Economics Letters 44 (1994) 181–190

0165-1765/94/$07.00

Risk aversion, imperfect competition, and long-lived information

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Received 1 June 1993
Accepted 9 August 1993

Abstract

This paper presents a methodology for characterizing the optimal dynamic behavior of risk-averse, strategic agents with private information, by building on Kyle (Econometrica, 1985, 53, 1315–1335). It is shown that both monopolistic and competing informed traders choose to exploit rents rapidly, causing market depth to be low in the initial periods and high in later periods, and causing information to be revealed rapidly, unlike in the case of a risk-neutral monopolist considered by Kyle.

JEL classification: G14

1. Introduction

This work presents a methodology for characterizing the dynamically optimal trading patterns of strategic, risk-averse traders who possess private information about the fundamental value of a security, using an extension of the framework of Kyle (1985). Risk aversion raises some interesting questions that have not yet been addressed within a dynamic framework. For example, what role does risk sharing play in limiting the extent of competition between the informed traders? What is the effect of risk aversion on the intertemporal patterns of market liquidity and the informational efficiency of prices? We attempt to answer such questions by way of our model.

Kyle (1985) considers the case of a single risk-neutral, privately informed agent. The main results obtained by Kyle are that information is incorporated into price at a slow, almost linear rate, and that market depth is approximately constant over time. Holden and Subrahmanyam (HS) (1992) and Foster and Viswanathan (FV) (1993) point out the dramatic contrast between the case of an informational monopolist and multiple non-cooperative informed agents. In HS and FV, when auctions are held sufficiently closely, markets are essentially strong-form efficient and market depth is infinite at almost all times, even in the case of only two informed agents. This result obtains because each informed trader trades aggressively to exploit his informational advantage before the trades of other informed agents reveal his private information to the market.

The purpose of this work is to embed the Kyle (1985) and the HS–FV frameworks in a richer

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SSDI 0165-1765(93)00288-Y

preference structure, namely to allow for risk aversion on the part of informed agents. To our knowledge, this is the first attempt to model risk aversion and strategic behavior in a dynamic framework. Thus, our dynamic model is characterized by three classes of agents: risk-averse informed traders with long-lived information, risk-neutral market makers, and 'liquidity' traders with exogenous motives for trading.

Our primary finding is that both monopolistic and competing informed traders choose to exploit rents rapidly, causing market depth to be low in the initial periods and high in later periods, and causing information to be revealed rapidly, unlike in the case of a risk-neutral monopolist considered by Kyle (1985). The intuition for this result is that although the risk-averse informed agent wishes to exploit his rents gradually, he is also concerned with the risk of the uncertain prices at which future trades will be conducted. As a result, to avoid bearing too much risk, the risk-averse monopolist trades substantially more aggressively than a risk-neutral one. Further simulations indicate that the HS–FV results, however, do not seem to be sensitive to the assumption of risk neutrality, for reasonable values of the risk-aversion coefficient.

In section 2 we discuss the structure of our model and derive its linear equilibrium. Section 3 presents some properties of the model's equilibrium. Section 4 concludes.

2. The model

2.1. Structure and notation

We conform to the notation of Kyle (1985). A security is traded in N sequential auctions in a time interval which begins at $t = 0$ and ends at $t = 1$. The security's value at the end of trading is denoted by v, which is assumed to be normally distributed with mean p_0 and variances Σ_0. Let M denote the number of informed traders, who are indexed by $i = 1, \dots, M$. All informed traders have identical initial wealth W_0. Let W_n denote the wealth at auction N. Informed traders have negative exponential utility with risk-aversion coefficient A, for terminal wealth (denoted by W_{N+1}), i.e.

$$U(W_{N+1}) = -\exp(-AW_{N+1}) . \tag{1}$$

Each informed trader observes the liquidation value, v, in advance (before commencement of trading). Let ΔX_n and Δx_n denote the *total* order by *all* informed traders and the *individual* order by the ith informed trader at the nth auction, respectively. (We suppress i subscripts from the parameters associated with the informed traders since we only examine the symmetric equilibrium.)

Each risk-averse informed trader determines his optimal trading strategy by a process of backward induction in order to maximize his expected utility given his conjectures about the trading strategies of the other informed traders. In the rational expectations equilibrium, the conjectures of each identical informed trader must be correct conditional on each trader's information at each auction.

At each auction orders are also submitted by liquidity traders. Let Δu_n be the aggregate order submitted by liquidity traders at the nth auction. We assume that Δu_n is serially uncorrelated and is normally distributed with zero mean and variance of $\sigma_u^2 \Delta t_n$, where Δt_n is the time interval between the nth auction and the previous auction. Let W_n denote the wealth of an informed trader at the nth auction and $J(W_n)$ denote the indirect utility from W_n.

Trading takes place through risk-neutral market makers who absorb the order flow while earning zero expected profits. [1] At each auction, market makers observe only the *combined* order flow $\Delta X_n + \Delta u_n$, whereupon they set p_n, the price at the nth auction. Equilibrium is defined by a market efficiency condition that p_n equals the expected value of v conditional on the information available to the market makers at the auction, by a utility-maximization condition that each informed trader selects the optimal strategy conditional on his conjectures and his information at each auction, and by a condition that all conjectures are correct.

2.2. Equilibrium

We now state a proposition which provides the difference equation system characterizing our equilibrium:

Proposition 1. There exists a recursive linear (symmetric) equilibrium in our model, in which there are constants α_n, β_n, δ_n, λ_n, Σ_n, and γ_n, characterized by the following:

$$\Delta X_n = M\beta_n(v - p_{n-1})\,\Delta t_n\,, \tag{2}$$

$$\Delta p_n = \lambda_n(\Delta X_n + \Delta u_n)\,, \tag{3}$$

$$\Sigma_n = \mathrm{var}(v|\Delta X_1 + \Delta u_1, \ldots, \Delta X_n + \Delta u_n)\,, \tag{4}$$

$$J(W_{n+1}) = -\gamma_n \exp[-A(W_{n+1} + \alpha_n(v - p_n)^2)]\,. \tag{5}$$

for all auctions $n = 1, \ldots, N$ and for all informed traders $i = 1, \ldots, M$. Given the prior variance Σ_0, the constants β_n, λ_n, α_n, Σ_n, and γ_n are the solution to the difference equation system

$$\beta_n\,\Delta t_n = \frac{1 - 2\alpha_n\lambda_n}{\lambda_n[M(1 - 2\alpha_n\lambda_n) + 1 + A\lambda_n\sigma_u^2\,\Delta t_n]}\,, \tag{6}$$

$$\alpha_{n-1} = \frac{1 - \alpha_n\lambda_n + \frac{1}{2}A\lambda_n\sigma_u^2\,\Delta t_n}{\lambda_n[M(1 - 2\alpha_n\lambda_n) + 1 + A\lambda_n\sigma_u^2\,\Delta t_n]^2}\,, \tag{7}$$

$$\lambda_n = \frac{M\beta_n\Sigma_{n-1}}{M^2\beta_n^2\,\Delta t_n\Sigma_{n-1} + \sigma_u^2}\,, \tag{8}$$

$$\Sigma_n = \frac{\sigma_u^2\Sigma_{n-1}}{M^2\beta_n^2\,\Delta t_n\Sigma_{n-1} + \sigma_u^2}\,, \tag{9}$$

for auctions $n = 1, \ldots, N - 1$, subject to the boundary condition

$$\alpha_N = 0 \tag{10}$$

and the second-order condition

[1] Apart from tractability, there is another reason to model market makers as being risk neutral, since market making is typically performed by large financial institutions on the ‘trading floor’. These institutions would have large capacity to bear risk and their behavior can therefore be adequately modeled by assuming risk neutrality. Moreover, since risk aversion is akin to a ‘cost’ of providing services, one would expect that, in general, it would be performed by the individuals possessing the lowest costs, i.e. by risk-neutral agents. This would be especially true if entry into market making were easy.

$$-2\lambda_n + 2\alpha_n\lambda_n^2 + A\left[\frac{(\lambda_n - 2\alpha_n\lambda_n^2)^2\sigma_u^2\,\Delta t_n}{1 + 2A\alpha_n\lambda_n^2\sigma_u^2\,\Delta t_n}\right] < 0\,. \tag{11}$$

The constant γ_n evolves according to

$$\gamma_{n-1} = \frac{\gamma_n}{(1 + 2A\alpha_n\lambda_n\sigma_u^2\,\Delta t_n)^{1/2}} \tag{12}$$

subject to the boundary condition

$$\gamma_N = 1\,. \tag{13}$$

Proof. We begin with the following lemma.

Lemma 1. Let $Y \sim \mathrm{N}(0, \sigma^2)$. Then, if $2a\sigma^2 < 1$,

$$\mathrm{E}[\exp(aY^2 + bY)] = \exp\left(\frac{b^2\sigma^2}{2(1 - 2a\sigma^2)}\right)\frac{1}{\sqrt{1 - 2a\sigma^2}}\,.$$

If $2a\sigma^2 > 1$, the above expectation is infinite.

Proof.

$$\mathrm{E}[\exp(aY^2 + bY)] = (2\pi\sigma^2)^{-1/2}\int_{-\infty}^{\infty}\exp\left[\left(a - \frac{1}{2\sigma^2}\right)Y^2 + bY\right]\mathrm{d}y\,. \tag{14}$$

The above expectation if finite only if $a - (1/2\sigma^2) > 0$. Suppose this condition holds. Then, write

$$v = \sqrt{2\left(\frac{1}{2\sigma^2} - a\right)}\,Y - \frac{b}{\sqrt{2\left(\frac{1}{2\sigma^2} - a\right)}}\,.$$

After a change of variables, (14) becomes

$$(2\pi\sigma^2)^{-1/2}\frac{1}{\sqrt{2\left(\frac{1}{2\sigma^2} - a\right)}}\int_{-\infty}^{\infty}\exp\left[-\frac{v^2}{2} + \frac{b^2}{4\left(\frac{1}{2\sigma^2} - a\right)}\right]\mathrm{d}v$$
$$= \exp\left(\frac{b^2\sigma^2}{2(1 - 2a\sigma^2)}\right)\frac{1}{\sqrt{1 - 2a\sigma^2}}\,,$$

thus completing the proof. □

Make the inductive hypothesis that $J(W_{n+1})$ is given by

$$J(W_{n+1}) = -\gamma_n\exp[-A(W_{n+1} + \alpha_n(v - p_n)^2)]\,. \tag{15}$$

Let Δx denote the control quantity of a particular informed trader at the nth auction. Then,

$$J(W_n) = \max_{\Delta x} \mathrm{E}_n[J(W_{n+1})]$$
$$= \max_{\Delta x} \mathrm{E}_n[-\gamma_n \exp(-A\{W_n + \Delta x(v - p_n) + \alpha_n(v - p_n)^2\})] . \tag{16}$$

Note that the total order ΔX_n can be written as $\Delta x + (M-1)\,\Delta\bar{x}_n$, where $\Delta\bar{x}_n$ represents the particular informed trader's conjecture of the average of the other informed traders' strategies. In a linear equilibrium,

$$p_n = p_{n-1} + \lambda_n(\Delta X_n + \Delta u_n) + h = p_{n-1} + \lambda_n(\Delta x + (M-1)\,\Delta\bar{x}_n + \Delta u_n) + h , \tag{17}$$

where h is some linear function of $\Delta X_1 + \Delta u_1, \ldots, \Delta X_{n-1} + \Delta u_{n-1}$.

Using Lemma 1 and (17), $J(W_n)$ can be written as

$$J(W_n) = \max_{\Delta x} -\gamma_n \exp\Big(-A\{W_n + \Delta x(v - p_{n-1} - \lambda_n(\Delta x + (M-1)\,\Delta\bar{x}) - h)$$
$$+ \alpha_n(v - p_{n-1} - \lambda_n(\Delta x + (M-1)\,\Delta\bar{x}_n) - h)^2\}$$
$$+ \frac{A^2[-\Delta x\lambda_n - 2\alpha_n\lambda_n(v - p_{n-1} - \lambda_n(\Delta x + (M-1)\,\Delta\bar{x}) - h)]^2\sigma_u^2\,\Delta t_n}{2(1 + 2A\alpha_n\lambda_n^2\sigma_u^2\,\Delta t_n)}\Big)$$
$$\times \frac{1}{\sqrt{1 + 2A\alpha_n\lambda_n^2\sigma_u^2\,\Delta t_n}} . \tag{18}$$

Denote the optimized value of Δx from the above expression as Δx_n. Now differentiate the RHS of (18) with respect to Δx and set the resulting expression to zero. Then, substituting $\Delta\bar{x}_n = \Delta x = \Delta x_n$ (thus solving for the symmetric equilibrium), we have

$$\Delta x_n = \frac{1 - 2\alpha_n\lambda_n}{\lambda_n[M(1 - 2\alpha_n\lambda_n) + 1 + A\lambda_n\sigma_u^2\,\Delta t_n]}(v - p_{n-1} - h) . \tag{19}$$

It is straightforward to verify that the second-order condition is given by (11). We now show that $h = 0$. First note from the market efficiency condition that

$$\mathrm{E}\{\Delta p_n | \Delta X_1 + \Delta u_1, \ldots, \Delta X_{n-1} + \Delta u_{n-1}\} = 0 .$$

However, from (19) and the linear pricing rule (17),

$$\mathrm{E}\{\Delta p_n | \Delta X_1 + \Delta u_1, \ldots, \Delta X_{n-1} + \Delta u_{n-1}\} = \mathrm{E}[\lambda_n(\Delta X_n + \Delta u_n) + h]$$
$$M\lambda_n(-h)\frac{1 - 2\alpha_n\lambda_n}{\lambda_n[M(1 - 2\alpha_n\lambda_n) + 1 + A\lambda_n\sigma_u^2\,\Delta t_n]} + h = 0 ,$$

thus implying that $h = 0$. Thus (6) follows from (19) and verifies (2), and (3) follows from (17). Using $\Delta x_n = \Delta\bar{x}_n = \beta_n\,\Delta t_n(v - p_{n-1})$, and Eq. (18), we get (7) and (12), and the functional form in (5) is also verified. Simple applications of the projection theorem for normally distributed random variables yield (8) and (9). The boundary condition (10) formalizes the obvious that no utility can be gained after trading is complete. Finally, evaluating the indirect utility at auction N yields (13). □

While the difference equation system does not permit the derivation of analytical results, we describe an explicit numerical method for solving the difference equation system. [2] First, solve (9) for Σ_{n-1} to get

$$\Sigma_{n-1} = \frac{\Sigma_n \sigma_u^2}{\sigma_u^2 - M^2 \beta_n^2 \Delta t_n \Sigma_n} . \tag{20}$$

Then substitute the RHS of the above equation for Σ_{n-1} into (8) to obtain

$$\lambda_n = \frac{M\beta_n \Sigma_n}{\sigma_u^2} . \tag{21}$$

Using (21) and (20), it can be shown that

$$\Sigma_{n-1} = (1 - M\beta_n \lambda_n \Delta t_n)^{-1} \Sigma_n . \tag{22}$$

Substituting for β_n from (21) into (6) and simplifying, we have the following cubic equation for λ_n in terms of the endogenous parameters Σ_n and α_n:

$$\lambda_n^3 \sigma_u^2 (2M\alpha_n - A\sigma_u^2 \Delta t_n) \Delta t_n - \lambda_n^2 \sigma_u^2 (M+1) \Delta t_n - 2M\alpha_n \Sigma_n \lambda_n + M\Sigma_n = 0 . \tag{23}$$

In all the numerical simulations we perform, the cubic equation (23) has a unique root that satisfies the second-order condition (11). The solution to the difference equation system is found as follows. Start from an arbitrary value for Σ_N, say Σ and the boundary condition $\alpha_N = 0$. Solve for λ_N using (23). Calculate Σ_{N-1} using (22) and β_N using (21). Then α_{N-1} can be calculated using (7) and we have thus iterated the system backwards one step. We can thus solve for the values of the endogenous parameters at the auctions $N-2, \ldots, 1$ in a straightforward manner. Denote the value for Σ_0 obtained after solving the system backward to be Σ'. If $\Sigma' > \Sigma_0$, iterate downward by decreasing the starting value Σ and repeatedly performing the above calculations till the value for Σ_0 is sufficiently close to Σ_0. If $\Sigma' > \Sigma_0$, iterate upwards by increasing the starting value Σ instead, while performing the same calculations as above.

3. A numerical analysis of the properties of the equilibrium

As in Kyle (1985), and in Holden and Subrahmanyam (1992), the parameters Σ_n and λ_n are measures of price efficiency and market depth, respectively. We now present some numerical simulations using the method of solution for the difference equation system described above. In all the simulations, we assume that $\Sigma_0 = 1$, $\sigma_u^2 = 1$, and $\Delta t_n = 1/N$, $\forall n$, i.e. that auctions occur at equally spaced intervals. The qualitative features of the simulations were found to be robust to a wide parameter range.

Figures 1 and 2 plot λ_n and Σ_n for the cases of a single risk-neutral and a single risk-averse informed trader, holding constant calendar time between commencement and end of trading. The figures effectively present the contrast between the cases of a risk-averse and a risk-neutral monopolist. Note that Σ_n declines at approximately a linear rate in the case of a risk-neutral informed trader, while it declines much more rapidly in the case of a risk-averse one. More interesting is the intertemporal pattern of market depth (λ_n) in Fig. 2. As can be seen from this

[2] Note that the system of equations for α_n, β_n, λ_n, and Σ_n in Proposition 1 reduces to Kyle's (1985) system of difference equations when $A = 0$ and $M = 1$.

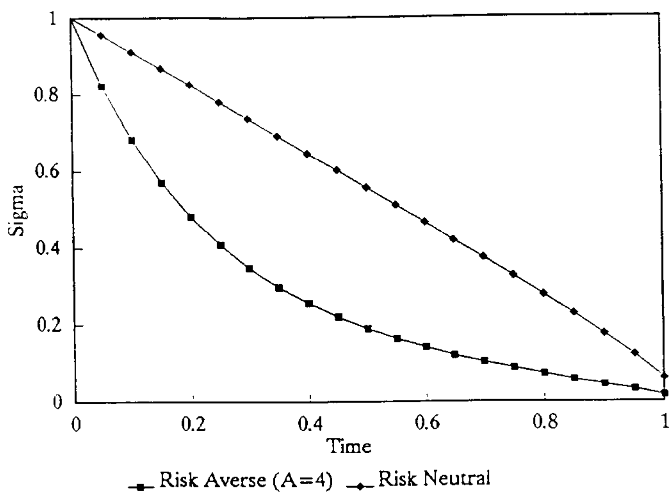

Fig. 1. Price efficiency parameter Σ_n over time for risk averse and risk neutral utility functions by a monopolist informed trader ($M = 1$).

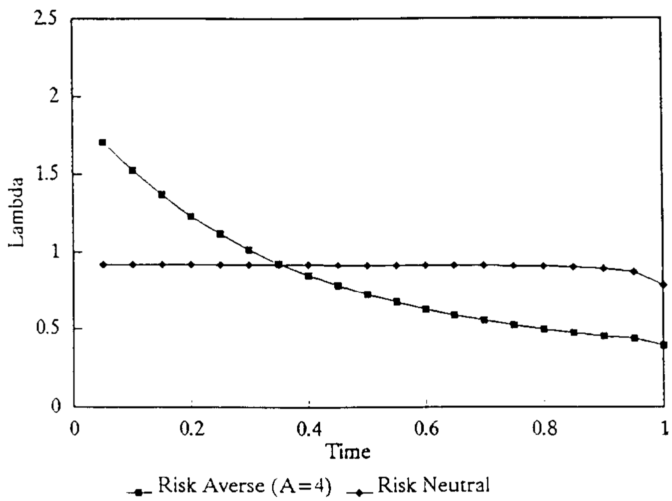

Fig. 2. Market liquidity parameter λ_n over time for risk averse and risk neutral utility functions by a monopolist informed trader ($M = 1$).

figure, λ_n is nearly constant in the Kyle (1985) case of a risk-neutral informed trader, while it declines quite sharply over time in the case of a risk-averse informed trader. The intuition for this result is that a risk-averse informed trader is concerned about future price risk, and this causes him to trade more rapidly than a risk-neutral informed trader. Consequently, the adverse selection (measured by λ_n) is high in the earlier periods because the information content of the order flow is high, and negligible in the later periods because the market maker has very little to fear from an informed trader who has already exploited most of his informational advantage.

An interesting feature of Fig. 2 is that market liquidity is *lower* at the initial auction in the case of the risk-averse insider than in the case of risk-neutral one. This seems surprising at first glance, since initial intuition suggests that risk-averse informed traders would trade less aggressively than

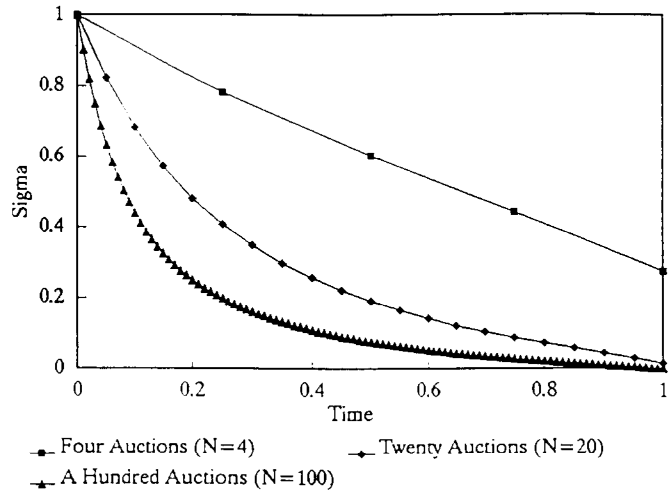

Fig. 3. Price efficiency parameter Σ_n over time for different values of N, the number of auctions, by a risk averse monopolist ($M = 1$, $A = 4$).

risk-neutral ones and thus lead to higher market liquidity. However, in this case the risk-averse agent trades more aggressively in the initial auctions to protect himself against future price risk and therefore causes the market to be less liquid than under risk neutrality. Note that the sign of the differences in liquidities reverses in later auctions, since the risk-averse informed trader exploits rents more rapidly over time than a risk-neutral one.

Figures 3, 4, 5, and 6 present the effect of increasing the number of auctions (i.e. approaching the case of continuous time) on λ_n and Σ_n. Consider first the case of a risk-averse monopolist, illustrated in Figs. 3 and 4. These figures again demonstrate the dramatic contrast between the case of a risk-neutral monopolist considered by Kyle (1985) and a risk-averse one. Unlike in Kyle (1985), as the number of auctions increases, λ_n does not tend to remain constant over time, but its value at the early auctions increases, and it drops to zero increasingly rapidly. Similarly, Σ_n drops

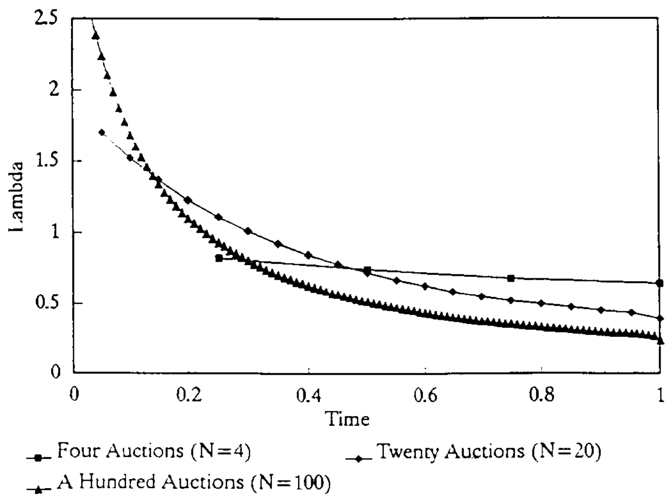

Fig. 4. Market liquidity parameter λ_n over time for different values of N, the number of auctions, by a risk averse monopolist ($M = 1$, $A = 4$).

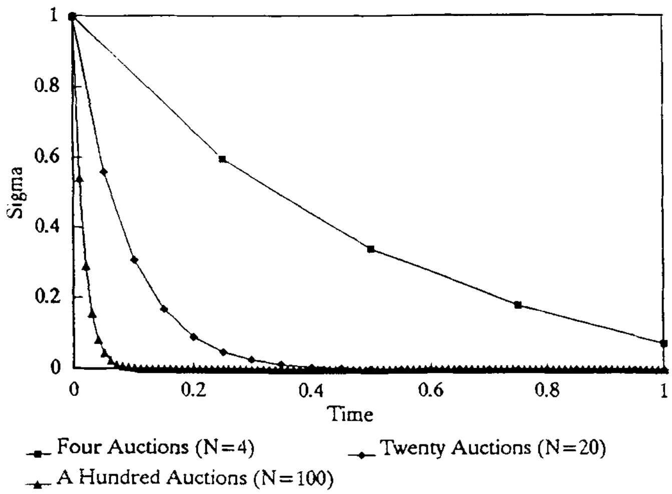

Fig. 5. Price efficiency parameter Σ_n over time for different values of N, the number of auctions, by risk averse competitors ($M = 2$, $A = 4$).

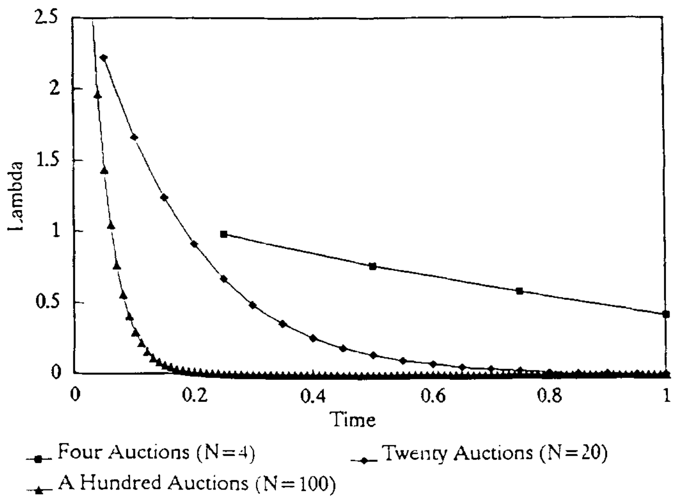

Fig. 6. Market liquidity parameter λ_n over time for different values of N, the number of auctions, by risk averse competitors ($M = 2$, $A = 4$).

to zero increasingly rapidly as N increases. Figures 5 and 6 illustrate the same phenomenon described above for the case of multiple privately informed agents. Note, however, that the phenomenon arises solely due to risk aversion in Figs. 3 and 4, but arises due to *both* risk aversion and competition in Figs. 5 and 6.

4. Concluding remarks

In this paper we have characterized the dynamically optimal trading strategies of risk-averse informed traders with a long-lived informational advantage. It should be noted that, for reasons of tractability, we have imposed a considerable amount of structure in our model. For example, we assumed that all informed traders possessed homogeneous information. This assumption may not be unreasonable for the case of corporate insiders, whose information would be likely to be highly correlated. The assumption, however, may be less appropriate for the case of security analysts, who are likely to possess more diverse signals. Modeling risk aversion in a dynamic framework is a difficult problem, however, and our model serves as a first attempt to address this issue.

From a regulatory perspective, our paper indicates that insider trading may be much less of a potential problem than the analysis of Kyle (1985) indicates. In the Kyle (1985) paper, information is incorporated into prices at a slow, almost linear, rate. Our analysis shows that under risk aversion, information gets disseminated to the market rapidly in calendar time, even in the case of the monopolistic insider.

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