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## Long-Lived Private Information and Imperfect Competition

CRAIG W. HOLDEN and AVANIDHAR SUBRAHMANYAM\*

### ABSTRACT

We develop a multi-period auction model in which multiple privately informed agents strategically exploit their long-lived information. We show that such traders compete aggressively and cause most of their common private information to be revealed very rapidly. In the limit as the interval between auctions approaches zero, market depth becomes infinite and all private information is revealed immediately. These results are in contrast to those of Kyle (1985) in which the monopolistic informed trader causes his information to be incorporated into prices gradually, and, when the interval between auctions is vanishingly small, market depth is constant over time.

FAMA (1970) DEFINES A “STRONG form” efficient market as one in which security prices fully reflect all available information, including both publicly and privately held information. In a pioneering and influential article, Kyle (1985) develops a model in which a *single* privately informed trader with long-lived information optimally exploits his monopoly power over time. Kyle’s (1985) main results are: (i) the informed trader trades in a gradual manner, so that his information is incorporated into prices at a slow, almost linear rate, and (ii) when auctions are held continuously, the depth of the market is constant over time. However, Kyle’s (1985) assumption of a single informed trader is strong, in the sense that in actual financial markets, it is reasonable to expect that at least a few players will have access to private information and will trade in the knowledge that they will face competition with other informed agents in the market.<sup>1</sup> We develop a multi-period auction model in which multiple (strategic) informed traders optimally exploit their long-lived informational advantage. We thus explore the restrictiveness of Kyle’s assumption of a single informed trader, and also examine

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<sup>1</sup>An indirect proxy for the number of informed traders per security is the number of security analysts per security who generate private information for sale to brokers and traders. Brennan and Hughes (1990) report that in 1987, on average there were 12.4, 4.0, and 4.8 security analysts making one-year forecasts on firms listed on the NYSE, AMEX, and NASDAQ, respectively.

how quickly the system approaches the perfect competition outcome characterized by the price fully reflecting the information of all privately informed agents.

Our basic finding is that in a unique linear equilibrium, informed traders trade very aggressively. Even just two informed traders cause nearly all of their common private information to be incorporated into prices almost immediately and cause the depth of the market to become extremely large almost immediately, provided the number of auctions is reasonably large. Hence, we show that a market with multiple informed traders approximates a strong-form efficient market quite accurately at almost all times.

In the case of at least two informed traders, as the number of auctions is increased, by subdividing each time interval into even smaller subintervals, more information is revealed by any cutoff point in calendar time. In the limit as the number of auctions goes to infinity all information is revealed immediately (in an arbitrarily small amount of calendar time). Market depth is small in the earlier periods when there is severe adverse selection and large in later periods when most of the private information has already been revealed.

It is also demonstrated that in the limit in which the number of informed traders goes to infinity all of their private information is revealed in the first trade. Thus the perfectly competitive outcome is strong form market efficiency, irrespective of the total number of auctions in the game.

The contrast in results between the case of a monopolistic informed trader and that of multiple informed traders is driven by aggressive competition between these traders. In the game in which private information lasts only one period, with a linear pricing rule, the unique Nash equilibrium is an equilibrium in which imperfect competitors acting noncooperatively choose larger quantities than a monopolist (or collusive agents) would choose. (See, for example, Admati and Pfleiderer (1988).) This aggressive trading is what drives our results. In the multi-period game with a linear pricing rule, the unique linear equilibrium consists of imperfect competitors trading aggressively in each period, in a manner analogous to their behavior in a single-period Nash equilibrium.

A recent paper by Back (1990) considers the continuous time version of Kyle's (1985) model with a single informed trader. Back allows for a very general class of distributions and a more general strategy space that includes the possibility of discrete trades. He derives conditions for equilibrium for the general case and provides closed-form solutions for the special case in which the asset value is lognormally distributed. In contrast to Back's paper, we assume normality and focus on the discrete time model, while allowing for multiple informed traders. However, we informally argue that in the case of continuous auctions, there can be no equilibrium in continuous, linear strategies, or in discrete strategies under our structure.

The intuition behind the results in our paper is analogous to a result in Holden (1990) in the context of intertemporal arbitrage trading. In Holden's model, a monopolist arbitrageur finds it optimal to trade arbitrage positions

at a nearly constant rate in order to exploit arbitrage opportunities over time. By contrast, imperfectly competitive arbitrageurs trade arbitrage positions very aggressively and thus drive the price difference between two securities very close to zero very rapidly.

In Section I, we present our model and characterize its unique linear equilibrium. Section II discusses properties of the equilibrium and derives some limiting results. Section III concludes the paper.

## I. The Model

### A. Structure and Notation

We conform to the notation of Kyle (1985). A security is traded in  $N$  sequential auctions in a time interval which begins at  $t = 0$  and ends at  $t = 1$ . The security's value at the end of trading is denoted by  $v$  which is assumed to be normally distributed with mean  $p_0$  and variance  $\Sigma_0$ . Let  $M$  denote the number of informed traders, who are indexed by  $i = 1, \dots, M$ . Each informed trader observes the liquidation value  $v$  in advance. Let  $\Delta X_n$  and  $\Delta x_{in}$  denote the total order by *all* informed traders and the *individual* order by the  $i$ th informed trader at the  $n$ th auction, respectively. Further, let  $\pi_{in}$  denote the total expected profits of the  $i$ th informed trader from positions acquired at all future auctions  $n, \dots, N$ .

Each risk neutral informed trader determines his optimal trading strategy by a process of backward induction in order to maximize his expected profits given his conjectures about the trading strategies of the other informed traders. In the rational expectations equilibrium, the conjectures of each identical informed trader must be correct conditional on each trader's information at each auction.

At each auction orders are also submitted by noise traders. Let  $\Delta u_n$  be the aggregate order submitted by noise traders at the  $n$ th auction. We assume that  $\Delta u_n$  is serially uncorrelated and is normally distributed with zero mean and variance of  $\sigma_u^2 \Delta t_n$ , where  $\Delta t_n$  is the time interval between the  $n$  auction and the previous auction.

Trading takes place through market makers who absorb the order flow while earning zero expected profits. At each auction, market makers observe only the *combined* order flow  $\Delta X_n + \Delta u_n$ , whereupon they set  $p_n$ , the price at the  $n$ th auction. Equilibrium is defined by a market efficiency condition that  $p_n$  equals the expected value of  $v$  conditional on the information available to the market makers at that auction, by a profit maximization condition that each informed trader selects the optimal strategy conditional on his conjectures and his information at each auction, and by a condition that all conjectures are correct.

### B. Equilibrium

We now state a proposition which provides the difference equation system characterizing our equilibrium:

**Proposition 1:** *There exists a unique linear equilibrium in our model, in which there are constants  $\alpha_n$ ,  $\beta_n$ ,  $\delta_n$ ,  $\lambda_n$ , and  $\Sigma_n$ , characterized by the following:*

$$\Delta X_n = M\beta_n(v - p_{n-1}) \Delta t_n \quad (1)$$

$$\Delta p_n = \lambda_n(\Delta X_n + \Delta u_n) \quad (2)$$

$$\Sigma_n = \text{var}(v | \Delta X_1 + \Delta u_1, \dots, \Delta X_n + \Delta u_n) \quad (3)$$

$$E(\pi_n | p_1, \dots, p_{n-1}, v) = \alpha_{n-1}(v - p_{n-1})^2 + \delta_{n-1} \quad (4)$$

for all auctions  $n = 1, \dots, N$  and for all informed traders  $i = 1, \dots, M$ .

The constants  $\beta_n$ ,  $\lambda_n$ ,  $\alpha_n$ , and  $\Sigma_n$  are the unique solution to the difference equation system

$$\alpha_n = \frac{1 - \alpha_{n+1}\lambda_{n+1}}{\lambda_{n+1}[M(1 - 2\alpha_{n+1}\lambda_{n+1}) + 1]^2} \quad (5)$$

$$\beta_n \Delta t_n = \frac{1 - 2\alpha_n\lambda_n}{\lambda_n[M(1 - 2\alpha_n\lambda_n) + 1]} \quad (6)$$

$$\lambda_n = \frac{M\beta_n\Sigma_n}{\sigma_u^2} \quad (7)$$

$$\Sigma_{n+1} = (1 - M\beta_{n+1}\lambda_{n+1}\Delta t_{n+1})\Sigma_n \quad (8)$$

for auctions  $n = 1, \dots, N - 1$ , subject to the boundary conditions

$$\alpha_N = 0 \quad (9)$$

$$\beta_N \Delta t_N = \frac{1}{\lambda_N(M + 1)} \quad (10)$$

$$\lambda_N = \frac{1}{\sigma_u} \left[ \frac{M\Sigma_N}{(M + 1)\Delta t_n} \right]^{1/2} \quad (11)$$

at the  $N$ th and final auction, the condition

$$\Sigma_1 = (1 - M\beta_1\lambda_1\Delta t_1)\Sigma_0 \quad (12)$$

at the first auction for a given value of  $\Sigma_0$  and the second order condition

$$\lambda_n(1 - \alpha_n\lambda_n) > 0 \quad (13)$$

for all auctions  $n = 1, \dots, N$ .

The constant  $\delta_n$  is calculated at auctions  $n = 1, \dots, N$  using the difference equation

$$\delta_{n-1} = \delta_n + \alpha_n\lambda_n^2\sigma_u^2\Delta t_n \quad (14)$$

given  $\delta_N = 0$ .

*Proof:* The proof is a modification of the proof in Kyle (1985). Recall that  $x_{in}$  and  $\pi_{in}$  denote the order and the profits of a particular informed trader at the  $n$ th auction. Using a symmetry argument, it is straightforward to show that, given linear pricing by the market maker, the only possible equilibrium between the informed traders is one in which they choose identical strategies. Accordingly, from this point, we suppress the ‘ $i$ ’ subscript from  $x_{in}$  and  $\pi_{in}$ . We proceed by backward induction. Suppose that for constants  $\alpha_n$  and  $\delta_n$ ,

$$E(\pi_{n+1} | p_1, \dots, p_n, v) = \alpha_n (v - p_n)^2 + \delta_n$$

We then have

$$\begin{aligned} E(\pi_n | p_1, \dots, p_{n-1}, v) \\ = \max_{\Delta x} E\{(v - p_n) \Delta x + \alpha_n (v - p_n)^2 + \delta_n | p_1, \dots, p_{n-1}, v\} \end{aligned} \quad (15)$$

A linear equilibrium implies that

$$p_n = p_{n-1} + \lambda_n (\Delta X_n + \Delta u_n) + h \quad (16)$$

where  $h$  is some linear function of  $\Delta X_1 + \Delta u_1, \dots, \Delta X_{n-1} + \Delta u_{n-1}$ . Now,  $\Delta X_n$  can be written as  $\Delta x + (M - 1) \Delta \bar{x}_n$ , where  $\Delta \bar{x}_n$  represents the particular informed trader’s conjecture of the average of the other informed traders’ strategies. Substituting (16) into (15) and evaluating the conditional expectation, we have

$$\begin{aligned} E(\pi_n | p_1, \dots, p_{n-1}, v) \\ = \max_{\Delta x} \{ (v - p_{n-1} - \lambda_n (\Delta x + (M - 1) \Delta \bar{x}_n) - h) \Delta x \\ + \alpha_n (v - p_{n-1} - \lambda_n (\Delta x + (M - 1) \Delta \bar{x}_n) - h)^2 \\ + \alpha_n \lambda_n^2 \sigma_u^2 \Delta t_n + \delta_n \} \end{aligned} \quad (17)$$

Solving the above maximization problem yields the maximizing value of  $\Delta x$ , which we denote as  $\Delta x_n$ . This value is

$$\Delta x_n = \frac{(v - p_{n-1} - h - \lambda_n (M - 1) \Delta \bar{x}_n) [1 - 2\alpha_n \lambda_n]}{2\lambda_n (1 - \alpha_n \lambda_n)} \quad (18)$$

To solve for the equilibrium, we set  $\Delta \bar{x}_n = \Delta x_n$  and solve for  $\Delta x_n$ . We then have

$$\Delta x_n = (v - p_{n-1} - h) \frac{1 - 2\alpha_n \lambda_n}{\lambda_n [M(1 - 2\alpha_n \lambda_n) + 1]} \quad (19)$$

It can easily be verified that the second order condition is given by (13). To prove that  $h = 0$ , first note that

$$E\{\Delta p_n | \Delta X_1 + \Delta u_1, \dots, \Delta X_{n-1} + \Delta u_{n-1}\} = 0$$

However, from the pricing function (16),

$$\begin{aligned} & E\{\Delta p_n | \Delta X_1 + \Delta u_1, \dots, \Delta X_{n-1} + \Delta u_{n-1}\} \\ &= \frac{h}{2(1 - \alpha_n \lambda_n) + (M - 1)(1 - 2\alpha_n \lambda_n)} \end{aligned}$$

thus proving that  $h = 0$ . Hence, (6) follows from (19), and (5) and (14) follow from (17).

The market efficiency condition implies that  $\lambda_n$  is a regression coefficient of  $v$  on  $\Delta x_n + \Delta u_n$ , conditional on  $\Delta X_1 + \Delta u_1, \dots, \Delta X_{n-1} + \Delta u_{n-1}$ . Due to normality, the regression is linear, and using the standard formula for the regression coefficient, we have

$$\lambda_n = \frac{M\beta_n \Sigma_{n-1}}{M^2 \beta_n^2 \Sigma_{n-1} \Delta t_n + \sigma_u^2} \quad (20)$$

Also,

$$\Sigma_n = \frac{\sigma_u^2 \Sigma_{n-1}}{M^2 \beta_n^2 \Sigma_{n-1} \Delta t_n + \sigma_u^2} \quad (21)$$

From these equations, (7) and (8) can easily be derived. Finally, the boundary conditions  $\alpha_N = \delta_N = 0$  formalize the obvious by stating that no new profits can be made after trading is complete.

We have thus proved that the difference equation system (5)–(13) describes a linear equilibrium of the model. The proof that this equilibrium is the unique linear equilibrium is completely analogous to Kyle's (1985) proof for the case of a monopolist (see Kyle (1985), pp. 1325–1326), and is therefore omitted.  $\square$

Benoit and Krishna (1985) show that for a finitely repeated game of complete information where the stage game has a unique Nash equilibrium, the unique subgame perfect strategy<sup>2</sup> is to play the unique Nash equilibrium at each date. In our model, informed traders have imperfect information (not incomplete information) due to the noise in observable prices that comes from

<sup>2</sup> In our model, information sets correspond to proper subgame nodes and all players are Bayesian updaters so extending the strategy to be part of the unique perfect Bayesian equilibrium is straightforward.

the unobservable realizations of noise trading. Informed traders cannot invert the pricing rule to precisely determine the number of shares previously traded by other informed traders. The backward induction argument of Benoit and Krishna carries through to our model with a noisy observable. In the last period, each player follows the strategy of the single-period Nash equilibrium based on information updated to the last period and irrespective of what previous deviations by other players may be inferred from past prices. Working recursively, all players know that no deviations in the next to last period will be “punished” or “rewarded” in the last period. Hence, they play the single-period Nash equilibrium based on information updated to the next to last period and so on.

In the Appendix we use a variable  $q_n \equiv \alpha_n \lambda_n$  to present an explicit method for solving the difference equation system in Proposition 1.<sup>3</sup> The following proposition characterizes this solution method.

**Proposition 2:** *Let  $q_n \equiv \alpha_n \lambda_n$ . The solution of the difference equation system in Proposition 1 is given by starting from  $q_N = 0$  and iterating backward for  $q_{N-1}, \dots, q_1$  by using the unique root of the cubic equation*

$$0 = 2M \left( \frac{\Delta t_{n-1}}{\Delta t_n} \right) q_{n-1}^3 - (M + 1) \left( \frac{\Delta t_{n-1}}{\Delta t_n} \right) q_{n-1}^2 - 2k_n q_{n-1} + k_n \quad (22)$$

where

$$k_n \equiv \frac{(1 - q_n)^2}{(1 - 2q_n)(M(1 - 2q_n) + 1)^2} \quad (23)$$

which lies in the interval  $(0, \frac{1}{2})$ . Then starting from the exogenous value  $\Sigma_0$ , iterate forward for each of the following variables in the order listed

$$\Sigma_n = \left( \frac{1}{M(1 - 2q_n) + 1} \right) \Sigma_{n-1} \quad (24)$$

$$\lambda_n = \left( \frac{M \Sigma_n (1 - 2q_n)}{\Delta t_n \sigma_u^2 [M(1 - 2q_n) + 1]} \right)^{1/2} \quad (25)$$

$$\beta_n = \left( \frac{M(1 - 2q_n) \sigma_u^2}{\Sigma_n \Delta t_n M [M(1 - 2q_n) + 1]} \right)^{1/2} \quad (26)$$

<sup>3</sup> The method we describe is *not* the analog of the method used by Kyle (1985) to solve the difference equation system for the case of a monopolist (see Section 4 of his paper). Kyle’s (1985) method cannot be generalized to the case of many informed traders. Our method can be characterized as an alternative method of solving Kyle’s (1985) difference equation system. Michener and Tighe (1991) derive a similar recursion. Foster and Viswanathan (1991) show that this recursion extends to elliptically distributed random variables.



at auctions  $n = 1, \dots, N$  and calculate  $\alpha_n$  and  $\delta_n$  using equations (5) and (14), respectively.

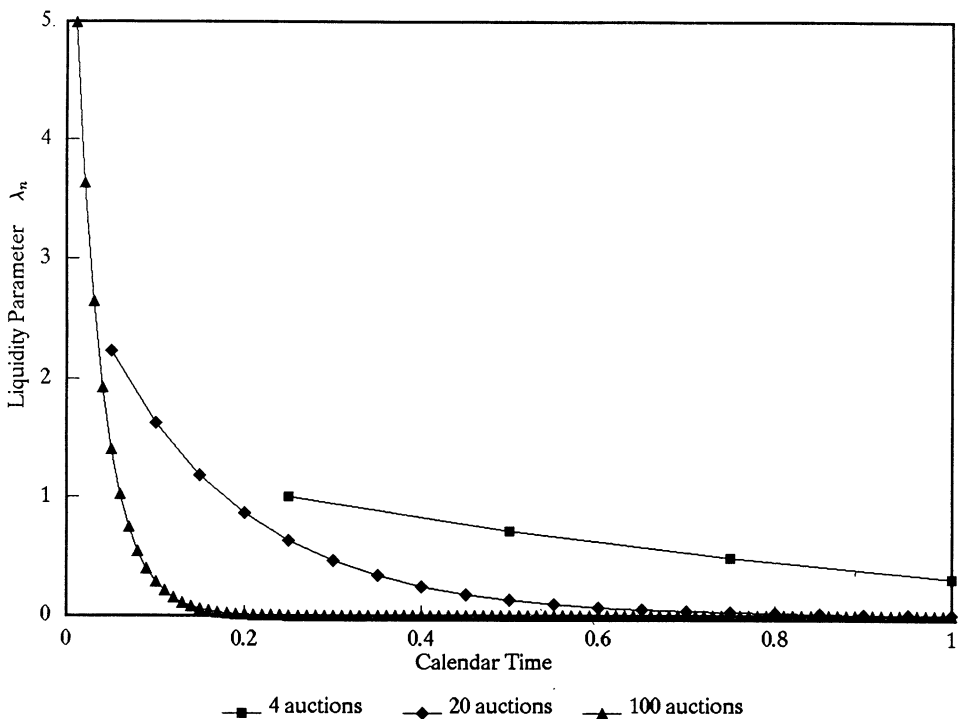
*Proof:* See the Appendix.

## II. Properties of the Equilibrium

### A. Numerical Illustrations

As in Kyle (1985), the parameters  $\Sigma_n$  and  $\lambda_n$  are inverse measures of price efficiency and market depth, respectively. The quantity  $E[\Delta X_n | v]$  is the informed traders' total ex ante expected order at the  $n$ th auction given a particular realization of  $v$ . To compare the imperfectly competitive case to the monopolist case, we present a series of numerical examples using the method of solution in Proposition 2. In all the numerical examples, we assume that  $\Sigma_0 = 1$ ,  $\sigma_u^2 = 1$ , and  $\Delta t_n = 1/N$ ,  $\forall n$ .

Figures 1, 2, and 3 plot  $\lambda_n$ ,  $\Sigma_n$ , and  $E[\Delta X_n | v]$ , respectively, for the particular cases of  $N = 4$ ,  $N = 20$ , and  $N = 100$ , while holding constant the



**Figure 1. Liquidity parameter ( $\lambda_n$ ) over time for different values of  $N$ , the number of auctions.** The number of informed traders is fixed at  $M = 2$  and the liquidity parameter at each auction is plotted for different values of  $N$ , the number of auctions. It is assumed that the variance of noise trading per unit time  $\sigma_u^2 = 1$  and the ex ante variance of the terminal value  $\Sigma_0 = 1$ , and that auctions occur at equally spaced intervals in the time interval  $[0, 1]$ .

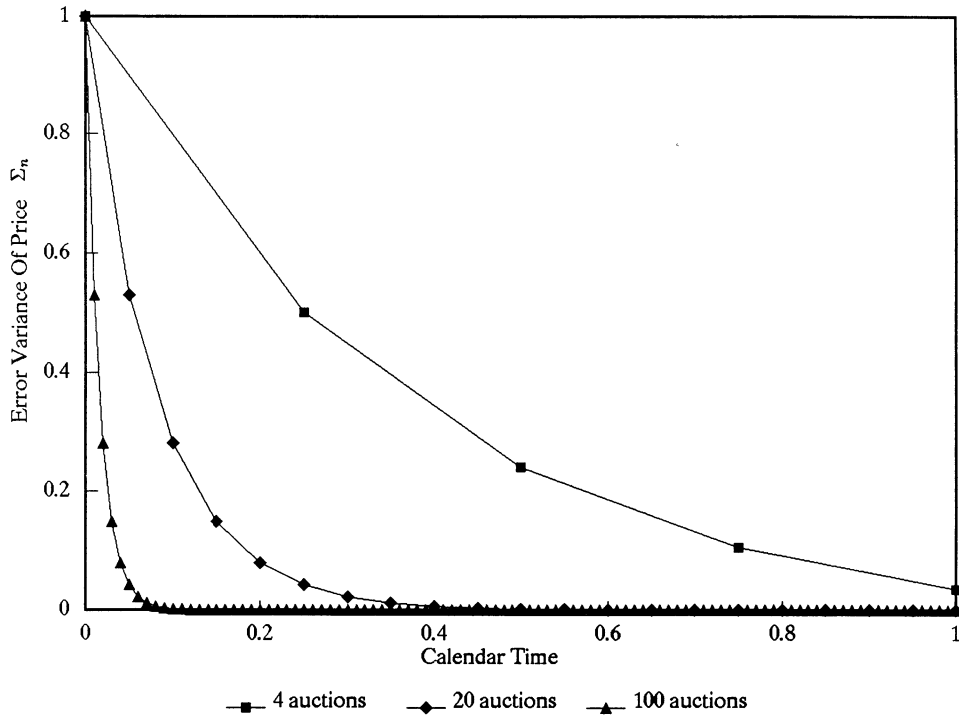
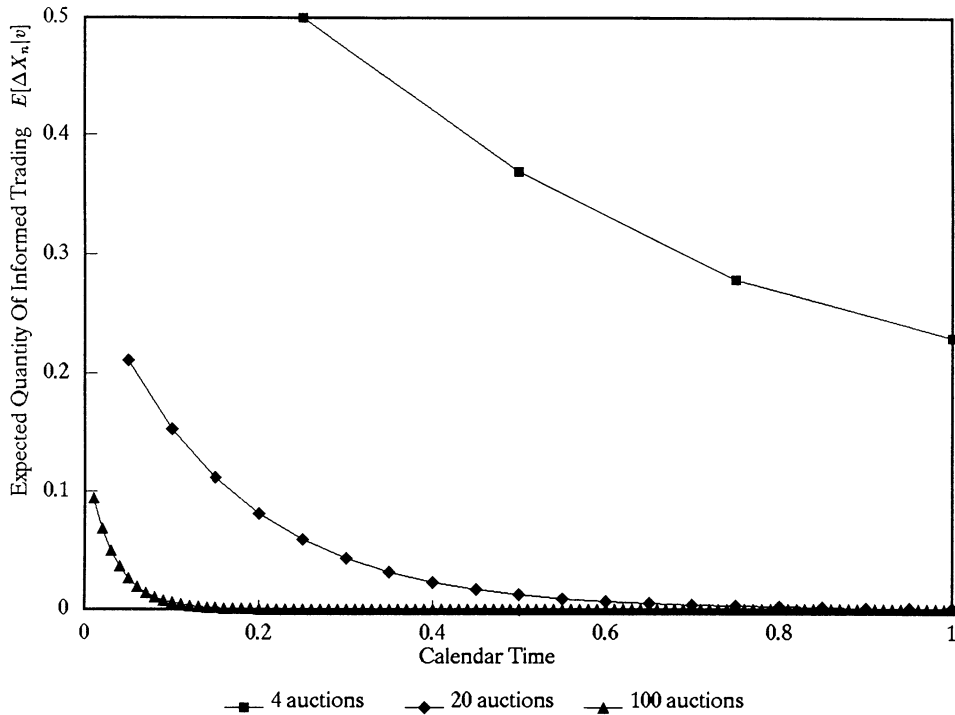


Figure 2. Error variance of the price ( $\Sigma_n$ ) over time for different values of  $N$ , the number of auctions. The number of informed traders is fixed at  $M = 2$  and the error variance of the price (a measure of the amount of unrevealed private information in the market) at each auction is plotted for different values of  $N$ , the number of auctions. It is assumed that the variance of noise trading per unit time  $\sigma_u^2 = 1$  and the ex ante variance of the terminal value  $\Sigma_0 = 1$ , and that auctions occur at equally spaced intervals in the time interval  $[0, 1]$ .

calendar time between commencement and end of trading. To present the sharpest contrast with the monopolist case, we set  $M = 2$ . As can be seen from Figures 1 and 2,  $\lambda_n$  and  $\Sigma_n$  decline nearly to zero very rapidly through time. As the number of auctions increases, the speed with which they drop increases dramatically. In fact, in the case of 100 auctions, less than 5% of the information (in the sense of an error variance) remains to be revealed by the 5th auction! This is because imperfectly competitive informed traders cannot collude to exploit their rents slowly. The noncooperative setting results in aggressive competition, which causes most of the private information to be revealed in the early periods.

Note also in Figure 1 the large  $\lambda_n$  in the initial periods when the interval between auctions is small. The adverse selection (measured by  $\lambda_n$ ) is high in the earlier periods because the information content of the order flow is high, and negligible in the later periods because the market maker has very little to fear from traders that have already exploited most of their informational advantage. As the number of auctions is increased, trading becomes more

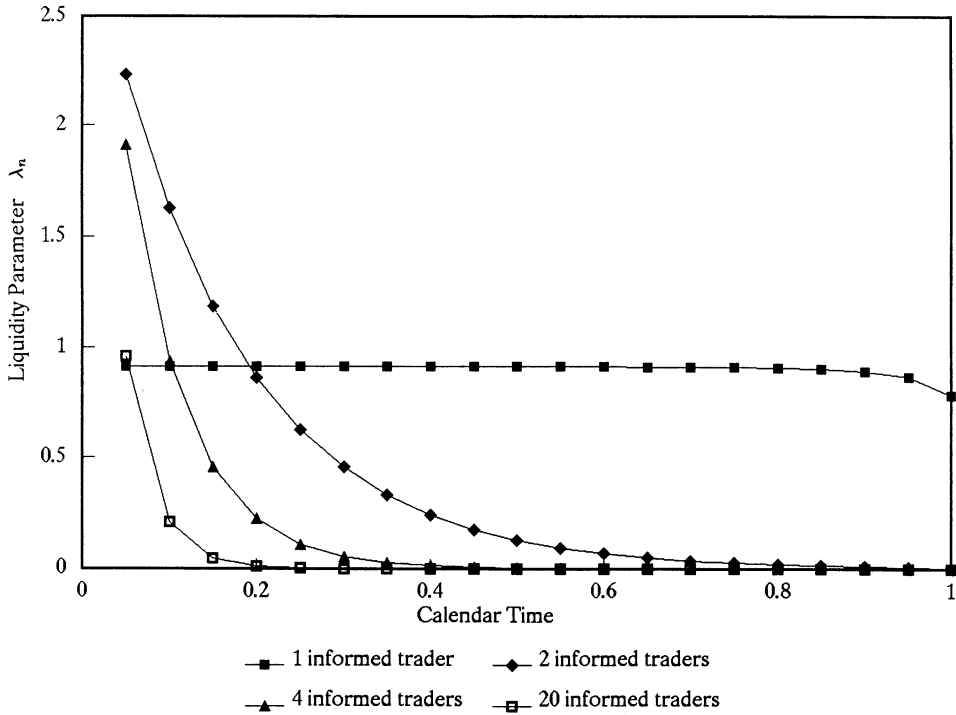


**Figure 3.** Total expected quantity of informed trading [ $E(\Delta X_n | v)$ ] over time for different values of  $N$ , the number of auctions. The number of informed traders is fixed at  $M = 2$  and of the total expected quantity of informed trading at each auction (conditional on a particular realization of the random terminal value  $v$ ) is plotted for different values of  $N$ , the number of auctions. It is assumed that  $v = 21$ , the ex ante mean of  $v$ ,  $p_0 = 20$ , the variance of noise trading per unit time  $\sigma_u^2 = 1$ , the ex ante variance of the terminal value  $\Sigma_0 = 1$ , and that auctions occur at equally spaced intervals in the time interval  $[0, 1]$ .

concentrated at earlier auctions and hence there is greater adverse selection at these auctions.

Figure 3 uses the additional assumption that  $v = 21$  and  $p_0 = 20$ . The figure demonstrates that the expected amount of trading is large in the first auction and drops close to zero in later auctions. As the number of auctions increases, the expected amount of trading decreases. Intuitively, as the time interval between auctions decreases, there is less liquidity trading at each auction to provide camouflage for the informed traders. Hence, the informed traders scale back the quantity traded at each auction.

Figures 4, 5, and 6 plot  $\lambda_n$ ,  $\Sigma_n$ , and  $E[\Delta X_n | v]$ , respectively, for the cases of  $M = 1$ ,  $M = 2$ ,  $M = 4$ , and  $M = 20$ , fixing the number of auctions  $N$  at 20. These graphs effectively illustrate the contrasts in the equilibria involving one and many informed traders. In the case of the single informed trader,  $\lambda_n$  is almost constant and  $\Sigma_n$  declines monotonically at a slow, approximately linear rate. This is because the single informed trader acts in a manner



**Figure 4. Liquidity parameter ( $\lambda_n$ ) over time for different values of  $M$ , the number of informed traders.** The number of auctions is fixed at  $N = 20$  and the liquidity parameter at each auction is plotted for different values of  $M$ , the number of informed traders. It is assumed that the variance of noise trading per unit time  $\sigma_u^2 = 1$  and the ex ante variance of the terminal value  $\Sigma_0 = 1$ , and that auctions occur at equally spaced intervals in the time interval  $[0, 1]$ .

similar to a perfectly discriminating monopolist who moves along his residual demand curve. In the case of imperfect competition,  $\lambda_n$  is larger in the earlier periods than the corresponding value for single informed trader in the first few auctions and declines sharply thereafter, and  $\Sigma_n$  declines sharply toward zero. As the number of informed traders increases,  $\lambda_n$  and  $\Sigma_n$  drop increasingly rapidly. This is because more informed traders act increasingly competitively and cause more information to be revealed in earlier auctions.

It is worth focusing on the contrast between the case of a monopolist and that of  $M = 2$ . From Figure 5, it can be seen that when  $M = 2$ ,  $\Sigma_n$  decreases to 0.05 by the 5th auction, whereas at that point it is about 0.8 for the case of the monopolist. This seems to indicate that informational advantage in financial markets is competed away extremely rapidly even in the case of just two informed traders.

In Figure 6, it is again assumed that  $v = 21$  and  $p_0 = 20$ . This figure shows the contrast between a monopolist informed trader who trades at a nearly constant rate and imperfectly competitive informed traders who trade very aggressively in early auctions. As the number of informed traders

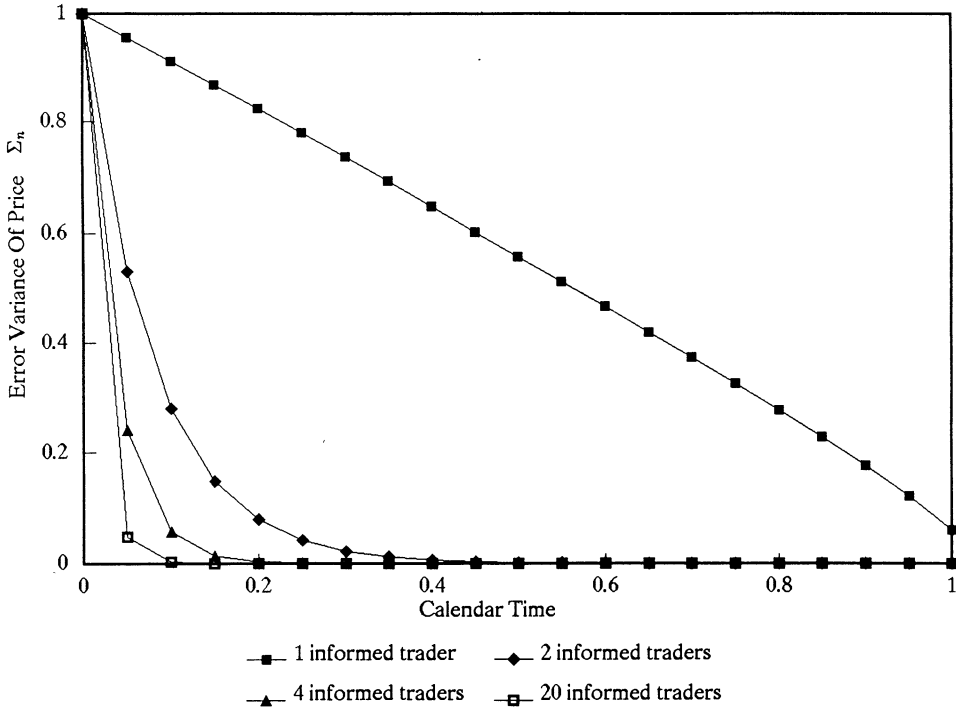


Figure 5. Error variance of the price ( $\Sigma_n$ ) over time for different values of  $M$ , the number of informed traders. The number of auctions is fixed at  $N = 20$  and the error variance of the price at each auction is plotted for different values of  $M$ , the number of informed traders. It is assumed that the variance of noise trading per unit time  $\sigma_u^2 = 1$ , the ex ante variance of the terminal value  $\Sigma_0 = 1$ , and that auctions occur at equally spaced intervals in the time interval  $[0, 1]$ .

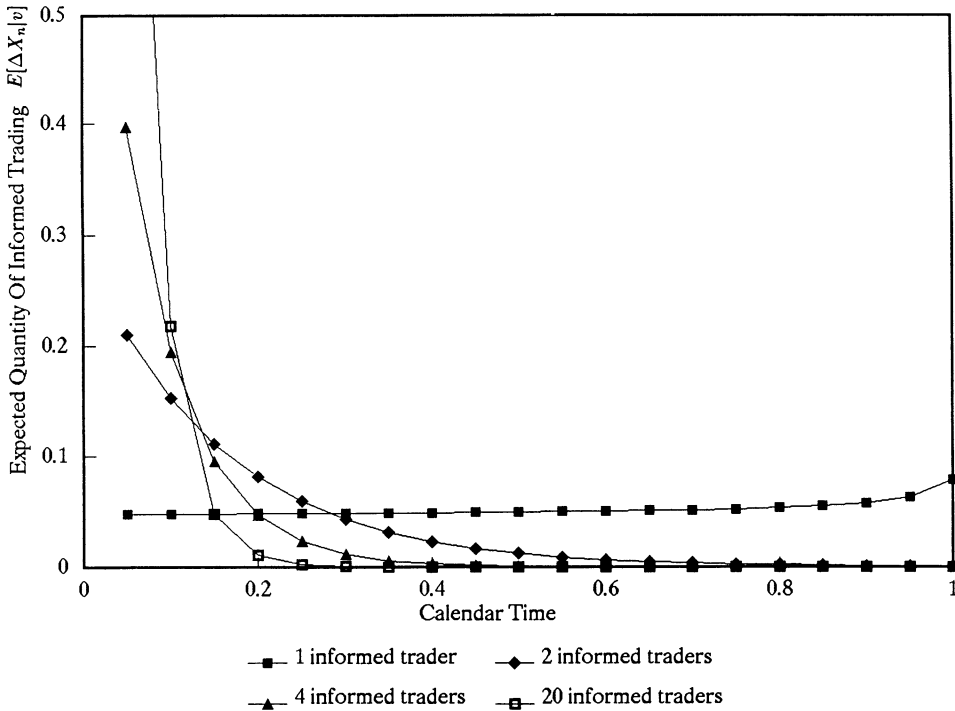
increases, the total expected quantity traded by them in the initial auction becomes larger because they compete more aggressively. This reveals their information earlier and hence their trading drops toward zero more rapidly in calendar time.

B. Limiting Results

We now examine three classes of analytic limits. We set  $\Delta t_n = 1/N \forall n$  (recall that the overall time period is fixed at  $[0, 1]$ ). Define  $\tau$  as the calendar time elapsed from the commencement of trading and  $n'$  as the last auction before a particular time  $\tau$ . That is,  $n'$  is defined by

$$\frac{n'}{N} \leq \tau \quad \text{and} \quad \frac{n' + 1}{N} > \tau.$$

For the first class of limits, we hold  $M (\geq 2)$  constant and take the limit as  $N$  goes to infinity (approaching the case of continuous auctions) for an arbitrary



**Figure 6.** Total expected quantity of informed trading  $[E(\Delta X_n | v)]$  over time for different values of  $M$ , the number of informed traders. The number of auctions is fixed at  $N = 20$  and the total expected quantity of informed trading at each auction (conditional on a particular realization of the terminal value  $v$ ) is plotted for different values of  $M$ , the number of informed traders. It is assumed that  $v = 21$ , the ex ante mean of  $v$ ,  $p_0 = 20$ , the variance of noise trading per unit time  $\sigma_u^2 = 1$ , the ex ante variance of the terminal value  $\Sigma_0 = 1$ , and that auctions occur at equally spaced intervals in the time interval  $[0, 1]$ .

calendar time cutoff  $\tau$ , for price efficiency and market depth. Notice that for any fixed  $\tau > 0$ , as  $N \rightarrow \infty$  then  $n' \rightarrow \infty$ .

In the second class of limits, we take the limit as  $N$  goes to infinity and examine what happens at the *very first* auction to market depth, expected total informed trading, and an informed trader's control  $\beta_1$ . The third class of limits involves holding  $N$  constant and taking the limit as  $M$  goes to infinity, i.e., as we approach perfect competition. We examine the behavior of price efficiency, market depth, expected total informed trading, and the price at the first auction.

**Proposition 3:** *Under the assumption that auctions occur at equally spaced intervals, the discrete time model has the following limits:*

- (i) *The last auction before an arbitrary calendar time cutoff approaching the case of continuous auctions. Holding  $M \geq 2$  constant, for any  $\tau > 0$*

we have

$$\lim_{N \rightarrow \infty} \Sigma_{n'} = 0 \quad \lim_{N \rightarrow \infty} \lambda_{n'} = 0$$

- (ii) *The first auction approaching the case of continuous auctions. Holding  $M \geq 2$  constant, then*

$$\lim_{N \rightarrow \infty} \lambda_1 = \infty \quad \lim_{N \rightarrow \infty} E_0[\Delta X_1 | v] = 0 \quad \lim_{N \rightarrow \infty} \beta_1 = \infty.$$

- (iii) *The first auction approaching perfect competition. Holding  $N$  constant, then*

$$\begin{aligned} \lim_{M \rightarrow \infty} \Sigma_1 &= 0 & \lim_{M \rightarrow \infty} \lambda_1 &= 0 \\ \lim_{M \rightarrow \infty} E_0[\Delta X_1 | v] &= \infty & \lim_{M \rightarrow \infty} p_1 &= v. \end{aligned}$$

*Proof:* See the Appendix.

The first class of limits yields the result that, as the number of auctions goes to infinity, all information is revealed immediately (in an arbitrarily small amount of calendar time). Also, market depth ( $1/\lambda_{n'}$ ) goes to infinity immediately. These results are in substantial contrast to those of Kyle (1985) in which information is revealed at a constant rate and market depth is constant in the limiting case of continuous auctions.

In the second class of limits we show that, as the total number of auctions goes to infinity, market depth at the first auction ( $1/\lambda_1$ ) goes to zero. The intuition for the contrast with the previous market depth limit is that market depth is small at early auctions when there is severe adverse selection and large at later auctions when most of the private information has already been revealed. We also find that the expected total quantity of informed trading goes to zero in the first auction and an informed trader's control  $\beta_1$  goes to infinity.

Conforming to intuition, the third class of limits demonstrates that as the number of informed traders goes to infinity, all information is revealed in the first auction, market depth at the first auction and the expected quantity traded by informed traders at the first auction both go to infinity, and the price at the first auction equals the terminal value  $v$ . These results follow from the fact that as the number of informed traders increases, they behave increasingly competitively. In the limit, we obtain perfect competition.

### C. The Case of Continuous Auctions

We now provide a discussion on the existence of an equilibrium in the case of continuous auctions, based on our limiting results.<sup>4</sup> The first limit in part (i) of Proposition 3 indicates that a continuous time equilibrium in continuous, linear strategies of the type considered by Kyle (1985) would have to be characterized by private information being revealed instantaneously. However, the third limit in part (ii) of Proposition 3 indicates that the

<sup>4</sup> The comments of the referee helped us immensely in formulating this discussion.

maximization problem of the informed traders at the first instant (time 0) would have no optimum, which indicates that there can be no equilibrium in continuous, linear strategies.

The question then arises as to whether there can be an equilibrium if the strategy space is expanded to include discrete trades. If discrete orders are placed at time 0, then, given that noise trades are infinitesimal at any instant, there is no camouflage for the informed traders and a necessary condition for equilibrium is that the price must jump to the terminal value as soon as the discrete order appears (i.e., an equilibrium, if it exists, must be fully revealing). Laffont and Maskin (1990) consider a one-period model with no noise traders, a single strategic, risk neutral informed trader, and a continuum of competitive, risk averse uninformed traders (who trade to hedge their risky endowments). They demonstrate that ‘separating’ equilibria, i.e., those in which the equilibrium price function is invertible in the informed trader’s information, always exist. However, a key difference between our model and that of Laffont and Maskin (1990) is that because of risk neutrality of market makers and informed traders, there is no risk-sharing motive for trade in our model. Thus, with infinitesimal noise trading and discrete orders by informed traders, by the well-known theorem of Milgrom and Stokey (1982), there will be no trade. If the distribution of the terminal value  $v$  had bounded support, there would be an equilibrium in which the bid price would be the lower bound and the ask price would be the upper bound. However, since the support of the distribution of  $v$  is unbounded, there can be no equilibrium in discrete strategies.

Of course, the discussion in the two preceding paragraphs provides only informal arguments. Attempting to prove a general nonexistence result for the case of continuous auctions, however, takes us beyond the scope of this paper, in large part due to the analytical difficulties involved in addressing the issue of nonlinear equilibria in Kyle’s (1985) model (and thus our model).

#### *D. Endogenous Information Acquisition*

To this point we have taken as exogenous the number of informed traders  $M$ . Now we consider how many risk neutral individuals will choose to become informed when there is a cost  $C > 0$  of observing  $v$ . The expected profits from becoming informed, found by taking an ex ante expectation of equation (4), are

$$E_0[\pi_1(M)] = E_0[\alpha_0(M)(v - p_0)^2 + \delta_0(M)] = \alpha_0(M)\Sigma_0 + \delta_0(M),$$

where  $\alpha_0$  and  $\delta_0$  are calculated from equation (5), evaluated at  $n = 0$ , and equation (14), evaluated at  $n = 1$ , respectively.

In equilibrium, risk neutral individuals continue to become informed as long as the expected profits from becoming informed are greater than or equal to the cost of becoming informed. This leads to the following proposition for the equilibrium number of informed traders  $M^*$ .



Proposition 4: *The equilibrium number of informed traders  $M^*$  is given by*

$$\alpha_0(M^*)\Sigma_0 + \delta_0(M^*) = C$$

*where  $M^*$  is constrained to be a non-negative integer.*

Kyle (1985), based on the insight of Black (1971), suggests three components of market liquidity: (i) tightness (the cost of turning around a position in a short time), (ii) depth (the order flow necessary to move prices a unit amount), and (iii) resiliency (the speed with which prices recover from an uninformative shock). It is worthwhile to contrast the liquidity characteristics of the market with many informed traders to those of a market with a single informed trader.

In the case of a single informed trader and continuous auctions, the market is infinitely tight, and market depth is a finite constant over time. Also, the market is finitely resilient at all times except at the last instant, when the informed trader's strategy parameter goes to infinity (see Kyle (1985), p. 1330). In a model with multiple informed traders in which the interval between the auctions is very small, the market is almost infinitely deep and almost infinitely tight at all times. Since prices practically cease to fluctuate after the first auction in such a case, resiliency is also infinite. Thus, such a market exhibits all the characteristics of a perfect one, despite the fact that it is characterized by only a few strategic traders who optimally exploit their informational advantage.

### III. Conclusion

A fundamental issue which has been addressed using different approaches in the literature on financial markets is the manner in which prices come to reveal the information of privately informed individuals. The contribution of this paper has been to develop a multi-period auction model of price formation which is characterized by multiple noncompetitive agents exploiting their long-lived informational advantage over time.

We have shown that the contrast between our results and those of Kyle (1985), which assumes the presence of a single informed trader, are far from trivial. In particular, in our case, as the interval between auctions becomes large, the market approaches the perfectly competitive outcome of full information revelation and infinite market depth almost immediately, in contrast to the case of a monopolist considered by Kyle (1985), wherein market depth is finite at all times and information is revealed only gradually. It is worth noting that the remarkable contrast between our results and those of Kyle (1985) obtains even when the presence of just two noncooperative privately informed agents is assumed.

Our model could be applied to explain intraday phenomena, such as the temporal variation in the adverse selection component of the bid-ask spread (measured by the adverse price impact of trades), documented in Foster and Viswanathan (1990). For example, if long-lived information arrives during

nontrading hours, then our model suggests that informed traders will concentrate their trading at the opening of the market, and therefore that the adverse selection problem will be most severe at the beginning of the day. This is consistent with the finding of Foster and Viswanathan (1990). We view our model as a first step toward understanding the trading patterns of strategic agents with a long-lived informational advantage, and the consequent impact of such patterns on parameters of regulatory and practical interest such as market liquidity and the informational efficiency of prices.

**Appendix**

*Proof of Proposition 2:* Define  $q_n \equiv \alpha_n \lambda_n$ . From (5), we have

$$\alpha_{n-1} = \frac{1 - q_n}{\lambda_n [M(1 - 2q_n) + 1]^2}$$

or

$$q_{n-1} = \frac{\lambda_{n-1}(1 - q_n)}{\lambda_n [M(1 - 2q_n) + 1]^2}$$

implying that

$$\frac{\lambda_n}{\lambda_{n-1}} = \frac{1 - q_n}{q_{n-1} [M(1 - 2q_n) + 1]^2}. \tag{27}$$

Now, from (7), we also have

$$\frac{\lambda_n}{\lambda_{n-1}} = \frac{\beta_n}{\beta_{n-1}} \frac{\Sigma_n}{\Sigma_{n-1}}$$

and, from (8), this is equivalent to

$$\frac{\lambda_n}{\lambda_{n-1}} = \frac{\beta_n}{\beta_{n-1}} [1 - M\beta_n \lambda_n \Delta t_n]. \tag{28}$$

From (6),

$$\beta_n \Delta t_n = \frac{1 - 2q_n}{\lambda_n [M(1 - 2q_n) + 1]}. \tag{29}$$

Substituting for  $\beta_n$  and  $\beta_{n-1}$  from (29) above into (28), we have

$$\frac{\lambda_n}{\lambda_{n-1}} = \left( \frac{1 - 2q_n}{[M(1 - 2q_n) + 1]^2} \right) \left( \frac{M(1 - 2q_{n-1}) + 1}{1 - 2q_{n-1}} \right) \left( \frac{\lambda_{n-1} \Delta t_{n-1}}{\lambda_n \Delta t_n} \right)$$

so that

$$\frac{\lambda_n^2}{\lambda_{n-1}^2} = \left( \frac{1 - 2q_n}{[M(1 - 2q_n) + 1]^2} \right) \left( \frac{M(1 - 2q_{n-1}) + 1}{1 - 2q_{n-1}} \right) \left( \frac{\Delta t_{n-1}}{\Delta t_n} \right). \tag{30}$$

Squaring the RHS of (27), equating the resulting expression to the RHS of (30), and rearranging yields the cubic equation in Proposition 2. Note that the boundary condition  $\alpha_N = 0$  implies that  $q_N \equiv \alpha_N \lambda_N = 0$ .

It remains to be shown that there is a unique economically sensible root to this cubic equation which lies in the interval  $(0, \frac{1}{2})$ . First, note from the second order condition (13) that  $q_n < 1$ . Then note from equations (24) and (25) that any root of this cubic equation which makes economic sense must lie in the interval  $(0, \frac{1}{2})$ . The cubic equation (22) may be rewritten as

$$[1 + M(1 - 2q_{n-1})]q_{n-1}^2 \frac{\Delta t_{n-1}}{\Delta t_n} = (1 - 2q_{n-1})k_n \tag{31}$$

Define  $f(q_{n-1})$  and  $g(q_{n-1})$  to be the LHS and RHS of (31), respectively. It is straightforward to show that these functions have a unique point of intersection in the interval  $q_{n-1} \in (0, \frac{1}{2})$ . Equations (24), (25), and (26) follow from the definition of  $q_n$  and from equations (6), (7), and (8).

*Proof of Proposition 3:* Define  $q_n^N \equiv \alpha_n \lambda_n$  in an economy with  $N$  auctions. There are four steps in the proof. The first three steps establish some preliminary results and then the fourth step uses these preliminary results to take all of the limits. In step one we show that for any positive constant  $n$ , there exists a limit for the sequence  $q_n^n, q_n^{n+1}, q_n^{n+2}, \dots$ . Denote the limit by  $q_n \equiv \lim_{N \rightarrow \infty} q_n^N$ . In step two we show that for any two positive constants  $n_1 \neq n_2$ , the sequences go to the same limit. That is,  $q_{n_1} = q_{n_2}$ . Denote this common limit by  $q \equiv q_n \equiv \lim_{N \rightarrow \infty} q_n^N \forall n < \infty$ . In step three we show that when  $M = 1$  then  $q = \frac{1}{2}$ , when  $M = 2$  then  $q = 0.278626 \dots$ , and when  $M \geq 3$  then  $q$  is positive and is strictly smaller than the  $q$  for  $M = 2$ . In step four we use the results of the first three steps to take all of the limits.

*Step One:* In order to show that a limit exists, we start by showing that  $q_n^N$  is bounded. We know by the definitions of  $\alpha_n$  and  $\lambda_n$  in equations (5) and (7) that they are non-negative. Hence  $q_n$  is non-negative. From the second order condition  $\lambda_n(1 - q_n^N) > 0$ , we know that  $q_n^N < 1$ . Hence,  $q_n^N$  is bounded in the interval  $[0, 1)$ .

Using the assumption that  $\Delta t_n = 1/N, \forall n$ , then from Proposition 2 we have  $q_N^N = 0$  and the sequence  $q_{N-1}^N, q_{N-2}^N, \dots, q_1^N$  is generated iteratively from

$$0 = 2M(q_{n-1}^N)^3 - (M + 1)(q_{n-1}^N)^2 - 2k_n^N(q_{n-1}^N) + k_n^N \tag{32}$$

where

$$k_n^N \equiv \frac{(1 - q_n^N)^2}{(1 - 2q_n^N)(M(1 - 2q_n^N) + 1)^2} \tag{33}$$

Note these equations are only functions of  $M$  and of the number of periods until information revelation  $N - (n - 1)$ , not of  $\Delta t_n$  or  $N$ . In other words, the sequence of  $q$ 's generated from the final auction backward will be the

same no matter what  $N$  is. Hence, comparing economies with different numbers of auctions we see

$$q_n^N = q_{n+1}^{N+1} = q_{n+2}^{N+2} = \dots \tag{34}$$

Each of the  $q$ 's in (34) is  $N - n$  auctions away from the final auction.

Now consider a given economy with a fixed  $N$ . Taking partial derivatives in equations (32) and (33) it is straightforward to show that

$$\frac{\partial q_{n-1}^N}{\partial k_n^N} > 0 \quad \text{and} \quad \frac{\partial k_n^N}{\partial q_n^N} > 0.$$

Starting from  $q_N^N = 0$  and particular values for  $k_{N-1}^N > 0$  and  $q_{N-1}^N > 0$ , then the partial derivatives above imply that as we iterate backward an increase in  $q$  will increase  $k$  and similarly an increase in  $k$  will increase  $q$ . Hence, both the  $q$ 's and  $k$ 's increase monotonically. That is

$$q_N^N < q_{N-1}^N < q_{N-2}^N < \dots < q_n^N < q_{n-1}^N < q_{n-2}^N < \dots \tag{35}$$

Substituting (34) into (35) we obtain

$$\dots < q_n^N < q_n^{N+1} < q_n^{N+2} < \dots \tag{36}$$

Hence we have proved that the sequence in (36) is bounded and monotonic. Using the fundamental property that any bounded, monotonic sequence converges to a limit we obtain the desired result. Denote that limit by  $q_n \equiv \lim_{N \rightarrow \infty} q_n^N$ .

*Step Two:* Let  $n_1$  and  $n_2$  be any two positive constants such that  $n_1 \neq n_2$ . Using the definition of a limit we know that for any  $\hat{\varepsilon} > 0$ , there exists  $N_1$  such that for all  $N > N_1$

$$|q_{n_1}^N - q_{n_1}| < \hat{\varepsilon}. \tag{37}$$

Similarly we know that for the same  $\hat{\varepsilon} > 0$ , there  $N_2$  such that for all  $N > N_2$

$$|q_{n_2}^N - q_{n_2}| < \hat{\varepsilon}. \tag{38}$$

For any  $\varepsilon > 0$ , set  $\hat{\varepsilon} = \varepsilon/3$  and determine  $N_1$  and  $N_2$  such that (37) and (38) obtain. Using (34), (37), and (38) below, we have for any  $N > \text{Max}(N_1, N_2)$

$$\begin{aligned} |(q_{n_1}^N - q_{n_1}) - (q_{n_2}^N - q_{n_2})| &< 2\hat{\varepsilon} \\ |(q_{n_1}^N - q_{n_2}^N) - (q_{n_1} - q_{n_2})| &< 2\hat{\varepsilon} \\ |(q_{n_1}^N - q_{n_1}^{N+(n_1-n_2)}) - (q_{n_1} - q_{n_2})| &< 2\hat{\varepsilon} \end{aligned}$$

Note that

$$\hat{\varepsilon} > |q_{n_1}^N - q_{n_1}^{N+(n_1-n_2)}| > 0.$$

Hence

$$\begin{aligned} |-(q_{n_1} - q_{n_2})| &< 3\hat{\varepsilon} \\ |-(q_{n_1} - q_{n_2})| &< \varepsilon, \end{aligned}$$

which is the desired result. Denote  $q \equiv q_n \equiv \lim_{N \rightarrow \infty} q_n^N, \forall n$ .

*Step Three:* From step two we know that in the limit as  $N \rightarrow \infty$ , we can substitute  $q_n^N = q_{n-1}^N = q$  into (32) and (33) and then substitute (33) into (32) to obtain a fifth order equation for the limit  $q$

$$(1 - 2q)(4M^3q^4 - 2M^2(2M + 3)q^3 + M(M^2 + 3M + 3)q^2 - 1) = 0 \quad (39)$$

In order to determine which of the five roots is relevant root, it is useful to obtain a tighter bound  $q$  when  $M = 1$ . Substituting  $h = 0$  into (19) to obtain

$$\Delta x_n = (v - p_{n-1}) \frac{1 - 2q_n^N}{\lambda_n [M(1 - 2q_n^N) + 1]} \quad (40)$$

Given  $\lambda_n \geq 0$  by definition and the previous bound of  $q_n^N \in [0, 1]$ , then when  $M = 1$  the denominator is strictly positive. In order for  $\Delta x_n$  to have the same sign as  $(v - p_{n-1})$ , that is the informed trader buys undervalued securities and sells overvalued securities, the numerator  $(1 - 2q_n^N)$  must be strictly positive. This implies that  $q_n^N$  is bounded in the interval  $[0, \frac{1}{2}]$ . Combining this with the monotonicity result in (35), we obtain the result that when  $M = 1$ , then  $q \in [0, \frac{1}{2}]$ . Of the five roots of the fifth order equation (39), when  $M = 1$  only one root is in the interval  $[0, \frac{1}{2}]$  and that is  $q = \frac{1}{2}$ .

It is clear by inspection of (33) and the appropriate cube root of (32) that  $\partial k_n^N / \partial M < 0$  and  $\partial q_n^N / \partial M < 0$ . Combining these partial derivatives with the fact that  $q_n^N = 0$  is the same for all values of  $M$ , it follows that the limit has the property  $\partial q / \partial M < 0$ . Hence, for  $M \geq 2$ ,  $q$  must be in the interval  $[0, \frac{1}{2}]$ . Of the five roots of the fifth order equation (39), when  $M \geq 2$  only one root is in the interval  $[0, \frac{1}{2}]$ . For  $M = 2$ ,  $q = 0.278626 \dots$ . Using the property  $\partial q / \partial M < 0$ , then for  $M \geq 3$  the limit  $q$  must be a strictly smaller positive number than the  $q$  for  $M = 2$ .

*Step Four:* Now we use the preliminary results of the first three steps to take all of the limits.

*Part (i):* From Proposition 2 we have that

$$\frac{\Sigma_n}{\Sigma_{n-1}} = \frac{1}{M(1 - 2q_n^N) + 1} < \frac{1}{M(1 - 2q) + 1} \quad (41)$$

Evaluating the RHS of (41) at  $q = \frac{1}{2}$  yields RHS = 1. Evaluating the RHS of (41) at  $M = 2$  and  $q = 0.278626 \dots$  yields RHS =  $0.530365 \dots < \frac{2}{3}$ . Evaluating the RHS of (41) at  $M \geq 3$  and the corresponding limit  $q$  yields RHS  $< \frac{2}{3}$ .

Substituting recursively we obtain

$$\begin{aligned} \Sigma_{n'} &= \Sigma_0 \left( \frac{\Sigma_1}{\Sigma_0} \right) \left( \frac{\Sigma_2}{\Sigma_1} \right) \cdots \left( \frac{\Sigma_{n'}}{\Sigma_{n'+1}} \right) \\ &= \Sigma_0 \left( \frac{1}{M(1 - 2q_1^N) + 1} \right) \\ &\quad \cdot \left( \frac{1}{M(1 - 2q_2^N) + 1} \right) \cdots \left( \frac{1}{M(1 - 2q_{n'}^N) + 1} \right) \\ &< \Sigma_0 \left( \frac{1}{M(1 - 2q) + 1} \right) \left( \frac{1}{M(1 - 2q) + 1} \right) \cdots \left( \frac{1}{M(1 - 2q) + 1} \right) \\ &= \Sigma_0 \left( \frac{1}{M(1 - 2q) + 1} \right)^{n'} \end{aligned}$$

When  $M = 1$ , the term in parentheses is equal to one, which is not a meaningful restriction on  $\Sigma_{n'}$ . However, when  $M \geq 2$  we can substitute the restriction that the term in parenthesis is less than  $\frac{2}{3}$  and  $n' < \tau N$  to obtain

$$\lim_{N \rightarrow \infty} \Sigma_{n'} \leq \lim_{N \rightarrow \infty} \Sigma_0 \left( \frac{2}{3} \right)^{(\tau N)} = 0.$$

From Proposition 2 we have

$$\begin{aligned} \lambda_{n'} &= \left( \frac{M \Sigma_{n'} (1 - 2q_{n'}^N)}{\frac{1}{N} \sigma_u^2 [M(1 - 2q_{n'}^N) + 1]} \right)^{1/2} \\ &= \left( \frac{M \Sigma_0 \left( \frac{1}{M(1 - 2q_1^N) + 1} \right) \left( \frac{1}{M(1 - 2q_2^N) + 1} \right) \cdots \left( \frac{1}{M(1 - 2q_{n'}^N) + 1} \right) (1 - 2q_{n'}^N)}{\frac{1}{N} \sigma_u^2 [M(1 - 2q_{n'}^N) + 1]} \right)^{1/2} \\ &< \left( \frac{NM \Sigma_0 \left( \frac{1}{M(1 - 2q) + 1} \right)^{\tau N} (1 - 2q)}{\sigma_u^2 [M(1 - 2q) + 1]} \right)^{1/2} \end{aligned}$$

When  $M \geq 2$  then  $q \in (0, \frac{1}{2})$  and hence  $1/(M(1 - 2q) + 1) \in (0, 1)$ . This implies

$$\lim_{N \rightarrow \infty} \lambda_{n'} \leq \lim_{N \rightarrow \infty} \left( \frac{NM \Sigma_0 \left( \frac{1}{M(1 - 2q) + 1} \right)^{\tau N} (1 - 2q)}{\sigma_u^2 [M(1 - 2q) + 1]} \right)^{1/2} = 0.$$

*Part (ii):* For each of the limits in this part, we use the result that when  $M \geq 2$  then  $q_1^N \in (0, \frac{1}{2})$ , which implies that  $(1 - 2q_1^N) \in (0, 1)$ . From Proposition 2, the first limit is

$$\lim_{N \rightarrow \infty} \lambda_1 = \lim_{N \rightarrow \infty} \left( \frac{M \left( \frac{1}{M(1 - 2q) + 1} \right) \Sigma_0 (1 - 2q_1^N)}{\frac{1}{N} \sigma_u^2 [M(1 - 2q_1^N) + 1]} \right)^{1/2} = \infty. \quad (42)$$

The second limit is

$$\begin{aligned} \lim_{N \rightarrow \infty} E[\Delta X_1 | v] &= \lim_{N \rightarrow \infty} \left( \frac{M(1 - 2q_n^N) \frac{1}{N} \sigma_u^2}{\Sigma_1 [M(1 - 2q_n^N) + 1]} \right)^{1/2} (v - p_0) \\ &= \lim_{N \rightarrow \infty} \left( \frac{M(1 - 2q_n^N) \frac{1}{N} \sigma_u^2}{\left( \frac{1}{M(1 - 2q_1^N) + 1} \right) \Sigma_0 [M(1 - 2q_n^N) + 1]} \right)^{1/2} (v - p_0) = 0. \end{aligned}$$

From (7) and Proposition 2, the third limit is

$$\lim_{N \rightarrow \infty} \beta_1 = \lim_{N \rightarrow \infty} \frac{\lambda_1 \sigma_u^2}{M \Sigma_1} = \lim_{N \rightarrow \infty} \frac{\lambda_1 \sigma_u^2}{M \left( \frac{1}{M(1 - 2q_1^N) + 1} \right) \Sigma_0} = \infty$$

where the last equality follows from (42).

*Part (iii):* For each of the limits in this part, we hold  $N$  constant and use the result that when  $M \geq 2$  then  $q_1^N \in (0, \frac{1}{2})$ , which implies that  $(1 - 2q_1^N) \in (0, 1)$ . We compute all limits using the relevant expressions in Proposition 2.

$$\begin{aligned} \lim_{M \rightarrow \infty} \Sigma_1 &= \lim_{M \rightarrow \infty} \left( \frac{1}{M(1 - 2q_1^N) + 1} \right) \Sigma_0 = 0. \\ \lim_{M \rightarrow \infty} \lambda_1 &= \lim_{M \rightarrow \infty} \left( \frac{M \left( \frac{1}{M(1 - 2q) + 1} \right) \Sigma_0 (1 - 2q_1^N)}{\frac{1}{N} \sigma_u^2 [M(1 - 2q_1^N) + 1]} \right)^{1/2} = 0. \end{aligned}$$

$$\begin{aligned}
 & \lim_{M \rightarrow \infty} E[\Delta X_1 | v] \\
 &= \lim_{M \rightarrow \infty} \left( \frac{M(1 - 2q_n^N) \frac{1}{N} \sigma_u^2}{\Sigma_1 [M(1 - 2q_n^N) + 1]} \right)^{1/2} (v - p_0) \\
 &= \lim_{M \rightarrow \infty} \left( \frac{M(1 - 2q_n^N) \frac{1}{N} \sigma_u^2}{\left( \frac{1}{M(1 - 2q_1^N) + 1} \right) \Sigma_0 [M(1 - 2q_n^N) + 1]} \right)^{1/2} \\
 &\quad \cdot (v - p_0) = \infty. \\
 \lim_{M \rightarrow \infty} p_1 &= \lim_{M \rightarrow \infty} [\Delta p_1 + p_0] \\
 &= \lim_{M \rightarrow \infty} \left( \frac{M \Sigma_1 (1 - 2q_1^N)}{\frac{1}{N} \sigma_u^2 [M(1 - 2q_1^N) + 1]} \right)^{1/2} \{\Delta X_1 + \Delta u_1\} + p_0 \\
 &= \lim_{M \rightarrow \infty} \left( \frac{M \Sigma_1 (1 - 2q_1^N)}{\frac{1}{N} \sigma_u^2 [M(1 - 2q_1^N) + 1]} \right)^{1/2} \\
 &\quad \cdot \left\{ \left( \frac{M(1 - 2q_1^N) \frac{1}{N} \sigma_u^2}{\Sigma_1 [M(1 - 2q_1^N) + 1]} \right)^{1/2} (v - p_0) + \Delta u_1 \right\} + p_0 \\
 &= \lim_{M \rightarrow \infty} \left( \frac{M(1 - 2q_1^N)}{M(1 - 2q_1^N) + 1} \right) (v - p_0) \\
 &\quad + \left( \frac{M \Sigma_1 (1 - 2q_1^N)}{\frac{1}{N} \sigma_u^2 [M(1 - 2q_1^N) + 1]} \right)^{1/2} \Delta u_1 + p_0 \\
 &= (v - p_0) + p_0 = v.
 \end{aligned}$$

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