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Performance share plans: Valuation and empirical tests

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ABSTRACT

Performance share plans are an increasingly important component of executive compensation. They are equity-based, long-term incentive plans where the number of shares to be awarded is a quasi-linear function of a performance result over a fixed time period. We derive closed-form formulas for the value of a performance share plan when the performance measure is: (1) a non-traded measure following an Arithmetic Brownian Motion (e.g., earnings per share), (2) a non-traded measure following a Geometric Brownian Motion (e.g., revenue), or (3) a rank-order tournament of traded asset returns that are following Arithmetic Brownian Motions (e.g., percentile of ranked stock returns). Then we empirically test our valuation formulas. We find that our valuation formulas are more accurate for performance share plans based on earnings per share when forecasting using analyst consensus prior to the grant date. We also find that the efficiency of our valuation model greatly depends on the method used to forecast future firm performance. The policy implication is that FASB should consider requiring that grant date fair value be estimated using valuation formulas such as ours.

1. Introduction

For several decades stock options have been the most widely-used incentive component of U.S. executive compensation (Murphy, 1999; Clementi and Cooley, 2010). Hall and Murphy (2003) cite tax laws enacted in 1994 (Internal Revenue Code 162(m)) as a major driver of the growth in stock options. The Financial Accounting Standards Board (FASB) regulation 123R 2004 changed the accounting treatment of stock options to require that they be expensed at the grant date fair value as estimated by one of several option pricing models. Since that time, the use of stock options has declined from 99% of the Forbes 250 in 2003 to 71% in 2014, according to Frederick W. Cook, a compensation consulting firm.[1] By contrast, performance share plans, which are an alternative incentive component, have risen from 26% of the Forbes 250 in 2003 to 81% in 2014. Fig. 1 illustrates the rapid rise of performance share plans to exceed stock options in recent years.[2] Given their growth, the economics of performance share plans in executive compensation is increasingly important for academics to model and test, for practitioners to know the true costs and incentive effects, and for regulators to guide disclosure requirements.

A performance share plan is an equity-based, long-term incentive component of executive compensation in which the number of shares to be awarded is a quasi-linear function of a performance result over a fixed time period. A plan can be based on

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[1] Hayes et al. (2012) also document similar results after FAS 123R.

[2] The data comes from *The Top 250 Survey* by Frederick W. Cook from 1997 to 2014. Similar findings are also reported in *2011 CEO Pay Strategies Report* by Equilar.

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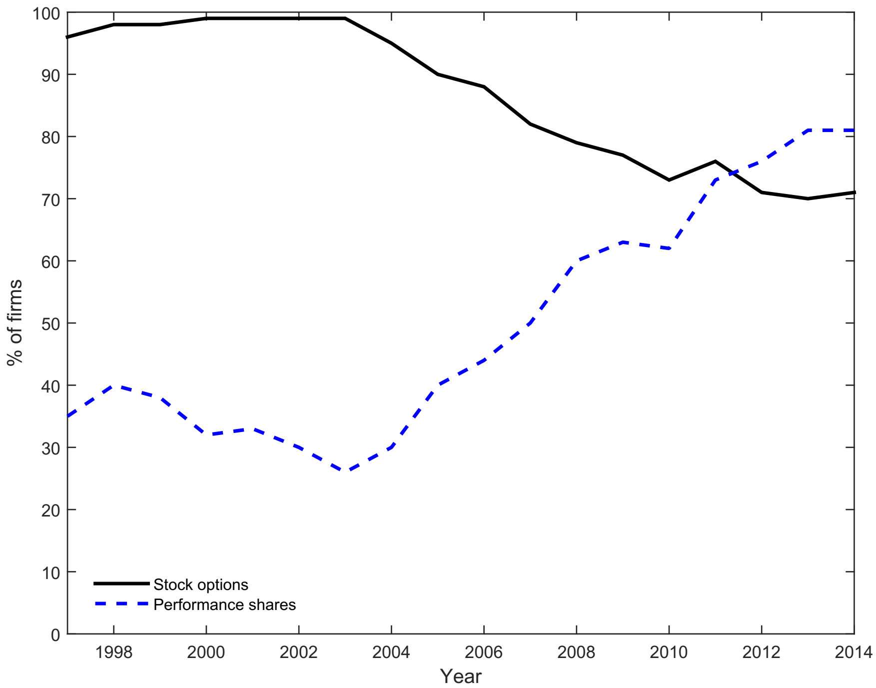


Fig. 1. Firms granting performance share plans and/or stock options. The percentage of firms that grant performance share plans and/or stock options to their executives. The sample period is from 1997 to 2014 and the top 250 Forbes firms are included. The data is collected from annual *The Top 250 Survey* by C. King of Frederick W. Cook (King, 2010).

a variety of alternative performance measures, such as earnings per share, revenue, percentile of ranked stock returns, return on invested capital, return on equity, etc. Once a performance measure is chosen, then the typical plan's payoff is determined by three performance goals and corresponding performance payoffs. For example, immediately prior to 2009, Coca Cola Enterprises (CCE) adopted a performance share plan for its CEO based on the coming year's earnings per share (EPS). If the EPS at maturity is below the *threshold goal* of $1.35/share, then the CEO's dollar payoff is zero. If EPS at maturity equals the threshold goal of $1.35/share, then the CEO's dollar payoff is (109,900 shares) X (CCE stock price at maturity). If EPS at maturity equals the *target goal* of $1.40/share, then the CEO's dollar payoff becomes (219,800 shares) X (CCE stock price at maturity). If EPS at maturity equals or exceeds the *stretch goal* of $1.45/share, then the CEO's dollar payoff becomes (439,600 shares) X (CCE stock price at maturity). If EPS falls between the threshold goal and the target goal, and between the target goal and the stretch goal, then the number of shares awarded increases linearly in performance.

Fig. 2 illustrates the CEO dollar payoff of the 2009 performance share plan at Coca Cola Enterprises as a function of: (1) stock price at maturity and (2) EPS at maturity. The shape of the CEO dollar payoff function is determined by the three performance goals (threshold goal, target goal, stretch goal) and the corresponding dollar payoffs (threshold payoff, target payoff, stretch payoff). Once the performance share plan has been designed, the dollar payoff function is locked in. Then the realized CCE dollar payoff at maturity depends strictly on: (1) the realized CCE stock price at maturity and (2) the realized CCE EPS at maturity.

Closely related are performance-vested share plans. A performance-vested share plan provides a fixed number of shares whenever performance exceeds a threshold goal and zero shares otherwise.[3] A performance-vested share plan can be thought of as a special case of a performance share plan in which the three performance goals are identical and the three performance payoffs are identical (i.e., the threshold goal equals the target goal equals the stretch goal and the threshold payoff equals the target payoff equals the stretch payoff). Appendix A provides some examples of proxy files for both performance shares and performance-vested share plans.

We begin by documenting the size and importance of performance share plans and performance-vested share plans. We analyze a large sample of S&P 500 firms from fiscal years ending on or after December 15, 2006 to fiscal years ending on or before December 31, 2012. We find that for those firms who use them, performance share plans have an average value of $2.53 million and performance-vested share plans have an average value of $3.00 million. Unconditionally on whether they are used or not, performance share plans represent 14.0% of total compensation and performance-vested share plans represent 3.1% of total compensation. We find that the four most popular performance measures for performance share plans are percentile

[3] There is some inconsistency in the way that different people use the terms "performance share plans" and "performance-vested share plans." We follow the most widespread convention.

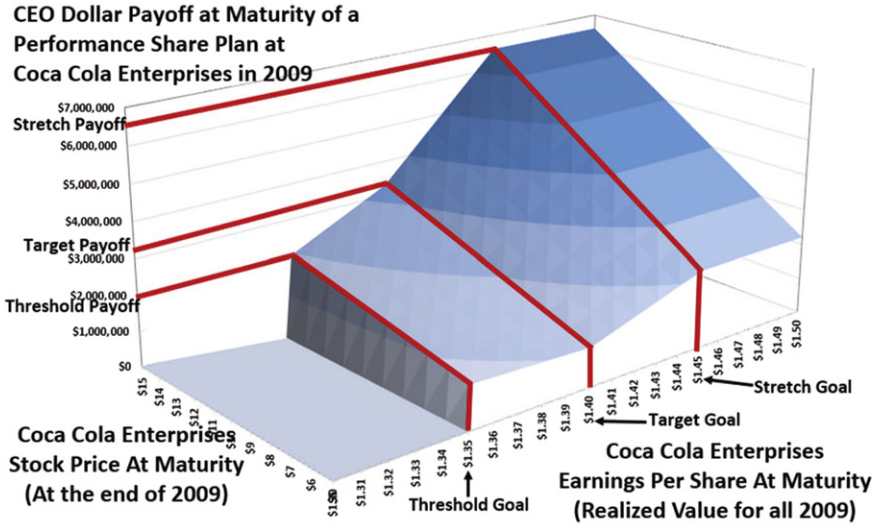


Fig. 2. CEO dollar payoff at maturity of the 2009 performance share plan at Coca Cola Enterprises. CEO dollar payoff at maturity of the 2009 performance share plan at Coca Cola Enterprises as a function of: (1) stock price at maturity and (2) earnings per share at maturity.

of ranked stock returns (24.7%), earnings per share (18.1%), EBIT (16.0%), and revenue (10.2%), but that a wide variety of other performance measures are used as well.

We derive closed-form formulas for the value of a performance share plan or a performance-vested share plan when the performance measure is: (1) a non-traded measure following an Arithmetic Brownian Motion (e.g., earnings per share),[4] (2) a non-traded measure following a Geometric Brownian Motion (e.g., revenue),[5] or (3) a rank-order tournament of traded asset returns that are following Arithmetic Brownian Motions (e.g., percentile of ranked stock returns).[6] Of course, the value depends on the six design parameters. But it also depends on environmental factors, such as the volatility of the performance measure, the beginning stock price, the beginning level of the performance measure, the length of the performance period, and the risk-neutral growth rate of the performance measure.[7]

We hand collect design parameters from proxy statements for S&P 500 firms with fiscal years ending on or after December 15, 2006 to fiscal years ending on or before December 31, 2012. We compare values generated by our new valuation formulas versus reported values on proxy statements versus heuristic values.[8] In most cases, reported values and heuristic values are insignificantly different from each other, providing evidence that heuristic value is the predominate basis for reported values on proxy statements. In most cases, we find that our new valuation formulas are statistically and economically different than reported value and heuristic value.

Next, we compare all three from above to the actual dollar payout from the sample plans. We find that our valuation formulas are more accurate for performance share plans based on EPS when forecasting based on analyst consensus prior to the grant date. We also find that the efficiency of our valuation model greatly depends on the method used to forecast future firm performance.

The policy implication is that FASB should consider changing the accounting treatment, especially for performance share plans based on EPS, to require that grant date fair value be estimated by valuation formulas such as ours.[9]

Our paper is related to three streams of literature within the extensive literature on executive compensation.[10] One stream of literature analyzes the optimal design of CEO compensation in a canonical principal-agent setting. DeMarzo and Sannikov (2006) and DeMarzo and Fishman (2007) analyze the optimal dynamic contract including the dynamic reoptimization of effort

[4] An Arithmetic Brownian Motion (ABM), when cumulated over a discrete time interval such as quarters or years, yields a normally distributed variable. Thus, ABM is consistent with the widespread use of normal distributions to model earnings as in Degeorge et al. (1999), Yu et al. (2006), and Beaver et al. (2007).

[5] A Geometric Brownian Motion (GBM), when cumulated over a discrete time interval, yields a lognormally distributed variable. Thus, GBM is consistent with the use of the lognormal distribution to model revenues, as in Babcock and Hennessy (1996), Stokes (2000) and Sherrick et al. (2004).

[6] Again, ABM cumulates to a normal distribution over time. Normal distributions are the standard model of stock returns. For example, Ingersoll (1987) notes that normally-distributed stock returns is often used as the basis to derive the CAPM.

[7] For a *non-traded* performance measure, the risk-neutral growth rate is the nominal growth rate less the risk premium for the systematic risk of the performance measure. When the performance measure is a rank-order tournament of *traded* asset returns, the risk neutral growth rate is the risk-free rate.

[8] The heuristic value is a simple approximate rule that is frequently used for valuation. Specifically, the heuristic value of a performance share plan is the target number of shares times the current stock price. The heuristic value of a performance-vested share plan is the threshold number of shares times the current stocks price.

[9] Currently, FASB requires that plans with market-based performance (e.g., percentile of ranked stock returns) be valued using Monte Carlo simulation or Binomial pricing models. However, plans with non-market based performance (e.g., revenue, earnings, etc.) do not have such a requirement.

[10] Surveys of the executive compensation literature include Abowd and Kaplan (1999), Murphy (1999), Prendergast (1999), Core et al. (2003a), Aggarwal (2008), Bertrand (2009), Edmans and Gabaix (2009), Hall and Murphy (2003), and Frydman and Jenter (2010).

over time. These studies have contributed to our knowledge of the optimal design of CEO compensation, where compensation may take any functional form and may be renegotiated at any time. However, their great generality abstracts away from many real-world compensation components, such as options, bonuses, performance shares, etc.

The second stream of literature takes the functional form of real-world compensation components as given and perform a theoretical analysis of the up-front executive compensation decision.[11] Representative of this approach are Hall and Murphy (2000, 2002), Dittmann and Maug (2007), and Dittmann et al. (2010), which take the functional form of stock options as given and perform a theoretical analysis of the up-front executive compensation decision in the presence of stock options.

The third stream of literature analyzes performance share plans or performance-vested share plans. Martellini and Urosevic (2005) value performance share plans when the performance measure is the firm's own stock price. This is a no-arbitrage result, because the underlying asset is traded. By contrast, we expand the scope to value performance shares when the performance measure is a non-traded measure following either an Arithmetic or a Geometric Brownian Motion or a rank-order tournament of traded asset returns following Arithmetic Brownian Motions. The first two cases are equilibrium results, precisely because the underlying assets are not traded.[12] Bettis et al. (2010) empirically investigate stock or option grants with performance-based vesting provisions. They find that these provisions provide meaningful incentives to executives and document that the firms with performance-based vesting provisions significantly outperform the control firms. Bizjak et al. (2012) develop *approximate* present value formulas for equity awards with performance-vesting provisions. Both Bettis et al. (2014) and Bettis et al. (2015) value performance-vesting provisions by numerically simulating future cash flows and discounting these cash flows back to the present using a risk-adjusted discount rate based on the static CAPM. By contrast with these papers, we develop closed-form formulas for the value of performance share plans and performance-vested share plans based on the Cox and Ross (1976) risk neutral valuation method and empirically test our theoretical formulas.

The remainder of the paper is organized as follows. Section 2 empirically documents the size and importance of performance share plans and performance-vested share plans. Section 3 derives closed-form formulas for the value of performance share plans and performance-vested share plans. Section 4 empirically compares the actual payout of plans to values generated by our new valuation formulas versus reported values on proxy statements versus heuristic values. Section 5 concludes. Appendix A contains examples of proxy filings for different types of awards. Appendix B contains the proofs.

2. The importance of performance share plans and performance-vested share plans

We begin by documenting the size and importance of performance share plans and performance-vested share plans. We collect data from proxy statements on all firms that were in the S&P 500 index as of January 2006. The sample period is from fiscal years ending on or after December 15, 2006 to fiscal years ending on or before December 31, 2012.

Table 1 describes the compensation structure of CEOs in our sample. Panel A presents the mean and median target amount[13] conditional on that component being granted and number of firms granting each compensation component. The amount of compensation granted in the form of performance shares is sizable. 1435 firm-years used performance share plans; granting a mean of $2.53 million and a median of $1.80 million. The mean amount is significantly larger than average salary ($1.11 million) and performance cash ($2.28 million), which includes annual cash bonus and long-term cash incentive plans.

Panel B reports the unconditional breakdown of the compensation components (i.e, not conditioning on whether that component is offered). Performance shares and performance-vested shares together represent 17.1% of total compensation. This approaches the size of stock options and performance cash, which represents 24.0% and 24.0% of total compensation, respectively.

Table 2 describes the performance measures, plan structure, and value derivations used by performance share plans. We hand-collected this information from definitive proxy statements of S&P 500 firms from the Edgar database.[14] We obtain 2881 firm-year observations of performance share plans. The frequency with which various performance measures are used is: percentile of ranked stock returns 24.7% of the time, earnings per share (EPS) 18.1%, earnings before interest and taxes (EBIT), 16.0%, revenue 10.2%, return on invested capital (ROIC) 9.7%, return on equity (ROE) 5.0%, and all other performance measures 16.2% of the time. There are many other measures used including profit measures (e.g., operating income, net income, and profit before tax) and cash flow measures (e.g., free cash flow, cash flow from operations, and economic value added).

Panel B describes the plan structures in our sample. Out of 2881 performance share plans in our sample, 152 plans employ triggers. We define a trigger as an extra layer of performance conditioning that must be satisfied before for the executive receives any payout. Many companies report that they set this extra requirement to satisfy IRS 162(m)'s tax deductibility condition, and the performance goal for triggers are often set at an easy level. 194 plans add a modifier to the plan. We define a modifier as any performance criteria that can adjust payout upward or downward. The modifiers are applied only when the primary performance condition is met. 134 plans use a matrix type of payout structure. We define a matrix type as any plan that has

11 In practice, renegotiation typically takes place when the executive contract comes up for renewal and seldom within the life of a contract.

12 A related stream of accounting literature studies different performance measures used in executive compensation. See Bushman and Indjejikian (1993), Kim and Suh (1993), Lambert (1993), Lambert and Larcker (1987), Sloan (1993), and Core et al. (2003a,b).

13 For performance cash, the *target amount* represents the amount of cash award the CEO will receive when the ending performance is exactly at performance target. For performance share plans, we obtain reported grant date fair value of equity awards.

14 Proxy statements can be found from http://edgar.sec.gov/edgar/searchedgar/companysearch.html.

Table 1
CEO compensation components. This table shows components of CEO compensation for CEOs at S&P 500 firms. The data comes from proxy statements. The sample spans fiscal years ending from December 15th, 2006 to December 31st, 2012. *Performance Share Plans*, *Performance-Vested Share Plans*, and *Performance Options* are equity-based compensation plans in which the number of shares or options awarded is tied to the performance of pre-specified measures. *Performance Cash* includes cash-based annual and long-term incentive pay.

Components	Mean	Median	# of obs.
Panel A. Mean and median target amount conditional on component being granted ($ MM)			
Salary	1.11	1.01	2920
Restricted Stocks	2.88	1.94	1432
Incentive Pay			
Performance Share Plans	2.53	1.80	1435
Performance Cash	2.28	1.58	2520
Performance Options	3.43	2.58	48
Performance-Vested Share Plans	3.00	2.00	307
Stock Options	3.41	2.43	2024
Panel B. Compensation components not conditioning on whether that component is offered (%)			
Salary	19.6	14.9	
Restricted Stocks	14.9	0.0	
Incentive Pay			
Performance Share Plans	14.0	0.0	
Performance Cash	24.0	0.0	
Performance Options	0.5	0.0	
Performance-Vested Share Plans	3.1	0.0	
Stock Options	24.0	23.5	

preset payout grid depending simultaneously on two performance measures. The payoff of plan structures with trigger, modifier, or matrix types depends on two or more performance measures. In our empirical analysis in Section 4, we exclude plans that are matrix type; however, we include plans with triggers and modifiers as these are secondary conditions.

There are some special payoff structures in our sample. 16 plans use step functions instead of linear interpolation between performance goals. 22 plans offer more than one way to earn a performance payoff. This is different from having the payoff be a quasi-linear function of multiple performance measures. Instead, they offer to award shares if any one of several alternative criteria are satisfied. Finally, 54 plans offer consolation, which offers executives second or third chance to earn shares when they fail to satisfy the main performance condition within the performance period.

Panel C describes the grant date fair value derivations. Among 2881 performance share plans in our sample, we classify 1173 plans as using simulated values to report the grant date fair value of their plans. This is 40.7% of the total. On top of the plans that explicitly state that they base their valuation on simulations, we classify every plan that did not use closing stock price, average of high and low price on the grant date, or did not explicitly state to have used non-simulated values. Thus some plans, such as the ones that used closing share price less expected dividend payout but did not state so in the proxy filings, may have been incorrectly included in the 40.7%. 53% of the plans use closing stock price, average of high and low price on the grant date, exercise price of stock options granted on the same date, or historical average of share prices as the value for the performance shares. Final 6.6% of plans used closing price less expected dividends, closing price of the fiscal year, or did not report the grant date fair value.

Among the firms that used simulated values, 107 plans explicitly used Monte Carlo simulation methodology for valuation and five plans used Black-Scholes based methodology. Interestingly, 97 plans employed market-based value (as defined above) but used the maximum number of shares to derive grant date fair value; however, only one plan used the minimum number of shares to derive the grant date fair value.

3. Valuation

3.1. Payoff at maturity

When designing a performance share plan, the board of directors first chooses the performance measure, such as percentile of ranked stock returns, earnings per share, revenue, etc. Next, the board of directors designs the dollar payoff function, which is illustrated in Fig. 3. Let P_T be the performance measure value at maturity, which is on the x-axis. Let S_T be the firm's stock price at maturity, which is on the y-axis. Let PSP_T be the performance share plan's dollar payoff at maturity, which is on the z-axis.

Now define the three performance goals on the x-axis. Let L be the threshold goal, M be the target goal, and H be the stretch goal. Next define the number of shares awarded in these three cases. Let N_L be the threshold shares, N_M be the target shares, and N_H be the stretch shares. Now we can define the three corresponding dollar payoffs on the z-axis. Let N_LS_T be the threshold payoff, N_MS_T be the target payoff, and N_HS_T be the stretch payoff.

Table 2

CEO performance share plans: measures, structure, and derivations. This table shows performance measures, plan structure, and grant date fair value derivations used in performance share plans for CEOs at S&P 500 firms. The data comes from proxy statements. The sample spans fiscal years ending from December 15th, 2006 to December 15th, 2012. *Performance Share Plans*, *Performance-Vested Share Plans*, and *Performance Options* are equity-based compensation plans in which the number of shares or options awarded is tied to the performance of pre-specified measures. *Performance Cash* includes cash-based annual and long-term incentive pay.

Compensation components	# of obs.	% of total
Panel A. Performance measures		
Stock Returns	713	24.7
Earnings Per share (EPS)	521	18.1
Earnings Before Interest and Taxes (EBIT)	461	16.0
Revenue	294	10.2
Return on Invested Capital	280	9.7
Return on Equity	145	5.0
Cash Flow	138	4.8
Annual Incentive Plan	49	1.7
Other Performance Measures	280	9.7
Total	2881	100.0
Panel B. Plan structure		
With trigger	152	5.3
With modifier	194	6.7
Matrix	134	4.7
Step function	16	0.6
Multiple options	22	0.8
Consolation	54	1.9
Panel C. Grant date fair value derivations		
Simulated value	1173	40.7
Market price	1519	52.7
Monte Carlo simulations	107	3.7
Black-Scholes pricing	5	0.2
Assume max number of shares	97	9.7
Assume min number of shares	1	0.0

If performance is below L, then dollar payoff is zero. If performance is between L and M, then the dollar payoff increases linearly with a slope $\lambda_L S_T$, where $\lambda_L = (N_M - N_L)/(M - L)$ is the corresponding rate of increase in shares. If performance is between M and H, then the dollar payoff increases linearly with a slope $\lambda_H S_T$, where $\lambda_H = (N_H - N_M)/(H - M)$ is the corresponding rate of increase in shares. If performance equals or exceeds H, then the dollar payoff of $N_H S_T$ is awarded. Typically, the two slopes are not equal ($\lambda_L S_T \neq \lambda_H S_T$), so there is a kink in the share reward function at M.

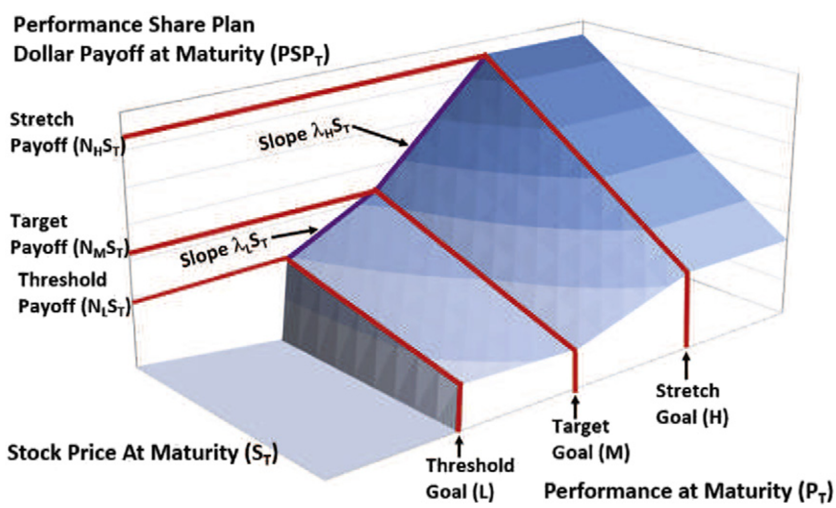


Fig. 3. Dollar payoff of performance share plan at maturity. Dollar payoff of performance share plan at maturity PSP_T as a function of: (1) stock price at maturity S_T and (2) performance at maturity P_T.

Based on this structure, the dollar payoff of a performance share plan can be summarized as

$$PSP_T = \begin{cases} 0 & \text{for} \quad P_T < L, & \text{(a)} \\ [N_L + \lambda_L (P_T - L)]\, S_T & \text{for} \quad L \le P_T < M, & \text{(b)} \\ [N_M + \lambda_H (P_T - M)]\, S_T & \text{for} \quad M \le P_T < H, & \text{(c)} \\ N_H S_T & \text{for} \quad H \le P_T. & \text{(d)} \end{cases} \tag{1}$$

The fact that dollar payoff of a performance share plan depends on *two* random variables (performance measure at maturity P_T and stock price at maturity S_T) stands a sharp contrast to standard call and put options, which depend on only a *single* random variable (stock price at maturity S_T). It immediately follows that a performance share plan is *not* a portfolio of options. As will be shown in the propositions below, the valuation formula for a performance share plan depends on the current value of *both* random variables (P_0 and S_0).

In Fig. 3, it is easy to see the influence of the performance measure at maturity by looking at the upper-left edge, where the monetary payoff is flat below L, jumps up at L, increases linearly between L and M, kinks at M to a new linear slope between M and H, and then is flat above H. It is also easy to see the influence of the stock price at maturity by looking at the upper-right edge, where the dollar payoff increases linearly in the stock price at maturity.

3.2. Decomposition

The dollar payoff function of a performance share plan can be decomposed into five simpler components. One component is a performance-vested share plan and the other four components are variations of what we call a “linear performance share plan”. We define a linear performance share plan with threshold goal X and rate of share increase λ as a simple plan where the dollar payoff when performance is below the threshold goal X is zero and the dollar payoff above the threshold goal is the following linear function $\lambda(P_T - X)S_T$.[15] Based on this structure, the dollar payoff of a linear performance share plan can be summarized as

$$LPS_T = \begin{cases} 0 & \text{for} \quad P_T < X & \text{(a)} \\ \lambda\,(P_T - X)\, S_T & \text{for} \quad X \le P_T, . & \text{(b)} \end{cases} \tag{2}$$

As before, the fact that dollar payoff of a linear performance share plan depends on *two* random variables (performance measure at maturity P_T and stock price at maturity S_T) stands a sharp contrast to standard call and put options, which depend on only a *single* random variable (stock price at maturity S_T). It immediately follows that a linear performance share plan is *not* a portfolio of options. As will be shown in the propositions below, the valuation formula for a linear performance share plan depends on the current value of *both* random variables (P_0 and S_0).

Importantly, we show that a performance share plan can be decomposed into five components. One of the components is based on a performance-vested share plan and the other four of the components are based on linear performance share plans.

Table 3 shows that the dollar payoff of the five components sum to the dollar payoff of a performance share plan. Specifically, the last four columns show the dollar payoffs for four regions of performance: (1) below L, (2) between L and M, (3) between M and H, and (4) above H. The first row shows the dollar payoff of a long position in performance-vested share (PVS) plan with a threshold L and the next four rows show the dollar payoff for various long or short positions in various linear performance share (LPS) plans.

The final row shows the total dollar payoff of the five components.[16] The total dollar payoff of the five components is identical to the dollar payoff of a performance share plan with threshold goal L, target goal M, stretch goal H, threshold shares N_L, target shares N_M, and stretch shares N_H as shown in Eq. (1). The tight connection between a performance share plan and the five components will carry over to the valuation formulas as well.

3.3. Stochastic processes

We observe that performance share plans are based on a wide variety of performance measures that have different attributes. Some performance measures have strictly non-negative realizations (e.g., revenue). However, other performance measures can be either positive or negative (e.g., earnings per share, return on equity). Most performance measures are not traded assets. However, an exception is when the performance measure is the percentile of ranked stock returns, where the underlying random variables are the firm's own traded stock return and the traded stock returns of peer firms.

To encompass all of these cases, we will value performance share plans under three alternative modeling assumptions: (1) a non-traded performance measure following an Arithmetic Brownian Motion, (2) a non-traded performance measure following

[15] A linear performance share plan can be thought of as a special case of a performance share plan, where the number of threshold shares is zero ($N_X = 0$), the two slopes are equal ($\lambda_L = \lambda_H$), the stretch shares stay on the line ($N_H = X + \lambda(H - X)$, and in the limit as the stretch goal goes to infinity ($H \to \infty$).

[16] The total takes advantage of the identities: $N_M = N_L + \lambda_L(M - L)$ and $N_H = N_M + \lambda_H(H - M)$.

Table 3
Five components sum to a performance share plan. This table shows that the dollar payoffs of following five components: (1) a long position in a performance-vested share (PVS) plan with a threshold goal L and threshold shares N_L, (2) a long position in a linear performance share (LPS) plan with a threshold L and slope λ_L, (3) a short position in a linear performance share (LPS) plan with a threshold M and slope λ_L, (4) a long position in a linear performance share (LPS) plan with a threshold M and slope λ_H, and (5) a short position in a linear performance share (LPS) plan with a threshold H and slope λ_H sum up to equal the dollar payoff of a performance share plan (PSP) with threshold goal L, target goal M, stretch goal H, threshold shares N_L, target shares N_M, and stretch shares N_H.

	$P_T < L$	$L \le P_T < M$	$M \le P_T < H$	$H \le P_T$
(1) Long a $PV\,S(L, N_L)$	0	$N_L S_T$	$N_L S_T$	$N_L S_T$
(2) Long a $LPS(L, \lambda_L)$	0	$\lambda_L(P_T - L)S_T$	$\lambda_L(P_T - L)S_T$	$\lambda_L(P_T - L)S_T$
(3) Short a $LPS(M, \lambda_L)$	0	0	$-\lambda_L(P_T - M)S_T$	$-\lambda_L(P_T - M)S_T$
(4) Long a $LPS(M, \lambda_H$	0	0	$\lambda_H(P_T - M)S_T$	$\lambda_H(P_T - M)S_T$
(5) Short a $LPS(H, \lambda_H)$	0	0	0	$-\lambda_H(P_T - H)S_T$
Total $= PSP(L, M, H, N_L, N_M, N_H)$	0	$[N_L + \lambda_L(P_T - L)]S_T$	$[N_M + \lambda_H(P_T - M)]S_T$	$N_H S_T$

a Geometric Brownian Motion, or (3) the performance measure is a rank-order tournament of traded asset returns that are following Arithmetic Brownian Motions. We begin by analyzing the first case and then turn to the latter two cases after that.

Let P_t be the performance measure at time t at any time during the performance period $[0, T]$. Assume that the performance measure is not traded, but evolves continuously based on an Arithmetic Brownian Motion as given by

$$dP = \alpha_P dt + \sigma_P dW_1, \tag{3}$$

where α_P is the instantaneous drift of performance, σ_P is the instantaneous standard deviation of performance, and dW_1 is the increment of a standard Wiener process. An Arithmetic Brownian Motion can have negative realizations, so this would be a candidate to represent performance measures that can go negative (e.g., earnings per share, free cash flow, operating income, etc.). When accumulated over a period of time, an Arithmetic Brownian Motion yields a normal distribution for the terminal value. Fig. 4 (a) shows a bar graph distribution of earnings per share for S&P 500 firms from 2000 to 2014 fiscal years. Indeed, we do see a substantial amount of both positive and negative realizations. The empirical distribution (bar graph) is compared to the normal distribution (solid blue curve) that best fits the data. We see that the empirical distribution is slightly skewed and its right tail is a little bit thick. Without the thick right tail, the best fit normal distribution would undoubtedly fit the empirical distribution better in the middle by having a smaller standard deviation, and thus, a higher frequency in the middle. Overall, the empirical distribution fits the normal distribution reasonably well. By reverse engineering, this implies that an Arithmetic Brownian Motion is a reasonable distribution for the implied continuous process of earnings per share. This is consistent with DeMarzo and Sannikov (2006), who develop a dynamic model of optimal capital structure in which they assume that firm cash flows, which are equivalent to earnings in their setting, follow an Arithmetic Brownian Motion.

Let S_t be the firm's stock price at time t. Assume that the stock price follows a Geometric Brownian Motion as given by

$$\frac{dS}{S} = hdP + \alpha_N dt + \sigma_N dW_2, \tag{4}$$

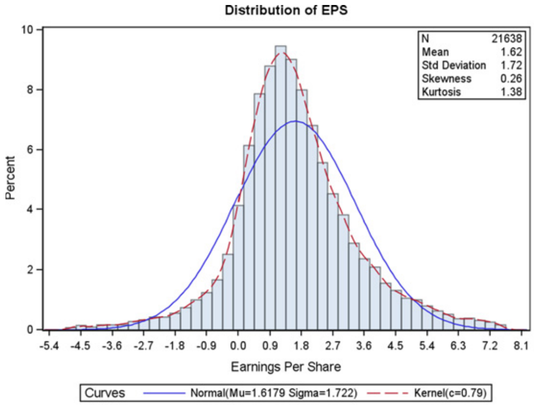
where h is the sensitivity of the stock price to the performance measure, α_N is the instantaneous drift that is not performance measure related, σ_N is the instantaneous standard deviation that is not performance measure related, and dW_2 is the increment of a standard Wiener process which is independent of dW_1. Substituting Eq. (3) into Eq. (4), we obtain

$$\frac{dS}{S} = (h\alpha_P + \alpha_N)\, dt + h\sigma_P dW_1 + \sigma_N dW_2 \tag{5}$$

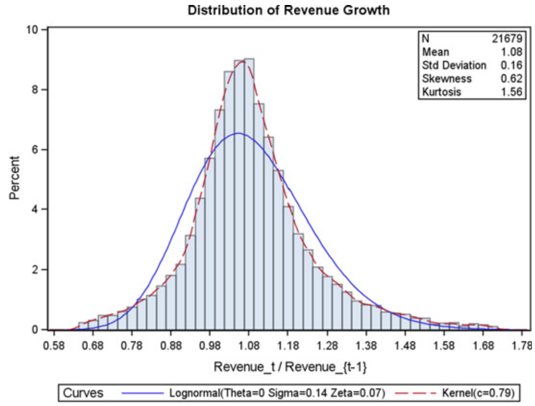
$$\equiv \alpha_S dt + \sigma_S dW, \tag{6}$$

where $\alpha_S = h\alpha_P + \alpha_N$ is the instantaneous drift of the stock and $\sigma_S = \sqrt{h^2\sigma_P^2 + \sigma_N^2}$ is the instantaneous standard deviation of the stock.

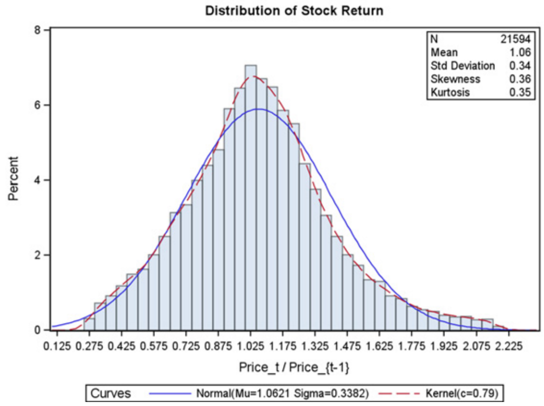
We value performance shares using the risk-neutral valuation method of Cox and Ross (1976). The risk-neutral valuation method was originally developed in the context of options that could be replicated by trading in the underlying stock and riskfree asset. But the risk-neutral method was subsequently extended to the valuation of option-like claims on non-traded variables. Under very general conditions, Cox et al. (1985) prove the equivalence of: (1) standard present-value valuation based on expected cash flows being discounted at an appropriate risk-adjusted discount rate and (2) risk-neutral valuation based on expected cash flows evaluated under the risk-neutral measure being discounted at the riskfree rate. Dixit and Pindyck (1994) and Hull (2012) show how the risk-neutral method can be applied to a wide range of "real options" problems (e.g., investments



(a) Distribution of EPS compared to the normal distribution.



(b) Distribution of revenue compared to the lognormal distribution.



(c) Distribution of stock returns compared to the normal distribution.

Fig. 4. Empirical distributions vs. normal or lognormal distributions. These figures show the distribution of three performance measures: (1) earnings per share, (2) revenue, and (3) stock returns for S&P 500 firms from 2000 to 2014 fiscal years compared to the normal or lognormal distribution.

with the option to shut down, investments with the option to delay initial investment, etc.) that depend on non-traded variables (e.g., sales revenue, production costs, etc.), rather than the price of a traded security.

Ingersoll (1987), page 295, shows that key differences when you value option-like claims on non-traded variables (and he specifically mentions executive compensation that depends on firm earnings as such an example) are that the resulting valuation includes variables that depend on the investor preferences (e.g., depend on their degree of risk aversion) and the value of these *preference-based variables* are determined in *equilibrium*. By contrast, in cases when it is possible to form a replicating portfolio (e.g., Black and Scholes, 1973 option pricing), then the resulting formula is always *preference-free* (e.g., the Black-Scholes formula does not depend on the instantaneous expected return of the stock α, nor on the market price of risk λ) and the value of the replicated claim is determined by *no-arbitrage* alone.

Our use of the risk-neutral method to value claims on non-traded variables is consistent with the advice provided by Brennan and Schwartz (1985) and Hodder et al. (2001), who show that static present-value valuation may be difficult-to-impossible to apply in settings with embedded options and/or non-linear payoffs; whereas risk-neutral valuation based on the dynamic option pricing paradigm may be much more tractable and useful. To implement the risk neutral valuation method, we transform the stochastic processes above to their corresponding risk-neutral processes. For a non-traded variable this is done by reducing the instantaneous growth rate by the market price of risk times the corresponding instantaneous standard deviation (σ_P).[17] Let ν be the market price of risk for this particular type of risk. Let $\hat{P}_t$ be the performance measure under the following risk-neutral process

$$d\hat{P} = (\alpha_P - \nu\sigma_P)\,dt + \sigma_P dW_1. \tag{7}$$

[17] See, Hull (2012), page 767.

For a traded asset, such as a stock, the instantaneous drift is adjusted to be the instantaneous riskfree rate r. Let $\hat{S}_t$ be the stock price under the following risk-neutral process

$$\frac{d\hat{S}}{\hat{S}} = rdt + \sigma dz, \tag{8}$$

where $\sigma \equiv \sqrt{h^2\sigma_P^2 + \sigma_S^2}$ and dz is an increment of a standard Wiener process. Based on these processes, the terminal value of the risk-neutral performance measure ($\hat{P}_T$) is normally distributed and the terminal value of risk-neutral stock price ($\hat{S}_T$) is log-normally distributed. For simplicity, let $\hat{Y}_T$ be the natural log of the risk-neutral stock return $\ln\left(\hat{S}_T/S_0\right)$, which is normally distributed. Thus, the distributions of $\hat{P}_T$ and $\hat{Y}_T$ are given by:

$$\hat{P}_T \sim \mathcal{N}\left(P_0 + (\alpha_P - \nu\sigma_P)\,T, \sigma_P^2 T\right) \text{ and} \tag{9}$$

$$\hat{Y}_T \sim \mathcal{N}\left(\left(r - \frac{1}{2}\sigma^2\right)T, \sigma^2 T\right), \tag{10}$$

where r is the riskfree rate that is assumed to be constant.

Finally, the correlation between $\hat{P}_T$ and $\hat{Y}_T$ is[18]

$$\rho = \frac{\sigma_P}{\sqrt{\sigma_P^2 + \frac{\sigma_S^2}{h^2}}}. \tag{11}$$

3.4. Performance share plans and performance-vested share plans

3.4.1. A non-traded measure following an arithmetic Brownian motion

Let $PSP_0^i\,(L, M, H, N_L, N_M, N_H)$ be the date 0 value of a performance share plan with threshold L, target M, stretch H, threshold shares N_L, target shares N_M, stretch shares N_H, and where the superscript $i \in \{A, G, R\}$ identifies one of the three types of performance measures. Let $PVS_0^i\,(L, N_L)$ be the date 0 value of a performance-vested share plan with a threshold L that pays N_L shares above the threshold and where the superscript $i \in \{A, G, R\}$ identifies one of the three types of performance measures. Let $LPS_0^i\,(X, \lambda_j)$ be the date 0 value of an linear performance share plan, where the first argument is the threshold $X \in \{L, M, H\}$, the second argument is the slope λ_j with $j \in \{L, H\}$), and where the superscript $i \in \{A, G, R\}$ identifies one of the three types of performance measures. In this subsection, an A superscript is used to identify variables in the case when a non-traded performance measure follows an Arithmetic Brownian Motion (e.g., earnings per share).

Proposition 1. *When a non-traded performance measure follows an Arithmetic Brownian Motion, the date 0 value of a performance share plan is*

$$PSP_0^A\,(L, M, H, N_L, N_M, N_H) = PVS_0^A\,(L, N_L) + LPS_0^A\,(L, \lambda_L) - LPS_0^A\,(M, \lambda_L) + LPS_0^A\,(M, \lambda_H) - LPS_0^A\,(H, \lambda_H), \tag{12}$$

the date 0 value of a performance-vested share plan is

$$PVS_0^A\,(L, N_L) = N_L S_0 N\left(d_1^A(L)\right), \tag{13}$$

and the date 0 value of an linear performance share plan is

$$LPS_0^A\,(X, \lambda_j) = S_0\lambda_j\left[\left\{P_0 + \left(\alpha_P - \nu\sigma_P + h\sigma_P^2\right)T - X\right\}N\left(d_1^A(X)\right) + \sigma_P\sqrt{T}n\left(d_1^A(X)\right)\right] \tag{14}$$

where $\lambda_L = \frac{N_M - N_L}{M - L}$, $\lambda_H = \frac{N_H - N_M}{H - M}$, $d_1^A(X) = \frac{P_0 - X + \left(\alpha_P - \nu\sigma_P + h\sigma_P^2\right)T}{\sigma_P\sqrt{T}}$ *and* $N(\cdot)$ *and* $n(\cdot)$ *are the cumulative distribution and density functions of the standard normal.*

[18] The correlation between $\hat{P}_T$ and $\hat{Y}_T$ can be derived as follows:

$$\rho = \frac{\sigma_{\hat{P}\hat{Y}}}{\sigma_{\hat{P}}\sigma_{\hat{Y}}} = \frac{h\sigma_P^2 T}{\sigma_P\sqrt{T}\sigma\sqrt{T}} = \frac{h\sigma_P}{\sigma} = \frac{h\sigma_P}{\sqrt{h^2\sigma_P^2 + \sigma_S^2}} = \frac{\sigma_P}{\sqrt{\sigma_P^2 + \frac{\sigma_S^2}{h^2}}}.$$

Intuitively, Eq. (12) shows that the value of a performance share plan is sum of current value of the five decomposition components from Table 3. Namely, the five components are: a performance-vested share plan with a threshold goal of L, a long position in an linear performance share plan with a threshold goal of L, a short position in an linear performance share plan with a threshold goal of M, a long position in an linear performance share plan with a threshold goal of M, and a short position in an linear performance share plan with a threshold goal of H. The performance-vested share plan formula has the intuitive interpretation of being the current stock price times the fixed number of shares times the probability that performance exceeds a threshold goal ($P_T > L$) under the risk neutral process. Furthermore, Eq. (14) shows that the value of a linear performance share plan is the product of the current stock price S_0, the slope of the payoff function λ_j, and the term in square brackets, which is the expected difference between the value of the performance measure and threshold goal X.

Note that the valuation formula for a linear performance share plan (Eq. (14)) depends on the current value of *two* random variables (P_0 and S_0). By contrast, the Black-Scholes option pricing model is based on the current value of *one* random variable (S_0). Since a linear performance share plan depends on non-traded variables, the resulting valuation formula (Eq. (14)) includes the preference-based variables α_P and ν, which depend on investor risk aversion and whose values are determined in equilibrium. By contrast, the Black-Scholes formula is preference-free and is determined by no-arbitrage alone.

Fig. 5 illustrates the date 0 value of a performance share plan. Fig. 5 (a) shows the date 0 value of a performance share as a function of: (1) current stock price S_0 and (2) current level of performance P_0. On the upper-left edge, we observe an S-shaped curve, where date 0 value increases non-linearly with current performance. On the upper-right edge, we observe that the date 0 value increases linearly with the current stock price. Fig. 5 (b) shows a cross-sectional slice for a given value of the current stock price. The solid blue curve is the date 0 value of a performance share plan, which rises rapidly from slightly below the threshold goal, continues rising in the incentive zone, slows down as current performance (P_0) approaches the stretch goal. The value asymptotically approaches $N_H S_0$. By analogy to the options literature, the red dashed line represents the *intrinsic value* of the performance share and the vertical gap between the date 0 value of a performance share and the intrinsic value represents the *time value* of the performance share plan. The time value is positive over most of the incentive zone, but turns slightly negative near the stretch goal H.

3.4.2. A non-traded measure following a geometric Brownian motion

Now we consider the case in which a non-traded performance measure follows a Geometric Brownian Motion. A Geometric Brownian Motion never goes negative, so this would be a candidate to represent performance measures that never go negative (e.g., revenue). Specifically, we assume that performance follows

$$\frac{dP}{P} = \alpha_P dt + \sigma_P dW_1. \tag{15}$$

and

$$\frac{dS}{S} = h\frac{dP}{P} + \alpha_N P dt + \sigma_N P dW_2, \tag{16}$$

where h is the sensitivity of the stock price to change in the performance measure, α_N is the instantaneous drift that is not performance measure related, σ_N is the instantaneous standard deviation that is not performance measure related, and dW_1 and dW_2 are increments of independent standard Wiener processes.

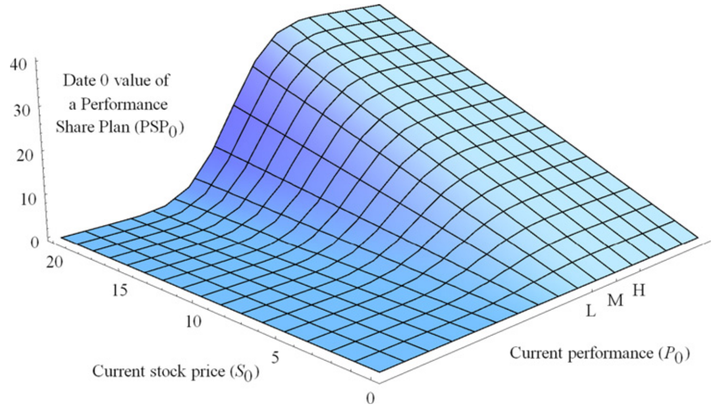
When accumulated over a period of time, a Geometric Brownian Motion yields a lognormal distribution for the terminal value. Fig. 4 (b) shows a bar graph distribution of revenue in year t divided by revenue on year $t-1$ for S&P 500 firms from 2000 to 2014 fiscal years. Indeed, we see that all observations are strictly positive. The empirical distribution (bar graph) is compared to the lognormal distribution (solid blue curve) that best fits the data. We see that the both tails of the empirical distribution are a bit thick. Without these thick tails, the best fit lognormal distribution would undoubtedly fit empirical distribution better in the middle by having a smaller standard deviation, and thus, a higher frequency in the middle. Overall, the empirical distribution fits the lognormal distribution reasonably well. By reverse engineering, this implies that a Geometric Brownian Motion is a reasonable distribution for the implied continuous process of revenue.

Substituting Eq. (15) into Eq. (16), we obtain

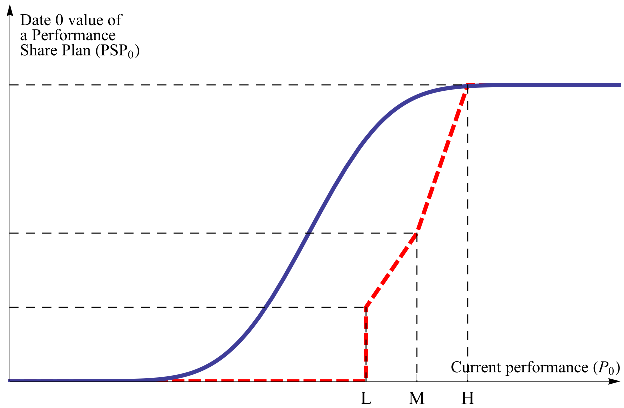
$$\frac{dS}{S} = (h\alpha_P + \alpha_N)\,dt + h\sigma_P dW_1 + \sigma_N dW_2 \tag{17}$$

$$\equiv \alpha_S dt + \sigma_S dW, \tag{18}$$

where $\alpha_S = h\alpha_P + \alpha_N$ is the instantaneous drift of the stock and $\sigma_S = \sqrt{h^2\sigma_P^2 + \sigma_N^2}$ is the instantaneous standard deviation of the stock. A G superscript is used to identify variables in the case when a non-traded performance measure follows a Geometric Brownian Motion.



(a) Date 0 value of a performance share plan PSP_0 as a function of: (1) current stock price S_0 and (2) current performance measure P_0



(b) A cross-sectional slice for a given value of the current stock price of the date 0 value of a performance share plan vs. its intrinsic value by the current performance measure P_0

Fig. 5. Date 0 value of a performance share plan. These figures show the date 0 value of a performance share plan. Panel (a) shows the date 0 value of a performance share plan PSP_0 as a function of: (1) current stock price S_0 and (2) current performance measure P_0. Panel (b) shows a cross-sectional slice for a given value of the current stock price. The solid blue curve shows how the date 0 value of a performance share PSP_0 changes with the current level of performance P_0. The red dashed line represents the intrinsic value of the performance share plan. The vertical gap between the date 0 value and the intrinsic value is the time value of the performance share plan.

Proposition 2. *When a non-traded performance measure follows a Geometric Brownian Motion, the date 0 value of a performance share plan is*

$$PSP_0^G\left(L,M,H,N_L,N_M,N_H\right) = PVS_0^G\left(L,N_L\right) + LPS_0^G\left(L,\lambda_L\right) - LPS_0^G\left(M,\lambda_L\right) + LPS_0^G\left(M,\lambda_H\right) - LPS_0^G\left(H,\lambda_H\right), \tag{19}$$

the date 0 value of a performance-vested share plan is

$$PVS_0^G\left(L,N_L\right) = N_L S_0 N\left(d_2^G(L)\right), \tag{20}$$

and the date 0 value of an linear performance share plan is

$$LPS_0^G\left(X,\lambda_j\right) = S_0\lambda_j\left[P_0 e^{\left(\alpha_P-\nu\sigma_P+h\sigma_P^2\right)T} N\left(d_1^G(X)\right) - XN\left(d_2^G(X)\right)\right] \tag{21}$$

where $\lambda_L = \frac{N_M-N_L}{M-L}$, $\lambda_H = \frac{N_H-N_M}{H-M}$, $d_1^G(X) = \frac{\ln\frac{P_0}{X}+\left(\alpha_P-\nu\sigma_P+\left(h+\frac{1}{2}\right)\sigma_P^2\right)T}{\sigma_P\sqrt{T}}$ *and* $d_2^G(X) = d_1^G(X) - \sigma_P\sqrt{T}$.

3.4.3. A rank-order tournament of traded asset returns

Now we consider the case in which a performance measure is a rank-order tournament of *traded* asset returns that are following Arithmetic Brownian Motions (e.g., percentiles of ranked stock returns). Specifically, performance is defined as the percentile value of one's own cumulative stock return over a specified time interval compared to the cumulative stock returns over the same time interval of a fixed set of peer firms.

Let k index a set of firms. Let $k = 1, 2, \ldots, K$ refer to a fixed set of K peer firms and let $k = K + 1$ be one's own firm. We assume that all $K + 1$ stock *prices* follow Geometric Brownian Motions. Thus, the kth instantaneous stock *return* dr_k follows an Arithmetic Brownian Motion as given by

$$dr_k = \alpha_k dt + \sigma_k dW_k \qquad k = 1, 2, \ldots, K + 1, \tag{22}$$

where α_k is the instantaneous drift of the kth stock, σ_k is the instantaneous standard deviation of the kth stock, and dW_k is an increment of a standard Wiener process for the kth stock.

When accumulated over a period of time, an Arithmetic Brownian Motion yields a normal distribution for the terminal value. Fig. 4 (c) shows a bar graph distribution of stock returns for S&P 500 firms from 2000 to 2014 fiscal years. The empirical distribution (bar graph) is compared to the normal distribution (solid blue curve) that best fits the data. The empirical distribution fits the normal distribution quite well with the main difference being a slightly thick right tail. By reverse engineering, this implies that an Arithmetic Brownian Motion is a reasonable distribution for the implied continuous process of stock returns. This is consistent with Martellini and Urosevic (2005), Dittmann and Maug (2007), and Dittmann et al. (2010), who develop models of executive compensation in which the terminal distribution of the stock price is lognormal. A lognormal distribution on the terminal date is the accumulated result of a continuous stock price following a Geometric Brownian Motion, or equivalently, a continuous stock return following an Arithmetic Brownian Motion.

Let Σ be a $(K + 1) \times (K + 1)$ matrix of instantaneous covariances between the $K + 1$ stock returns. Define r_t as a $K + 1$ vector of the cumulative stock returns over the time interval $[0, t]$ (where $t \in [0, T]$) for the $K + 1$ stocks.

Since all of the underlying random variables are the returns of traded assets, the corresponding kth risk neutral process $d\hat{r}_k$ is obtained by setting the instantaneous drift equal to the riskfree rate r as given by

$$d\hat{r}_k = r dt + \sigma_k dW_k \qquad k = 1, 2, \ldots, K + 1, \tag{23}$$

where the kth risk neutral process has the same starting point on date 0 ($\hat{r}_k = r_k = 1$) and maintains the same instantaneous covariance matrix Σ.

For a given realization of the cumulative, risk neutral returns at maturity, let h be the ranking at maturity (in ascending order) of the own risk neutral, cumulative return compared to the K peer firms' risk neutral, cumulative returns. To illustrate, if the own risk neutral, cumulative return is the lowest, then the rank would be $h = 1$. At the opposite extreme, if the own risk neutral, cumulative return is the highest, then the rank would be $h = K + 1$.

The performance measure at maturity P_T is the percentile value that corresponds to the maturity-date ranking h is given by the formula

$$P_T(h) = \frac{h - 1}{K}. \tag{24}$$

Since there are $K + 1$ possible ranks, it follows that the formula above yields $K + 1$ possible percentile values: $0 = \frac{0}{K}, \frac{1}{K}, \frac{2}{K}, \ldots, \frac{K}{K} = 1$. Let $\pi_h(r_t)$ be the probability that rank h will be obtained at maturity conditional on the current state of the cumulative stock return vector r_t.

Since there are $K + 1$ possible percentile values, it follows that there are $K + 1$ possible share payoffs at maturity (some of which could be the same). For a linear performance share (LPS) plan with threshold $X \in \{L, M, H\}$ and slope λ_j (where $j \in \{L, H\}$), the share payoff at maturity based on a rank h at maturity is

$$N_T(h) = \lambda_j (P_T(h) - X) = \lambda_j \left(\frac{h - 1}{K} - X \right). \tag{25}$$

For a performance-vested share (PVS) plan with a threshold L that pays N_L shares when performance is greater than or equal to the threshold, the share payoff at maturity based on a h ranking at maturity is $N_L I_L$ shares, where I_L is an indicator variable that equals one when $P_T(h) \geq L$ and zero otherwise.

Since the underlying assets are traded assets, the performance share plan can be valued by the no arbitrage approach of Black and Scholes (1973). A R superscript is used to identify variables in the case where the performance measure is a rank-order tournament of traded asset returns that follow Arithmetic Brownian Motions.

Proposition 3. *When a performance measure is a rank-order tournament of traded asset returns that follow Arithmetic Brownian Motions, the date 0 value of a performance share plan is*

$$PSP_0^R(L, M, H, N_L, N_M, N_H) = PVS^R(L, N_L) + LPS_0^R(L, \lambda_L) - LPS_0^R(M, \lambda_L) + LPS_0^R(M, \lambda_H) - LPS_0^R(H, \lambda_H) \tag{26}$$

the date 0 value of a performance-vested share plan is

$$PVS_0^R(L, N_L) = N_L S_0 e^{-rT} \sum_{h=1}^{K+1} \pi_h(r_0) I_L, \tag{27}$$

and the date 0 value of an linear performance share plan is

$$LPS_0^R(X, \lambda_j) = S_0 e^{-rT} \lambda_j \sum_{h=1}^{K+1} \pi_h(r_0) \max\left(\frac{h-1}{K} - X, 0\right), \tag{28}$$

where $\lambda_L = \frac{N_M - N_L}{M-L}$ *and* $\lambda_H = \frac{N_H - N_M}{H-M}$.

This formula is notably different than before as it is based on a discrete set of share payoffs, whereas the formulas in Propositions 1 and 2 are based on a continuous set of share payoffs. Further, the formula is a no arbitrage result and there are no investor preference parameters in it, because the underlying assets are traded assets. By contrast, Propositions 1 and 2 are equilibrium results and the formulas contain the investor preference parameters (α_P and ν), because the underlying assets are non-trading assets.

3.4.4. Factors affecting the value of performance shares

Fig. 6 (a) –(c) shows how the date 0 value of a performance share plan is affected by the contractual terms of the performance share plan. Fig. 6 (a) shows the date 0 value of a performance share plan for different widths of incentive zone. L and H represent a narrow incentive zone and L' and H' represent a wide incentive zone. A wide incentive zone plan is more valuable at low levels of current performance and a narrow incentive zone plan is more valuable at high levels of current performance. Notice that both curves have two humps, not just one hump. A variety of shapes are possible.

Fig. 6 (b) shows the date 0 value of a performance share plan for different times to maturity (meaning different lengths of a performance period). More time increases the value of a performance share plan, because it increases the time value component (the extra value above the intrinsic value).

Fig. 6 (c) shows the date 0 value of a performance share plan when there is a convex kink in the payoff function at the target vs. a concave kink in the payoff function at the target. We can see that the value of two performance share plans with different type of kinks have huge price gap around the kinks, although the gap narrows as you move away from the target. It implies that choosing the type of kink at the performance target will change the shape of the value function.

Fig. 6 (d) –(f) shows how the date 0 value of a performance share plan is affected by various environmental factors. Fig. 6 (d) shows the date 0 value of a performance share for different volatilities of the performance measure. For high (low) values of current performance, higher volatility decreases (increases) the value of a performance share plan because there are limited potential gains (losses) on the upside (downside) and greater potential losses (gains) on the downside (upside).

Fig. 6 (e) shows the date 0 value of a performance share plan for different stock sensitivities to performance. The case of a higher stock sensitivity h has a higher date 0 value of a performance share plan. Intuitively, for a given performance distribution, a higher stock sensitivity shifts the entire distribution of stock prices higher and thus increases the date 0 value of a performance share plan. A higher stock sensitivity h maps directly into a higher correlation ρ between performance under the risk neutral measure and stock return under the risk neutral measure (see Eq. (11)). Thus have the equivalent relationship that a higher correlation between performance and stock return leads to a higher date 0 value of a performance share plan.

Finally, Fig. 6 (f) shows how the date 0 value changes as the risk premium on non-traded assets change. Because the risk premium negatively affects the attractiveness of a plan with non-traded asset as a performance measure, we can observe that the date 0 value of a performance share plan decreases as the risk premium is higher.

3.5. A generalized performance share plan

The base model can be generalized to fit real-world payoff structures of performance share plans. While many firms offer plans with one kink at the target, many other firms have more complicated structure with multiple kinks within the incentive zone. Our model can be generalized to value any performance share plans with multiple kinks. Let X_1 be the threshold goal with N_1 being the jump in payoff at the threshold goal. Suppose a performance share plan has C kinks. Let X_{c+1}, where $c = 1, 2, \ldots, C$, be the $(c+1)^{st}$ intermediate goal that pays N_{c+1} shares. Let X_{C+2} be the stretch goal that pays N_{C+2} shares. Let λ_c be the slope of the payoff structure between two adjacent goals X_c and X_{c+1}, where $c = 1, 2, \ldots, C+1$.

Proposition 4 (*Generalized Performance Share Plan*). *Under three alternative assumptions about the performance measure, the date 0 value of a generalized performance share plan is*

$$PSP_0^i = PVS_0^i(X_1, N_1) + \sum_{c=1}^{C+1} \left[LPS_0^i(X_c, \lambda_c) - LPS_0^i(X_{c+1}, \lambda_c) \right] \tag{29}$$

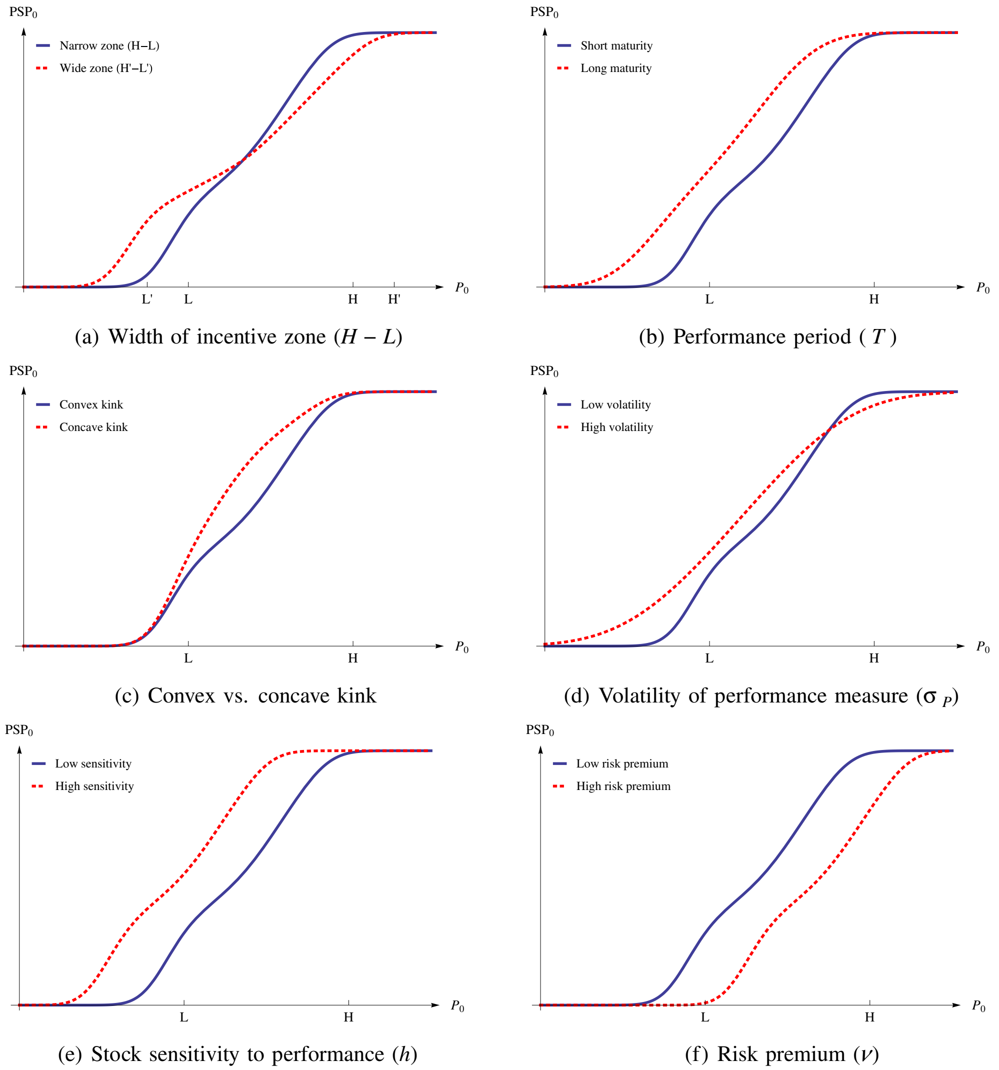


Fig. 6. Factors affecting the date 0 value of a performance share plan. These figures show how different factors affect the value of a performance share plan. All of the figures are cross-sectional slices for a given value of the current stock price and show the value of a performance share plan as a function of the current level of performance measure P_0. Panels (a), (b), and (c) show the value of a performance share plan PSP_0 by the contractual terms: the width of incentive zone $(H - L)$, performance period T, and whether the payoff function has convex kink or concave kink. Panels (d), (e), and (f) show the value of a performance share plan PSP_0 by various environmental factors: the volatility of performance measure (σ_P), the stock sensitivity to performance (h), and the risk premium on non-traded performance measures (ν).

where $i = \{A, G, R\}$ *and where* $PVS_0^i\ (X_1, N_1)$ *and* $LPS_0^i\ (X_c, \lambda_c)$ *are calculated following* Propositions 1–3.

In the special case where $C = 1$, we obtain Propositions 1–3.

4. Empirical tests

4.1. Ex-ante tests

In this subsection, we empirically test our valuation formulas by comparing them to other ex-ante measures, namely, reported value on proxy statements and heuristic value. We hand-collect plan parameters whenever they are reported on the firms' definitive proxy statements. We collect data on all firms that were in the S&P 500 index as of January 2006. The sample period is from fiscal years ending on or after December 15, 2006 to fiscal years ending on or before December 31, 2012. We limit the scope to performance measures whose definitions are standard across firms, which includes earnings per share (EPS), revenue, and percentile of ranked stock returns. By contrast, the definitions of ROIC and ROE are quite different across firms.

To value the plans, we need to have six design parameters for performance share plans or two parameters (threshold goal and threshold shares) for performance-vested share plans. We are able to identify 711 firm-years of performance share plans and 57 firm-years of performance-vested share plans with all required data.

Table 4 reports the design parameters of performance share plans and performance-vested share plans based on earnings per share (EPS), revenue, or percentile of ranked stock returns. The first three columns report the threshold goal (L), target goal (M), and stretch goal (H) divided by the performance level in the previous year (P_0) for the EPS and revenue performance measures, but not normalized by the prior year performance for the percentile of ranked stock return measure. The next three columns report threshold shares (N_L), target shares (N_M), and stretch shares (N_H) divided by the target shares (N_M). Panel A shows the subsample of performance share plans using Earnings Per Share, Revenue, and Percentile of Ranked Stock Returns. The ratio of the threshold goal/prior performance has a mean value of 1.19 for EPS plans (this value is inflated due to a huge outlier) and 1.02 for revenue plans and the threshold percentile is 0.31 for percentile plans. The ratio of stretch goal/prior performance has a mean value of 1.38 for EPS plans, 1.11 for revenue plans and the stretch percentile is 0.84 for percentile plans. Panel B reports the threshold goal (which equals target and stretch goals) divided by prior performance for performance-vested share plans using Earnings Per Share and Revenue (there weren't any based on percentile of ranked stock returns). We find that the ratio of the threshold goal/prior performance has a mean value of 0.88 for EPS plans and 1.08 for revenue plans.

Next, we compare our new valuation formulas versus the reported value on proxy statements versus heuristic value. Firms report the grant date fair value of equity awards in the *Grants of Plan-based Award Table* in their annual proxy statements. In the majority of cases, the reported value is the same as heuristic value, which is defined below.

For performance share plans, heuristic value is arrived at by supposing that the performance outcome at maturity will exactly equal the target value ($P_T = M$). In this case, the number of shares that will be awarded will be N_M. Then, the heuristic value method values that number of shares at S_0, the current stock price at time 0. Let PSP_0^H be the heuristic value of a performance share plan at time 0, which is given by

$$PSP_0^H = N_M S_0. \tag{30}$$

For performance-vested share plans, recall that the manager receives the threshold shares (N_L) when the performance threshold goal is achieved. Heuristic value assumes that the performance will equal or exceed the threshold goal *L for certain* and then values the fixed number of shares at S_0. Let PVS_0^H be the heuristic value of a performance-vested share plan at time 0, which is given by

$$PVS_0^H = N_L S_0. \tag{31}$$

Table 5 compares the formula value based on Propositions 1, 2, and 3 versus the reported value on proxy statements versus the heuristic value for performance share plans and performance-vested share plans based on EPS, revenue, or percentile of

Table 4
Design parameters. This table reports the design parameters of performance share plans and performance-vested share plans with earnings per share (EPS), revenue, and percentile of ranked stock return as a performance measure. The data is hand-collected whenever they are reported on the proxy statements of S&P 500 firms for fiscal years ending from December 15th, 2006 to December 31st, 2012. This results in six parameters for 711 firm-years of performance share plans and two parameters for 57 firm-years of performance-vested share plans. Threshold Goal L, Target Goal M, and Stretch Goal H are divided by the previous year's performance level P_0 for the EPS and revenue plans.

	Goals			Shares		
	Threshold (L/P_0)	Target (M/P_0)	Stretch (H/P_0)	Threshold (N_L/N_M)	Target (N_M/N_M)	Stretch (N_H/N_M)
Panel A. Performance share plans						
Using EPS (198 firm-years)						
Mean	1.19	1.30	1.38	0.32	1.00	1.69
Median	1.03	1.09	1.14	0.33	1.00	2.00
Using revenue (93 firm-years)						
Mean	1.02	1.07	1.11	0.35	1.00	1.80
Median	1.03	1.06	1.10	0.49	1.00	2.00
Using percentile of ranked stock returns (420 firm-years)						
Mean	0.31	0.54	0.84	0.34	1.00	1.93
Median	0.30	0.50	0.84	0.33	1.00	2.00
Panel B. Performance-vested share plans						
Using EPS (47 firm-years)						
Mean	0.88					
Median	1.05					
Using revenue (10 firm-years)						
Mean	1.08					
Median	1.06					

Table 5
Formula value versus reported value versus heuristic value. This table compares the formula value based on Propositions 1, 2, and 3 versus the reported value on proxy statements versus the heuristic value for a set of performance share plans and performance-vested share plans with earnings per share (EPS), revenue, or percentile of ranked stock returns as the performance measure. The input values are hand-collected whenever they are reported on the proxy statements of S&P 500 firms for fiscal years ending from December 15th, 2006 to December 31st, 2012. * and ** mean statistical significance at the 10% and 5% level, respectively, based on the *t*-test for the difference in means and the Wilcoxon test for the difference in medians.

	Formula Value ($Mil.)	Reported Value ($Mil.)	Heuristic Value ($Mil.)	% Difference (Formula-reported)	% Difference (Formula-heuristic)	% Difference (Reported-heuristic)
Panel A. Performance share plans						
Using EPS (198 firm-years)						
Mean	2.85	2.81	2.70	1.22%	7.21%*	5.99%**
Median	1.96	1.88	1.86	3.47%	7.96%**	0.00%
Using revenue (93 firm-years)						
Mean	1.81	1.80	1.75	8.49%	13.99%**	5.51%
Median	1.55	1.41	1.34	5.55%*	8.28%**	0.00%
Using percentile of ranked stock returns (420 firm-years)						
Mean	1.98	2.36	2.22	−16.0%***	−10.7%**	6.3%**
Median	1.42	1.72	1.69	−17.3%***	−15.8%**	1.7%**
Panel B. Performance-vested share plans						
Using EPS (47 firm-years)						
Mean	2.06	2.61	2.60	−22.77%**	−23.07%**	−0.30%
Median	1.94	2.50	2.48	−12.34%**	−14.88%**	0.00%
Using revenue (10 firm-years)						
Mean	2.45	3.21	3.25	−22.70%**	−25.60%**	−2.90%
Median	1.17	1.38	1.40	−24.21%**	−24.41%**	−0.50%

ranked stock returns. For EPS measure firms, we use price-to-earnings ratio as the sensitivity of stock price to performance improvement (h), and for revenue measures, we use return-to-revenue growth (previous year's stock return divided by previous year's revenue growth) as h. We calibrate α_P to the prior five-year average of EPS increments or revenue growth rates, σ_P to the prior five-year standard deviation of EPS increments or revenue growth rates, ν to $(\alpha_P - r_f P_{(t-1)})/\sigma_P$, and r_f to the yield on 3-month US Treasury Bills.

Panel A covers performance share plans using EPS, revenue, or percentile of ranked stock return. Panel B covers performance-vested share plans using EPS or revenue.[19] Looking at the fourth column, our formula values are relatively close to reported values for performance share plans based on both EPS and revenue (i.e, the percentage differences are small and insignificant in three out of four cases). However, continuing down the fourth column, our formula values are large and significantly lower than reported values for performance share plans based on the percentile of ranked stock returns and for performance-vested share plans, where the differences range from −12.34% to −24.21%. In the fifth column, our formula values are significantly different than heuristic values in all cases. Given the large size of the differences, the differences are economically significant, as well as being statistically significant.

Why are there differences? One reason is that the differences in Panel A for performance share plans are driven by the fact that reported values (typically based on heuristic value) and heuristic values make the counterfactual assumption that hitting the performance target is the most likely outcome. By contrast, our new valuation formulas account for the true distribution of the performance measure. Similarly, the difference in Panel B for performance-vested share plans are driven by the fact that reported values (typically based on heuristic value) and heuristic value make the extremely optimistic assumption that performance *always* exceeds the threshold goal. By contrast, our new valuation formulas account for the true probability of exceeding the threshold goal, which is realistically lower than 100%.

The category of performance share plans based on the percentile of ranked stock returns is a bit more of a puzzle since firms are required to adopt a formal valuation model and in practice most firms use a Monte Carlo simulation-based approach. So why would firms tend to systematically report *higher* grant date values than our formula values? We think that a plausible explanation for this finding is that if human resource personnel or hired consultants have some discretion in the choice of model parameters and assumptions, then the strong economic incentive for firms is to report as *high of an accounting value* as they can legitimately justify so as to minimize taxes. That is, holding the actual economic value of the executive compensation contract fixed, if they can use modeling discretion to report, say, a half million dollars higher in accounting/grant date value, then the firm's taxable income would be a half million dollars less. Such a tax minimization strategy would lead to reported values being higher than formula values.

[19] There aren't any performance-vested share plans based on percentile of ranked stock return.

Table 6
Formula, reported, and heuristic values compared to actual payout using historical average. This table compares formula value, reported value, and heuristic value to actual payout. Formula value is based on Propositions 1, 2, and 3, assuming expected performance follows previous 5-year historical average. Reported value is based on proxy statements. Heuristic value are simple rule of thumb that are currently used, which is the product of the closing share price on the grant date and the target number of shares. Actual payout is the actual number of share awarded (based on the pre-defined payoff formula) times the stock price at maturity. The sample is a set of performance share plans and performance-vested share plans with earnings per share (EPS), revenue, or rank order of stock returns as the performance measure for which we can obtain realized performance outcome. Negative values of dif-in-absolute-dif imply that the absolute valuation errors using our model are smaller than the absolute valuation errors using reported/heuristic values. The input values are hand-collected whenever they are reported on the proxy statements of S&P 500 firms for fiscal years ending from December 15th, 2006 to December 31st, 2012. * and ** mean statistical significance at the 10% and 5% levels, respectively, based on the *t*-test (Wilcoxon test) for the difference in means (median).

					Dif-in-dif		Dif-in-absolute-dif	
	Actual payout ($Mil.)	Formula value ($Mil.)	Reported value ($Mil.)	Heuristic value ($Mil.)	Formula vs. reported (%)	Formula vs. heuristic (%)	Formula vs. reported (%)	Formula vs. heuristic (%)
Panel A. Performance share plans								
Using EPS (159 observations)								
Mean	3.04	2.81	2.89	2.76	−7.92%	4.92%	8.69%**	5.62%
Median	1.79	2.03	1.90	1.89	−7.36%	−0.12%	7.81%*	4.13%
Using revenue (84 observations)								
Mean	1.59	2.00	1.84	1.78	15.80%	21.66%	8.90%	16.43%**
Median	0.94	1.64	1.45	1.37	15.56%**	19.52%**	16.13%**	24.47%**
Using percentile of ranked stock returns (141 firm-years)								
Mean	2.41	1.98	2.36	2.22	−16.0%**	−10.7%**	−6.72%	2.93%
Median	0.78	1.42	1.72	1.69	−17.3%**	−15.8%**	−0.14%	1.84%
Panel B. Performance-vested share plans								
Using EPS (31 observations)								
Mean	1.95	1.99	2.57	2.57	−58.35%**	−58.56%**	16.23%**	16.12%**
Median	1.94	1.94	2.50	2.50	−41.88%**	−46.14%**	6.45%**	6.45%**
Using revenue (6 observations)								
Mean	4.51	3.75	4.52	4.60	−77.45%*	−84.95%*	12.79%	17.63%
Median	1.43	0.87	1.28	1.40	−62.91%*	−63.35%**	26.52%	30.03%

Looking at the sixth column, the differences between reported values and heuristic values are relatively small and mostly insignificantly different from each other. This provides evidence that heuristic value is typically the basis for reported values on proxy statements.

4.2. Ex-post tests

For 421 firm-years, we are able to determine the value of the performance measure at the plan maturity date. For these firm-years, we compute the actual dollar payout by plugging the actual performance outcome of EPS, revenue, or percentile of ranked stock return into the payoff structure of the plan to obtain the actual shares awarded and then multiplying by the realized stock price at the end of the performance period S_T.[20]

We perform formal statistical tests of the difference-in-difference. We first calculate valuation errors of our formula value, reported value, and heuristic values, where valuation error is defined as the difference between the valuation and the actual payout. And then we subtract valuation error of reported/heuristic values from the valuation error of our formula. We scale the valuation error by heuristic value to normalize across firms. The difference-in-difference statistic is thus

$$\begin{aligned} Dif-in-Dif &= \frac{(Formula-Actual)-(Reported-Actual)}{Heuristic} \\ &= \frac{Formula-Reported}{Heuristic}. \end{aligned} \tag{32}$$

As such, the dif-in-dif analysis is actually just testing whether the two valuation is statistically different or not. We thus added another analysis which is difference-in-absolute-difference. In this analysis, we compare the absolute valuation errors across three valuation methodologies.

$$Dif-in-Absolute-Dif = \frac{|\text{Formula}-\text{Actual}|-|\text{Reported}-\text{Actual}|}{\text{Heuristic}}. \tag{33}$$

[20] In unreported results and in the spirit of Shiller (1981) who compares stock prices to a "perfect foresight dividend series", we also compare our formula value, reported value, and heuristic value to a "perfect foresight value". Perfect foresight value is the actual share award times the date 0 stock price. The perfect foresight results are qualitatively similar to the actual payout results in Table 6 below.

Table 7
Formula, reported, and heuristic values compared to actual payout using analyst consensus. This table compares formula value, reported value, and heuristic value to actual payout. Formula value is based on Propositions 1, 2, and 3, assuming expected performance follows analyst consensus just prior to the grant date. If there doesn't exist analyst consensus prior to the grant date, we take the very first analyst consensus of the year as the proxy. Reported value is based on proxy statements. Heuristic value are simple rule of thumb that are currently used, which is the product of the closing share price on the grant date and the target number of shares. Actual payout is the actual number of share awarded (based on the pre-defined payoff formula) times the stock price at maturity. The sample is a set of performance share plans and performance-vested share plans with earnings per share (EPS), revenue, or rank order of stock returns as the performance measure for which we can obtain realized performance outcome. Negative values of the dif-in-absolute-dif imply that the absolute valuation errors using our model are smaller than the absolute valuation errors using reported/heuristic values. The input values are hand-collected whenever they are reported on the proxy statements of S&P 500 firms for fiscal years ending from December 15th, 2006 to December 31st, 2012. * and ** mean statistical significance at the 10% and 5% levels, respectively, based on the *t*-test (Wilcoxon test) for the difference in means (median).

					Dif-in-dif		Dif-in-absolute-dif	
	Actual payout ($Mil.)	Formula value ($Mil.)	Reported value ($Mil.)	Heuristic value ($Mil.)	Formula vs. reported (%)	Formula vs. heuristic (%)	Formula vs. reported (%)	Formula vs. heuristic (%)
Panel A. Performance share plans								
Using EPS (159 observations)								
Mean	3.04	2.88	2.89	2.76	−0.77%	12.07%	−0.29%	−3.36%
Median	1.79	1.85	1.90	1.89	0.37%	7.60%	2.08%	−2.62%
Using revenue (84 observations)								
Mean	1.59	1.75	1.84	1.78	−9.01%	−3.15%	−0.66%	6.87%*
Median	0.94	1.43	1.45	1.37	2.19%	1.66%	6.91%	5.91%**
Panel B. Performance-vested share plans								
Using EPS (31 observations)								
Mean	1.95	1.79	2.57	2.57	−78.03%**	−78.24%**	15.42%**	15.32%**
Median	1.94	1.96	2.50	2.50	−44.90%**	−42.95%**	7.76%**	17.83%**
Using revenue (6 observations)								
Mean	4.51	3.86	4.52	4.60	−66.00%**	−73.50%**	0.96%	15.81%
Median	1.43	0.85	1.28	1.40	−62.97%*	−63.41%**	25.45%	25.75%

Statistical analysis in difference-in-absolute-difference tests whether our formula value generates valuation error that is statistically smaller than reported/heuristic values. The results are provided in Tables 6, 7 and 8.

We note that ex-post comparison of our ex-ante formula values to the ex-post dollar payout is inherently very noisy. For example, suppose you wanted to test whether the mean value of two thrown dice is equal to seven by comparing to a sample of ex-post rolls. The ex-post rolls will range in value from two to twelve and the most of rolls will have values different than seven. It is certainly possible to do the test this way, but the difference between ex-ante and ex-post values includes a noisy error term that may be especially important in smaller samples.

Tables 6 to 8 show the formula value based on Propositions 1, 2, and 3, the reported value, and the heuristic value compared to the ex-post actual dollar payout. The sample is a subset of performance share plans and performance-vested share plans for which we are able to determine the value of the performance measure at the plan maturity date. Tables 6 to 8 provide analysis based on different methods to forecast future firm performance. In Table 6, we use historical 5-year average to estimate future firm performance. In Tables 7 and 8, we use analyst consensus to estimate future firm performance. In Table 7, if the analyst consensus data prior to the grant date is missing, we take the very first analyst consensus of the year as the proxy for market belief on the firm performance. In Tables 8, we only include plans with available analyst consensus data prior to the grant date.

The first four columns report actual payout, formula value, reported value, and heuristic value in millions of dollars. The next two columns report the difference-in-difference results. The last two columns report the difference-in-absolute difference results to test the efficiency of the model.

Interestingly, the results are drastically different across tables. Table 6 shows that, when historical average is used, our formula yields valuations with larger valuation errors than reported or heuristic values. Table 7 shows that when we use analyst consensus as the proxy for firm performance, the valuation errors of our formulas are roughly equivalent to the valuation errors from reported or heuristic values for performance share plans. Table 8 show that when we use analyst consensus issued before the grant date, our valuation formulas have lower valuation errors for performance share plans based on EPS. From these tables, we can conclude that the efficiency of our valuation model greatly depends on the method used to forecast future firm performance.

The policy implication is that FASB should consider changing the accounting treatment, especially for performance share plans based on EPS, to require that grant date fair value be estimated by valuation formulas such as ours. This is analogous to the way that FASB 123R requires that stock options be valued on the grant date by one of several option pricing models. This change would provide shareholders with a more accurate assessment of the true cost of these plans. An extensive accounting literature establishes that more accurate accounting disclosure yields real economic benefits, including the more efficient allocation of resources (for example, see Healy and Wahlen, 1999).

Table 8
Formula, reported, and heuristic values compared to actual payout using analyst consensus prior to the grant date. This table repeats the analysis in the previous table with formula value derived using subsample of firms with exact pre-grant date analyst consensus values. Reported value is based on proxy statements. Heuristic value are simple rule of thumb that are currently used, which is the product of the closing share price on the grant date and the target number of shares. Actual payout is the actual number of share awarded (based on the pre-defined payoff formula) times the stock price at maturity. The sample is a set of performance share plans and performance-vested share plans with earnings per share (EPS), revenue, or rank order of stock returns as the performance measure for which we can obtain realized performance outcome. Negative values of the dif-in-absolute-dif imply that the absolute valuation errors using our model are smaller than the absolute valuation errors using reported/heuristic values. The input values are hand-collected whenever they are reported on the proxy statements of S&P 500 firms for fiscal years ending from December 15th, 2006 to December 31st, 2012. * and ** mean statistical significance at the 10% and 5% levels, respectively, based on the *t*-test (Wilcoxon test) for the difference in means (median).

					Dif-in-dif		Dif-in-absolute-dif	
	Actual payout ($Mil.)	Formula value ($Mil.)	Reported value ($Mil.)	Heuristic value ($Mil.)	Formula vs. reported (%)	Formula vs. heuristic (%)	Formula vs. reported (%)	Formula vs. heuristic (%)
Panel A. Performance share plans								
Using EPS (89 observations)								
Mean	3.06	2.67	2.60	2.53	7.08%	13.82%	−7.83%	−9.02%*
Median	1.68	1.42	1.50	1.48	−3.62%	5.36%	−4.03%	−5.84%**
Using revenue (45 observations)								
Mean	1.15	1.40	1.55	1.40	−15.41%	−0.03%	−7.55%	3.86%
Median	0.84	1.42	1.35	1.30	6.71%	9.67%	6.11%	5.38%
Panel B. Performance-vested share plans								
Using EPS (14 observations)								
Mean	2.30	1.70	2.46	2.45	−75.77%**	−75.06%**	29.91%**	30.25%**
Median	2.31	2.04	2.33	2.33	−64.52%**	−61.27%**	23.17%**	25.74%**
Using revenue (4 observations)								
Mean	1.45	0.85	1.32	1.43	−47.34%	−58.58%	32.71%	39.99%*
Median	1.43	0.85	1.28	1.40	−54.03%	−55.49%	46.28%	51.04%

5. Conclusion

We document the size and importance of performance share plans and performance-vested share plans. Next, we derive closed-form formulas for the value of a performance share plan or performance-vested share plan when the performance measure is: (1) a non-traded measure following an Arithmetic Brownian Motion (e.g., earnings per share), (2) a non-traded measure following a Geometric Brownian Motion (e.g., revenue), or (3) a rank-order tournament of traded asset returns that following Arithmetic Brownian Motions (e.g., percentile of ranked stock returns). Finally, we compare the actual payout of plans to our new valuation formulas, the reported values on proxy statements, and heuristic values. We find that our valuation formulas are more accurate for performance share plans based on EPS when forecasting using analyst consensus prior to the grant date. We also find that the efficiency of our valuation model greatly depends on the method used to forecast future firm performance. The policy implication is that FASB should consider requiring that grant date fair value be estimated using valuation formulas such as ours.

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Appendix A. Examples of proxy filings for different types of awards

The main source of the data used in our analysis is definitive proxy filings by our sample firms. Starting from fiscal years ending in December 2016, firms are required to include more detailed information about executive compensation in their proxy filings. In addition to the Summary Compensation Table, firms are now required to disclose the Grants of Plan-based Awards Table, which includes detailed information about incentive plans granted to executives. Together with narrative disclosure in Compensation Discussion and Analysis, the new disclosure rule enables investors to better understand the design and real economic value of executive compensation. We include below some examples of the equity incentive plans included in our study.

A.1. Performance share plan based on EPS growth rate of McDonald's in FY2011

Grants of Plan-based Awards - Fiscal 2011

		Estimated future payouts under equity incentive plan awards (1)			
Name	Grant date	Threshold	Target	Maximum	Grant date fair value (2)
James A. Skinner	2/9/11	5,268	21,071	21,071	$1,429,035

1. Reflects grants of RSUs subject to performance-based vesting conditions under the Equity Plan. The RSUs vest on February 9, 2014, subject to achievement of a specified EPS growth target during the performance period ending on December 31, 2013. The performance target for all RSU awards granted to the NEOs in 2011 is compounded annual EPS growth of 6% on a cumulative basis, adjusted to exclude certain items as described on page 14. If target is achieved, 100% of the RSUs will vest. If no compounded EPS growth is achieved, no RSUs will vest. If compounded EPS growth is achieved, but below target, the awards will vest proportionally.
2. The values in this column for RSUs and options were determined based on the assumptions described in proxy footnotes 2 and 3, respectively, to the Summary Compensation Table on page 19.

This plan of McDonald's is classified as a performance share plan as the number of shares that may finally be awarded to the CEO may range from zero shares (when compounded EPS growth is 0% or less) to target number of shares (21,071 shares) depending on the EPS growth rate during the performance period. If very small positive compounded EPS growth rate is achieved, 5268 shares will be awarded. Proxy footnote 2 directs readers to a footnote to Summary Compensation Table, which states "Represents the aggregate grant date fair value, as computed in accordance with FASB ASC Topic 718, based on the probable outcome of the applicable performance conditions and excluding the effect of estimated forfeitures during the applicable vesting periods, of RSUs granted under the McDonald's Corporation Amended and Restated 2001 Omnibus Stock Ownership Plan, as amended (Equity Plan)."

A.2. Performance-vested share plan of COSTCO Wholesale Corp in FY2011

Fiscal 2011 Grants of Plan-based Awards

		Estimated future payouts under equity incentive plan awards (1)			
Name	Grant date	Threshold	Target	Maximum	Grant date fair value (2)
James D. Sinegal	10/22/10		25,000		$1,560,015

1. Represents the number of performance-based RSUs granted to the Named Executive Officers during fiscal 2011, subject to attainment of the performance criteria described under " Compensation Discussion and Analysis - Equity Compensation ". After the end of fiscal 2011, the Committee determined that the performance criteria had been met and the awards were earned. The earned awards vest 20% on the first anniversary of the grant date and an additional 20% vest over each of the ensuing four years, with acceleration of vesting for long service.
2. Represents the grant-date fair value of RSU awards granted, computed as described in proxy footnote 2 to the Summary Compensation Table above.

This plan is classified as a performance-vested restricted share plan as the number of shares that may finally be awarded to the CEO is either zero (when the performance is below the target goal) or the target number of shares (25,000 shares) when the performance is at or above the target performance goal. However, from the Grants of Plan-based award table alone, we don't have any information on the performance measure or goals. Proxy footnote 2 directs readers to a footnote to Summary Compensation Table, which state "The grant-date fair value is calculated as the market value of the common stock on the measurement date less the present value of the expected dividends forgone during the vesting period." To obtain information on the performance measures and goals of this plan, we must read the Compensation Discussion and Analysis section (CD&A). "For Mr. Sinegal, the criteria were a 3% increase (versus fiscal 2010) in total sales or pretax income (with both measures based on local currency). After the end of fiscal 2011, the Committee determined that both goals were achieved." From this statement, we know that the company used two performance measures: growth in total sales and growth in profit before tax. The target goal was 3% increase for both measures. However, it is unclear how the target number of shares are split between the two performance measures, and we assume that they are divided equally between the two performance measures.

A.3. Performance share plan based on percentile of ranked stock returns compared to a peer group for KIMCO Realty Corp in FY 2012

Grants of Plan-based Award for 2012

		Estimated future payouts under equity incentive plan awards (1)			
Name	Grant date	Threshold	Target	Maximum	Grant date fair value (2)(3)
David B. Henry	2/16/2012	31,100	62,200	93,300	$1,289,406

1. All awards are granted under the Kimco Realty Corporation 2010 Equity Participation Plan. (2) Represents the grant-date fair value of RSU awards granted, computed as described in proxy footnote 2 to the Summary Compensation Table above.
2. Fair value is determined, depending on the type of award, using the Black-Scholes option pricing formula, the Monte Carlo method or the closing price per share of our Common Stock on the date of grant, which are intended to estimate the grant date fair value of the options, the performance shares and restricted stock, respectively. The assumptions used by the Company in calculating these amounts are incorporated herein by reference to Note 23 to Consolidated Financial Statements in the Company's 2012 Form 10-K. See proxy footnote 6 for a discussion of Mr. Cohen's retention award.
3. The actual awards are set out in the 2012 Performance Share Awards Table.

This is a typical performance share plan with threshold, target, and maximum number of shares clearly stated in the table. Again, other than the number of shares to be awarded for each performance level, no other information on the performance measures or goals are provided in the table. Proxy footnote 3 additionally tells us that the fair value for the performance shares is derived using Monte Carlo simulation method. In CD & A, additional information on the performance share plan is provided.

	Company's 1 year total stockholder Return percentile in peer group			
	$< 25\%$	25%	50%	$\geq 75\%$
Restricted stocks granted	0%	50%	100%	150%

Restricted stock is granted on a linear scale between the 25% and 75% performance percentile. The company uses total stockholder return relative to its peer group of companies, which is also provided in CD&A and below. The potential payout ranges from zero shares (if one-year TSR percentile rank is below 25th percentile), 50% of target number of shares, if the percentile rank is 25th percentile, to 150% of target number of shares if the percentile rank is 75th percentile or above. The specific list of peer firms is:

Acadia Realty Trust	The Macerich Company
Agree Realty Corp.	National Retail Properties Inc.
Alexander's Inc.	Pennsylvania Real Estate Investment Trust
CBL & Associates Properties Inc.	Ramco-Gershenson Properties Trust
Cedar Shopping Centers Inc.	Realty Income Corporation
Developers Diversified Realty Corp.	Regency Centers Corp.
Equity One Inc.	Retail Opportunity Investment Corp.
Excel Trust	Saul Centers Inc.
Federal Realty Investment Trust	Simon Property Group Inc.
General Growth Properties, Inc.	Tanger Factory Outlet Centers Inc.
Getty Realty Corp.	Taubman Centers Inc.
Glimcher Realty Trust	Urstadt Biddle Properties Inc.
Inland Real Estate Corp.	Weingarten Realty Investors
Kite Realty Group Trust	

Appendix B. Proofs

B.1. Proof of Proposition 1

Table 3 shows that the five components have the same payoff at maturity as a performance share plan. Therefore, as shown in Eq. (12), the date 0 value of a performance share plan must be equal to the date 0 value of the five components in the absence of arbitrage.

The date 0 value of a performance-vested share plan is the expected value of the payoff at maturity under the risk-neutral growth rate discounted back to date 0 at the riskfree rate as given by

$$PVS_0^A(L,N_L) = e^{-rT}\int_{-\infty}^{L}\int_{-\infty}^{+\infty} 0\times S_0 e^{Y_T} f(\bullet)dY_T dP_T + e^{-rT}\int_{L}^{+\infty}\int_{-\infty}^{+\infty} N_L S_0 e^{Y_T} f(\bullet)dY_T dP_T \tag{B.1.1}$$

$$= N_L S_0 e^{-rT}\int_{L}^{+\infty}\int_{-\infty}^{+\infty} e^{Y_T} f(\bullet)dY_T dP_T. \tag{B.1.2}$$

Denote the PDF of conditional distribution of Y_T given P_T as $f(Y_T|P_T)$ and the PDF of P_T as $f(P_T)$. Then the value of a performance-vested share plan is given by

$$PVS_0^A(L,N_L) = N_L S_0 e^{-rT}\lambda\int_{L}^{+\infty}\int_{-\infty}^{+\infty} e^{Y_T} f(P_T)f(Y_T|P_T)dY_T dP_T. \tag{B.1.3}$$

Conditional distribution of Y given P is

$$Y_T|P_T \sim \mathcal{N}\left(\mu_Y + \frac{\sigma_Y}{\sigma_P}\rho\left(P_T-\mu_P\right), \left(1-\rho^2\right)\sigma_Y^2\right), \tag{B.1.4}$$

where ρ is the correlation coefficient between Y_T and P_T given in Eq. (11).[21]

$$e^{Y_T} f(Y_T|P_T) = \frac{1}{\sqrt{2\pi\left(1-\rho^2\right)\sigma_Y^2}}\exp\left(Y_T - \frac{\left(Y_T - \left(\mu_Y + \frac{\sigma_Y}{\sigma_P}\rho\left(P_T-\mu_P\right)\right)\right)^2}{2\left(1-\rho^2\right)\sigma_Y^2}\right) \tag{B.1.5}$$

$$= \exp\left(\frac{\left(1-\rho^2\right)\sigma_Y^2}{2} + \mu_Y - h\mu_P\right)\exp\left(hP_T\right) f\left(Y_T + \left(1-\rho^2\right)\sigma_Y^2|P_T\right). \tag{B.1.6}$$

where $h = \frac{\sigma_Y}{\sigma_P}\rho$. Thus,

$$PVS_0^A(L,N_L) = N_L S_0 e^{-rT}\lambda C_1\int_{L}^{+\infty} e^{hP_T} f(P_T)\int_{-\infty}^{+\infty} f\left(Y_T + \left(1-\rho^2\right)\sigma_Y^2|P_T\right)dY_T dP_T \tag{B.1.7}$$

where $C_1 \equiv \exp\left(\frac{(1-\rho^2)\sigma_Y^2}{2} + \mu_Y - h\mu_P\right)$. This simplifies to

$$PVS_0^A(L,N_L) = N_L S_0 e^{-rT}\lambda C_1 e^{\left(h\mu_P + \frac{1}{2}h^2\sigma_P^2\right)}\int_{L}^{+\infty} f\left(P_T + h\sigma_P^2 T\right)dP_T, \tag{B.1.8}$$

because $\int_{-\infty}^{+\infty} PDF = CDF(+\infty) = 1$. Plugging in $\mu_Y = rT - \frac{1}{2}\sigma_Y^2$, we have $e^{-rT}C_1 e^{\left(h\mu_P + \frac{1}{2}h^2\sigma_P^2\right)} = 1$.

$$PVS_0^A(L,N_L) = N_L S_0 e^{-rT}\int_{L}^{+\infty} f\left(P_T + h\sigma_P^2 T\right)dP_T \tag{B.1.9}$$

$$= N_L S_0\frac{1}{\sqrt{2\pi}\sigma_P}\int_{L}^{+\infty}\exp\left[-\frac{\left(P_T - h\sigma_P^2 T - \mu_P\right)^2}{2\sigma_P^2}\right]dP_T. \tag{B.1.10}$$

Let $H_T = \frac{P_T - h\sigma_{P_T}^2 - \mu_{P_T}}{\sigma_{P_T}}$. Then $dH_T = \frac{dP_T}{\sigma_{P_T}}$, and $dP_T = \sigma_{P_T} dH_T$.

$$PVS_0^A(L,N_L) = N_L S_0\frac{1}{\sqrt{2\pi}}\int_{\frac{L - h\sigma_{P_T}^2 - \mu_{P_T}}{\sigma_{P_T}}}^{+\infty}\exp\left[-\frac{1}{2}H_T^2\right]dH_T. \tag{B.1.11}$$

[21] Please see Greene (2003, pp. 868).

Using $\int_c^{\infty} n(x)dx = N(-c)$ and $n(c) = n(-c)$, when $n(x)$ and $N(x)$ are the PDF and CDF of a standard normal variable x, we can rewrite the equation as

$$PVS_0^A(L, N_L) = N_L S_0 N\left(\frac{\mu_{P_T} - L + h\sigma_{P_T}^2}{\sigma_{P_T}}\right), \tag{B.1.12}$$

where $d_1 = \frac{P_0 + (\alpha - \nu\sigma_P + h\sigma_P^2)T - L}{\sigma_P\sqrt{T}}$ after plugging in $\mu_{P_T} = P_0 + (\alpha - \nu\sigma_P)T$ and $\sigma_{P_T} = \sigma_P\sqrt{T}$. We finally have Eq. (13)

$$PVS_0^A(L, N_L) = N_L S_0 N(d_1). \tag{B.1.13}$$

The value of an linear performance share plan with strike level L and slope of payoff λ_L can be split into two terms

$$LPS_0^A(L, \lambda_L) = S_0 e^{-rT}\lambda_L \int_L^{+\infty}\int_{-\infty}^{+\infty} P_T e^{Y_T} f(\bullet) dY_T dP_T - L S_0 e^{-rT}\lambda_L \int_L^{+\infty}\int_{-\infty}^{+\infty} e^{Y_T} f(\bullet) dY_T dP_T. \tag{B.1.14}$$

Denote the PDF of conditional distribution of Y_T given P_T as $f(Y_T|P_T)$ and the PDF of P_T as $f(P_T)$. Then the value of a linear performance share plan is given by

$$LPS_0^A(L, \lambda_L) = S_0 e^{-rT}\lambda_L \int_L^{+\infty}\int_{-\infty}^{+\infty} P_T e^{Y_T} f(P_T) f(Y_T|P_T) dY_T dP_T - L S_0 e^{-rT}\lambda_L \int_L^{+\infty}\int_{-\infty}^{+\infty} e^{Y_T} f(P_T) f(Y_T|P_T) dY_T dP_T. \tag{B.1.15}$$

Conditional distribution of Y given P is

$$Y_T|P_T \sim \mathcal{N}\left(\mu_Y + \frac{\sigma_Y}{\sigma_M}\rho(P_T - \mu_P), \left(1-\rho^2\right)\sigma_Y^2\right), \tag{B.1.16}$$

where ρ is the correlation coefficient between Y_T and P_T given in Eq. (11).[22]

$$e^{Y_T} f(Y_T|P_T) = \frac{1}{\sqrt{2\pi(1-\rho^2)\sigma_Y^2}} \exp\left(Y_T - \frac{\left(Y_T - \left(\mu_Y + \frac{\sigma_Y}{\sigma_P}\rho(P_T - \mu_P)\right)\right)^2}{2(1-\rho^2)\sigma_Y^2}\right) \tag{B.1.17}$$

$$= \exp\left(\frac{(1-\rho^2)\sigma_Y^2}{2} + \mu_Y - h\mu_P\right)\exp(hP_T) f\left(Y_T + \left(1-\rho^2\right)\sigma_Y^2 | P_T\right). \tag{B.1.18}$$

where $h = \frac{\sigma_Y}{\sigma_P}\rho$. Thus,

$$\begin{aligned} LPS_0^A(L, \lambda_L) = {} & S_0 e^{-rT}\lambda_L C_1 \int_L^{+\infty} P_T e^{hP_T} f(P_T) \int_{-\infty}^{+\infty} f\left(Y_T + \left(1-\rho^2\right)\sigma_Y^2 | P_T\right) dY_T dP_T \\ & - L S_0 e^{-rT}\lambda_L C_1 \int_L^{+\infty} e^{hP_T} f(P_T) \int_{-\infty}^{+\infty} f\left(Y_T + \left(1-\rho^2\right)\sigma_Y^2 | P_T\right) dY_T dP_T \end{aligned} \tag{B.1.19}$$

$$\begin{aligned} = {} & S_0 e^{-rT}\lambda_L C_1 e^{\left(h\mu_P + \frac{1}{2}h^2\sigma_P^2\right)} \int_L^{+\infty} P_T f\left(P_T + h\sigma_P^2 T\right) dP_T \\ & - L S_0 e^{-rT}\lambda_L C_1 e^{\left(h\mu_P + \frac{1}{2}h^2\sigma_P^2\right)} \int_L^{+\infty} f\left(P_T + h\sigma_P^2 T\right) dP_T, \end{aligned} \tag{B.1.20}$$

because $\int_{-\infty}^{+\infty} PDF = CDF(+\infty) = 1$.[23]

[22] Please see Greene(2003, pp. 868).
[23] We followed the same steps from Eqs. (B.1.17) to (B.1.18) to derive (B.1.20).

Plugging in $\mu_Y = rT - \frac{1}{2}\sigma_Y^2$, we have $C_1 e^{\left(h\mu_P + \frac{1}{2}h^2\sigma_P^2\right)} = 1$. Therefore

$$LPS_0^A(L,\lambda_L) = S_0 e^{-rT}\lambda_L \int_L^{+\infty} P_T f\left(P_T + h\sigma_P^2 T\right) dP_T - LS_0 e^{-rT}\lambda_L \int_L^{+\infty} f\left(P_T + h\sigma_P^2 T\right) dP_T \tag{B.1.21}$$

$$= S_0\lambda_L \frac{1}{\sqrt{2\pi}\sigma_P}\int_L^{+\infty} P_T \exp\left[-\frac{\left(P_T - h\sigma_P^2 T - \mu_P\right)^2}{2\sigma_P^2}\right] dP_T - LS_0\lambda_L \frac{1}{\sqrt{2\pi}\sigma_P}\int_L^{+\infty} \exp\left[-\frac{\left(P_T - h\sigma_P^2 T - \mu_P\right)^2}{2\sigma_P^2}\right] dP_T. \tag{B.1.22}$$

Let $H_T = \frac{P_T - h\sigma_{P_T}^2 - \mu_{P_T}}{\sigma_{P_T}}$. Then $dH_T = \frac{dP_T}{\sigma_{P_T}}$, and $dP_T = \sigma_{P_T} dH_T$.

$$LPS_0^A(L,\lambda_L) = S_0\lambda_L \frac{\sigma_{P_T}}{\sqrt{2\pi}}\int_{\frac{L - h\sigma_{P_T}^2 - \mu_{P_T}}{\sigma_{P_T}}}^{+\infty} H_T \exp\left[-\frac{1}{2}H_T^2\right] dH_T + S_0\lambda_L\left(\mu_{P_T} + h\sigma_{P_T}^2 - L\right) N\left(\frac{\mu_{P_T} - L + h\sigma_{P_T}^2}{\sigma_{P_T}}\right) \tag{B.1.23}$$

$$= S_0\lambda_L \frac{\sigma_{P_T}}{\sqrt{2\pi}}\left[-\exp[-\infty] + \exp\left[-\frac{\left(\mu_{P_T} - L + h\sigma_{P_T}^2\right)^2}{2\sigma_{P_T}^2}\right]\right] + S_0\lambda_L\left(\mu_{P_T} + h\sigma_{P_T}^2 - L\right) N\left(\frac{\mu_{P_T} - L + h\sigma_{P_T}^2}{\sigma_{P_T}}\right) \tag{B.1.24}$$

$$= S_0\lambda_L\left[\left(\mu_{P_T} + h\sigma_{P_T}^2 - L\right) N(d_1) + \sigma_{P_T} n(d_1)\right], \tag{B.1.25}$$

where $d_1 = \frac{P_0 + \left(\alpha - \nu\sigma_P + h\sigma_P^2\right)T - L}{\sigma_P\sqrt{T}}$ after plugging in $\mu_{P_T} = P_0 + (\alpha - \nu\sigma_P)T$ and $\sigma_{P_T} = \sigma_P\sqrt{T}$. We finally have Eq. (14)

$$LPS_0^A(L,\lambda_L) = S_0\lambda_L\left[\left(P_0 + \left(\alpha - \nu\sigma_P + h\sigma_P^2\right)T - L\right) N(d_1) + \sigma_P\sqrt{T} n(d_1)\right]. \tag{B.1.26}$$

By combination $\{L, H\}$ and $\{\lambda_L, \lambda_H\}$, we can obtain the values of other linear performance share components.

B.2. Proof of Proposition 2

Table 3 shows that the five components have the same payoff at maturity as a performance share plan. Therefore, as shown in Eq. (19), the date 0 value of a performance share plan must be equal to the date 0 value of the five components in the absence of arbitrage.

When the performance measure follows geometric brownian motion, the above derivation of performance-vested share plan can be slightly modified.

$$PVS_0^G(L, N_L) = N_L S_0 e^{-rT}\lambda \int_L^{+\infty}\int_{-\infty}^{+\infty} e^{Y_T} f(P_T) f(Y_T|P_T) dY_T dP_T. \tag{B.2.1}$$

Introduce a change of variables for the terminal value $\hat{P}_T$. Specifically, define $X_T = ln\left(\frac{\hat{P}_T}{P_0}\right)$. Then X_T is normally distributed as follows

$$X_T \sim \mathcal{N}\left(\left(\alpha_P - \nu\sigma_P - \frac{1}{2}\sigma_P^2\right)T, \sigma_P^2 T\right). \tag{B.2.2}$$

Denote the PDF of conditional distribution of Y_T given X_T as $f(Y_T|X_T)$ and the PDF of X_T as $f(X_T)$. Then the value of a performance-vested share plan is given by

$$PVS_0^G(L, N_L) = N_L S_0 e^{-rT}\lambda \int_{\ln\frac{L}{P_0}}^{+\infty}\int_{-\infty}^{+\infty} e^{Y_T} f(X_T) f(Y_T|X_T) dY_T dX_T. \tag{B.2.3}$$

Following same steps from Eqs. (B.1.5)–(B.1.10), we obtain Eq. (20)

$$PVS_0^G(L, N_L) = LS_0 N\left(d_2^G\right), \tag{B.2.4}$$

where $d_2^G = \frac{\ln\frac{P_0}{L} + \left(\alpha_P - \nu\sigma_P + \left(h - \frac{1}{2}\right)\sigma_P^2\right)T}{\sigma_P\sqrt{T}}$.

Introduce a change of variables for the terminal value $\hat{P}_T$. Specifically, define $\hat{X}_T = ln\left(\frac{\hat{P}_T}{P_0}\right)$. Then $\hat{X}_T$ is normally distributed as follows

$$\hat{X}_T \sim \mathcal{N}\left(\left(\alpha_P - \nu\sigma_P - \frac{1}{2}\sigma_P^2\right)T, \sigma_P^2 T\right). \tag{B.2.5}$$

Denote the PDF of conditional distribution of Y_T given X_T as $f(Y_T|X_T)$ and the PDF of X_T as $f(X_T)$. Then the value of an linear performance share plan with strike level L and slope of payoff λ_L is given by

$$\begin{aligned} LPS_0^G(L, \lambda_L) = {} & S_0 P_0 e^{-rT}\lambda_L \int_{\ln\frac{L}{P_0}}^{+\infty}\int_{-\infty}^{+\infty} e^{X_T} e^{Y_T} f(X_T) f(Y_T|X_T)\, dY_T\, dX_T \\ & - LS_0 e^{-rT}\lambda_L \int_{\ln\frac{L}{P_0}}^{+\infty}\int_{-\infty}^{+\infty} e^{Y_T} f(X_T) f(Y_T|X_T)\, dY_T\, dX_T. \end{aligned}$$

We know that $\mu_X = \left(\alpha_P - \nu\sigma_P - \frac{1}{2}\sigma_P^2\right)T$, so $e^{\left(h+\frac{1}{2}\right)\sigma_P^2 T + \mu_X} = e^{\left(\alpha_P - \nu\sigma_P + h\sigma_P^2\right)T}$, and by standardizing $X_T + (h+1)\sigma_X^2$, it becomes

$$LPS_0^G(L, \lambda_L) = S_0 P_0 \lambda_L \int_{\ln\frac{L}{P_0}}^{+\infty} e^{X_T} f\left(X_T + h\sigma_X^2\right) dX_T \tag{B.2.6}$$

$$-LS_0\lambda_L \int_{\ln\frac{L}{P_0}}^{+\infty} f\left(X_T + h\sigma_X^2\right) dX_T \tag{B.2.7}$$

$$= S_0 P_0 e^{\left(\alpha_P - \nu\sigma_P + h\sigma_P^2\right)T} \lambda_L N\left(d_1^G\right) \tag{B.2.8}$$

$$-LS_0\lambda_L N\left(d_2^G\right), \tag{B.2.9}$$

where $d_1^G = \frac{\ln\frac{P_0}{L} + \left(\alpha_P - \nu\sigma_P + \left(h + \frac{1}{2}\right)\sigma_P^2\right)T}{\sigma_P\sqrt{T}}$ and $d_2^G = \frac{\ln\frac{P_0}{L} + \left(\alpha_P - \nu\sigma_P + \left(h - \frac{1}{2}\right)\sigma_P^2\right)T}{\sigma_P\sqrt{T}} = d_1^G - \sigma_P\sqrt{T}$.

Thus, the value of an linear performance share plan is:

$$LPS_0^G(L, \lambda_L) = S_0\lambda_L\left[P_0 e^{\left(\alpha_P - \nu\sigma_P + h\sigma_P^2\right)T} N\left(d_1^G\right) - LN\left(d_2^G\right)\right], \tag{B.2.10}$$

where $d_1^G = \frac{\ln\frac{P_0}{L} + \left(\alpha_P - \nu\sigma_P + \left(h + \frac{1}{2}\right)\sigma_P^2\right)T}{\sigma_P\sqrt{T}}$ and $d_2^G = \frac{\ln\frac{P_0}{L} + \left(\alpha_P - \nu\sigma_P + \left(h - \frac{1}{2}\right)\sigma_P^2\right)T}{\sigma_P\sqrt{T}}$. This is the Eq. (21). **Q.E.D.**

B.3. Proof of Proposition 3

For a performance-vested share plan, the expected share payoff at maturity under the risk neutral process is

$$N_L \sum_{h=1}^{K+1} \pi_h(r_0) I_L \tag{B.3.1}$$

Discounting at the riskfree rate and multiplying by the current stock price yields (27).

For a linear performance share plan, the expected share payoff at maturity under the risk neutral process is

$$\lambda_j \sum_{h=1}^{K+1} \pi_h(r_0) \max\left(\frac{h-1}{K} - X, 0\right), \tag{B.3.2}$$

Discounting at the riskfree rate and multiplying by the current stock price yields (28). **Q.E.D.**

B.4. Proof of Proposition 4

The same decomposition logic as used in Table 3 extends the same way to multiple kinks within the incentive zone. **Q.E.D.**

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