mu}=M[-\frac{1}{4k}\frac{\partial u}{\partial \mu}+[\overline{G}-\underline{G}+\frac{1}{4k}\frac{\partial u}{\partial \mu}]\left(1-\overline{G}\right)M]. \tag{15}$$

Hence

$$\begin{aligned}\frac{d\overline{\pi}}{d\mu} &= \left[1-\overline{G}\right]\left\{-\frac{1}{2}\frac{\partial u}{\partial \mu}M+2k\frac{\partial \overline{Q}}{\partial \mu}\right\}\\ &= \left[1-\overline{G}\right]M\left\{-\frac{1}{2}\frac{\partial u}{\partial \mu}+2k[-\frac{1}{4k}\frac{\partial u}{\partial \mu}+[\overline{G}-\underline{G}+\frac{1}{4k}\frac{\partial u}{\partial \mu}]\left(1-\overline{G}\right)M]\right\}\end{aligned}$$

Thus

$$\frac{d\overline{\pi}}{d\mu}=\left[1-\overline{G}\right]M\left\{-\frac{\partial u}{\partial \mu}+[2k\left(\overline{G}-\underline{G}\right)+\frac{1}{2}\frac{\partial u}{\partial \mu}]\left(1-\overline{G}\right)M\right\} \tag{16}$$

and similarly

$$\frac{d\underline{\pi}}{d\mu}=[1-\underline{G}]M\left\{-\frac{\partial u}{\partial \mu}+[2k\left(\overline{G}-\underline{G}\right)+\frac{1}{2}\frac{\partial u}{\partial \mu}](1-\underline{G})M\right\} \tag{17}$$

where recall $1-\overline{G}>1-\underline{G}$. Thus:

$$\begin{aligned}\frac{d\overline{\pi}}{d\mu}-\frac{d\underline{\pi}}{d\mu} &= \left(\overline{G}-\underline{G}\right)M\frac{\partial u}{\partial \mu}+\left(\left[1-\overline{G}\right]^2-[1-\underline{G}]^2\right)M^2[2k\left(\overline{G}-\underline{G}\right)+\frac{1}{2}\frac{\partial u}{\partial \mu}]\\ &= \left(\underline{G}-\overline{G}\right)M\underbrace{(-\frac{\partial u}{\partial \mu}+M\left(2-\underline{G}-\overline{G}\right)[2k\left(\overline{G}-\underline{G}\right)+\frac{1}{2}\frac{\partial u}{\partial \mu}])}_{\Psi}\end{aligned}$$

Thus, $sgn\left(\frac{d\overline{\pi}}{d\mu}-\frac{d\underline{\pi}}{d\mu}\right)=sgn\left(\Psi\right)$. Observe that the sign is clearly negative if $2k\left(\overline{G}-\underline{G}\right)+\frac{1}{2}\frac{\partial u}{\partial \mu}<0$. We find $sgn\left(\Psi\right)$ in the following three steps.

Step 1: Calculating $\frac{\partial u}{\partial \mu}$

Substituting $u=\overline{v}+\overline{\varepsilon}-\overline{p}=\underline{v}+\underline{\varepsilon}-\underline{p}$ in (1) gives the condition

$$\mu\int_{\overline{\varepsilon}}^{k}\left(\varepsilon-\overline{\varepsilon}\right)g\left(\varepsilon\right)d\varepsilon+(1-\mu)\int_{\underline{\varepsilon}}^{k}\left(\varepsilon-\underline{\varepsilon}\right)g\left(\varepsilon\right)d\varepsilon=s.$$

Substituting $g(\varepsilon)=\frac{1}{2k}$ gives:

$$\mu\int_{\overline{\varepsilon}}^{k}\left(\varepsilon-\overline{\varepsilon}\right)d\varepsilon+(1-\mu)\int_{\underline{\varepsilon}}^{k}\left(\varepsilon-\underline{\varepsilon}\right)d\varepsilon=\mu[\int_{\overline{\varepsilon}}^{k}\varepsilon d\varepsilon\ -\overline{\varepsilon}\int_{\overline{\varepsilon}}^{k}d\varepsilon\ ]+\ (1-\mu)\ [\int_{\underline{\varepsilon}}^{k}\varepsilon d\varepsilon\ -\underline{\varepsilon}\int_{\underline{\varepsilon}}^{k}d\varepsilon\ ]\ =\ 2ks$$

Carrying out the integration gives

$$\begin{aligned}
\mu[\frac{k^2}{2}+\frac{\overline{\varepsilon}^2}{2}-\overline{\varepsilon}k]+(1-\mu)\,[\frac{k^2}{2}+\frac{\underline{\varepsilon}^2}{2}-\underline{\varepsilon}k] &= 2ks \\
\mu[k^2-2\overline{\varepsilon}k+\overline{\varepsilon}^2]+(1-\mu)\,[k^2-2\underline{\varepsilon}k+\underline{\varepsilon}^2] &= 4ks \\
\mu[k-\overline{\varepsilon}]^2+(1-\mu)\,[k-\underline{\varepsilon}]^2 &= 4ks
\end{aligned} \tag{18}$$

Differentiating the implicit equation gives

$$-2\left\{\mu[k-\overline{\varepsilon})]\frac{\partial\overline{\varepsilon}}{\partial u}+(1-\mu)\,[k-\underline{\varepsilon})]\,\frac{\partial\underline{\varepsilon}}{\partial u}\right\}du+\left\{[k-\overline{\varepsilon}]^2-[k-\underline{\varepsilon})]^2\right\}d\mu=0$$

Since, by (8), $\frac{\partial\overline{\varepsilon}}{\partial u}=\frac{\partial\underline{\varepsilon}}{\partial u}=0.5$,

$$-\left\{\mu[k-\overline{\varepsilon})]+(1-\mu)\,[k-\underline{\varepsilon})]\right\}du+\left\{[k-\overline{\varepsilon}]^2-[k-\underline{\varepsilon})]^2\right\}d\mu=0$$

and thus

$$\frac{\partial u}{\partial\mu}=\frac{[k-\overline{\varepsilon}]^2-[k-\underline{\varepsilon})]^2}{\mu[k-\overline{\varepsilon})]+(1-\mu)\,[k-\underline{\varepsilon})]}=\frac{(\underline{\varepsilon}-\overline{\varepsilon})\,(2k-\overline{\varepsilon}-\underline{\varepsilon})}{k-\mu\overline{\varepsilon}-(1-\mu)\,\underline{\varepsilon}}=2\,(\underline{\varepsilon}-\overline{\varepsilon})\,\frac{k-0.5\overline{\varepsilon}-0.5\underline{\varepsilon}}{k-\mu\overline{\varepsilon}-(1-\mu)\,\underline{\varepsilon}}>0 \tag{19}$$

Step 2: Evaluating $2k\left(\overline{G}-\underline{G}\right)+\frac{1}{2}\frac{\partial u}{\partial\mu}$

Substituting $\overline{G}=\dfrac{\overline{\varepsilon}+k}{2k}$, $\underline{G}=\dfrac{\underline{\varepsilon}+k}{2k}$ gives $2k\left(\overline{G}-\underline{G}\right)=\overline{\varepsilon}-\underline{\varepsilon}$ and thus, using the preceding formula:

$$\begin{aligned}
2k\left(\overline{G}-\underline{G}\right)+\frac{1}{2}\frac{\partial u}{\partial\mu} &= \overline{\varepsilon}-\underline{\varepsilon}+\frac{1}{2}\frac{\partial u}{\partial\mu}=-\left(\underline{\varepsilon}-\overline{\varepsilon}\right)\;+\left(\underline{\varepsilon}-\overline{\varepsilon}\right)\frac{k-0.5\overline{\varepsilon}-0.5\underline{\varepsilon}}{k-\mu\overline{\varepsilon}-(1-\mu)\,\underline{\varepsilon}} \\
&= \left(\underline{\varepsilon}-\overline{\varepsilon}\right)\left[\frac{k-0.5\overline{\varepsilon}-0.5\underline{\varepsilon}}{k-\mu\overline{\varepsilon}-(1-\mu)\,\underline{\varepsilon}}-1\right]
\end{aligned}$$

The expression is negative if and only if $k-0.5\overline{\varepsilon}-0.5\underline{\varepsilon}<k-\mu\overline{\varepsilon}-(1-\mu)\,\underline{\varepsilon}$ or $\mu\overline{\varepsilon}+(1-\mu)\,\underline{\varepsilon}<0.5\overline{\varepsilon}+0.5\underline{\varepsilon}$, which, since $\underline{\varepsilon}>\overline{\varepsilon}$, is satisfied if $0.5>1-\mu$. Thus, $2k\left(\overline{G}-\underline{G}\right)+\frac{1}{2}\frac{\partial u}{\partial\mu}<0$ if and only if $\mu>0.5$

It follows from (15) and (19) that $\frac{\partial\overline{Q}}{\partial\mu}<0$ for $\mu>\mu^*$, where $\mu^*<0.5$.

Step 3: Evaluating $\Psi=-\dfrac{\partial u}{\partial\mu}+M\left(2-\underline{G}-\overline{G}\right)\left(2k\left(\overline{G}-\underline{G}\right)+\frac{1}{2}\frac{\partial u}{\partial\mu}\right)$

Clearly $\Psi<0$ for all $\mu\geq 0.5$. Next, observe that $2-\underline{G}-\overline{G}=1-\dfrac{\underline{\varepsilon}+\overline{\varepsilon}}{2k}$.

$$\begin{aligned}
\Psi &= -2\left(\underline{\varepsilon}-\overline{\varepsilon}\right)\frac{k-0.5\overline{\varepsilon}-0.5\underline{\varepsilon}}{k-\mu\overline{\varepsilon}-(1-\mu)\,\underline{\varepsilon}}+M\left(1-\frac{\underline{\varepsilon}+\overline{\varepsilon}}{2k}\right)\left(\underline{\varepsilon}-\overline{\varepsilon}\right)\left[\frac{k-0.5\overline{\varepsilon}-0.5\underline{\varepsilon}}{k-\mu\overline{\varepsilon}-(1-\mu)\,\underline{\varepsilon}}-1\right] \\
&\propto \frac{k-0.5\overline{\varepsilon}-0.5\underline{\varepsilon}}{k-\mu\overline{\varepsilon}-(1-\mu)\,\underline{\varepsilon}}\left[M\left(1-\frac{\underline{\varepsilon}+\overline{\varepsilon}}{2k}\right)-2\right]-M\left(1-\frac{\underline{\varepsilon}+\overline{\varepsilon}}{2k}\right)
\end{aligned}$$

Thus $\Psi < 0$ if and only if

$$\frac{k - 0.5\overline{\varepsilon} - 0.5\underline{\varepsilon}}{k - \mu\overline{\varepsilon} - (1-\mu)\,\underline{\varepsilon}} < \frac{M\left(1 - \dfrac{\underline{\varepsilon} + \overline{\varepsilon}}{2k}\right)}{M\left(1 - \dfrac{\underline{\varepsilon} + \overline{\varepsilon}}{2k}\right) - 2}$$

Substituting for M and simplifying, the RHS is shown equal to

$$\frac{k - 0.5\overline{\varepsilon} - 0.5\underline{\varepsilon}}{(0.5 - \mu)\,(\underline{\varepsilon} - \overline{\varepsilon})}$$

The inequality is therefore,

$$\frac{k - 0.5\overline{\varepsilon} - 0.5\underline{\varepsilon}}{k - \mu\overline{\varepsilon} - (1-\mu)\,\underline{\varepsilon}} < \frac{k - 0.5\overline{\varepsilon} - 0.5\underline{\varepsilon}}{(0.5 - \mu)\,(\underline{\varepsilon} - \overline{\varepsilon})}$$

which is equivalent to

$$\begin{aligned} (0.5 - \mu)\,(\underline{\varepsilon} - \overline{\varepsilon}) &< k - \mu\overline{\varepsilon} - (1-\mu)\,\underline{\varepsilon} \\ \underline{\varepsilon} + (0.5 - 2\mu)\,(\underline{\varepsilon} - \overline{\varepsilon}) &< k \end{aligned}$$

where, from (8), $\underline{\varepsilon} - \overline{\varepsilon} = (\overline{v} - \underline{v})\,/2$. Since $\underline{\varepsilon} < k$ the condition holds irrespective of k provided $\mu > 0.25$.

Rearranging and substituting $\underline{\varepsilon} = (k + u - \underline{v} + c)\,/2$, we obtain that $\Psi < 0$ if and only if

$$(0.5 - 2\mu)\,(\overline{v} - \underline{v}) < k - u + \underline{v} - c \tag{20}$$

From equation (18) above it follows that

$$\mu[k - u + \overline{v} - c]^2 + (1-\mu)\,[k - u + \underline{v} - c]^2 = 4ks$$

It is clear from the proceeding equation that $k - u$ is strictly increasing and unbounded in k. Thus, if k is sufficiently high, $\Psi < 0$ for all μ and $\frac{d(\overline{\pi} - \pi)}{d\mu} < 0$ everywhere. ■

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