# The Economics of Super Managers

**Nina Baranchuk**
School of Management, University of Texas

**Glenn MacDonald**
Olin Business School, Washington University in Saint Louis

**Jun Yang**
Kelley School of Business, Indiana University



We study a competitive model in which managers differ in ability and choose unobservable effort. Each firm chooses its size, how able a manager is to hire, and managerial compensation. The model can be considered an amalgam of agency and Superstars, where optimizing incentives enhances the firm's ability to provide a talented manager with greater resources. The model delivers many testable implications. Preliminary results show that the model is useful for understanding interesting compensation trends, for example, why CEO pay has recently become more closely associated with firm size. Allowing for firm productivity differences generally strengthens our results. (*JEL* G30, J21, J31, M12)

Recent growth in CEO compensation, especially the astronomical pay of top CEOs (e.g., in 2006 Steve Jobs realized nearly $650 million from vested restricted stock[1]), has led many to question whether CEOs have too much influence over their own compensation. The low pay for performance sensitivity in large firms (i.e., the inverse relation between CEO incentive compensation and firm size) has been used as evidence of CEO entrenchment (Bebchuk and Fried 2003, 2004). More important, empirically, the degree to which variation in size can explain variation in CEO compensation has increased (Frydman and Saks 2010), challenging existing theories of CEO compensation (e.g., Gabaix and Landier 2008), in which firm size is an exogenously given determinant of compensation.

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Send correspondence to Glenn MacDonald, Olin Business School, Washington University in Saint Louis, Campus Box 1133, One Brookings Drive, Saint Louis, MO 63130-4899; telephone: (314)-935-7768. E-mail: macdonald@wustl.edu.

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To improve our understanding of these observations, we study a Superstars model in which each firm faces an agency problem. Specifically, the model includes optimal incentives for CEOs (due to the agency problem), a competitive output market, and a competitive labor market with a limited supply of managerial talent (as in the Superstars model). Firm size and CEO compensation are equilibrium phenomena, and the distribution of salary and the incentive components of CEO compensation across firms is the consequence of firms choosing incentives that are tailored to CEO skills and activities. The equilibrium generates an extremely nonlinear cross-sectional relation between firm size and CEO incentives. Despite the many restrictions that equilibrium imposes, the model easily delivers a cross-sectional relation similar to the observed one (see Section 4). The endogeneity of firm size and managerial incentives allows us to investigate how changes in the economic environment (e.g., an increase in product demand or an inflow of workers as globalization progresses in recent years) alter the composition of CEO compensation, increase the level and dispersion of firm size and CEO compensation, and increase the elasticity of CEO compensation with respect to firm size.

The building blocks of the basic model are not unusual. Indeed, much of the originality in the article, as well as the wide variety of new results, stems from the novel combination of these standard features. Firms operate in a competitive product market with free entry. There is a limited supply of managerial talent, where talent (also referred to as *ability*) is observable and varies from agent to agent. Each firm employs just one manager, but its output is the aggregate of a number of "projects," which might be interpreted as plants or divisions, or components of a product line. Each project has managerial talent and a share of managerial effort as inputs, and is also subject to a firm-specific random shock. Managerial ability and effort influence firm output more when the firm is larger (i.e., has more projects). Firms differ in their choices of managers, and compensate their managers with salary and a proportional share of profits (we refer to the proportion as "incentive pay").

In equilibrium, more talented managers exert greater effort and manage more projects since they are employed by firms that, recognizing the complementarity between talent, effort, and projects, choose more projects. Higher-ability managers, in equilibrium, grow firms much larger and receive much greater pay. One reason for this to occur is that managing more projects means exerting greater total effort. Another is that having more projects causes firm profits to be more volatile, which, since managers have profit-based incentives, increases the risk managers bear. Altogether, to compensate for greater effort and more risk-bearing, higher-ability managers receive greater salaries and more total incentive compensation, albeit via smaller profit shares. We show that moral hazard magnifies the convexity of firm size and compensation familiar from the Superstars literature.

Endogeneity of firm size (in contrast to, for example, Gabaix and Landier 2008; Tervio 2008; Edmans, Gabaix, and Landier 2009) is important for three reasons. First, as a practical matter, firm size is not exogenous, and changes in firm size are routinely accompanied and facilitated by changes in managerial talent and responsibilities. New CEOs often undertake asset sales and restructure their labor force, reshaping firms to suit their strengths. For example, Carlos Brito's considerable international experience working for Daimler Benz, Shell Oil, and the Brazilian soft drink company Brahma enabled InBev to pursue its ambitious global plans, including acquiring Anheuser-Busch. In the mutual fund industry, it is the manager's talent that determines the size of assets under management, not vice versa; changes in managers are normally followed by substantial changes in fund size (e.g., Berk and Green 2004).

Second, when firm size is treated as exogenous, the predicted impact of changing firm size on, for example, CEO pay, assumes that fundamentals, such as product demand, technology, and factor prices, are all constant. Thus, the predicted effect must be interpreted as the response of CEO pay to some unspecified or unobserved change in the economic environment that causes firm size to vary despite all fundamentals remaining unchanged. In our model, predictions about firm size and CEO compensation follow from changes in the economic environment that lead to corresponding adjustments in the behavior of consumers, firms, CEOs, workers, and other input suppliers, all of which are interpretable and conceivably measurable. Specifically, we study how the distributions of firm size, along with fixed and incentive-based components of CEO compensation, change in response to shocks to product demand, managerial risk aversion, cost of effort, and supply of workers and managerial talent.[2] Consider, for example, an increase in demand for the industry's product. If labor supply is elastic, so that workers' wages are not much affected by the increase in product demand, then, in response to increasing demand, firms expand, with large firms growing more and new firms entering the industry. Since greater effort complements increasing firm size, managers receive greater incentive-based compensation and total compensation for working harder and bearing more risk, albeit via lower profit shares. Moreover, all components of compensation are convex in ability, and become even more convex when demand increases, tending to augment the cross-sectional dispersion of compensation.

Third, while we model size and CEO incentives as *simultaneous* firm choices, it is formally equivalent if the firm first selects incentives, then determines how large to be (similar to the mutual fund example), or the firm first selects size or capacity, then determines the structure of CEO incentives (reminiscent of Anheuser-Busch Inbev), provided the earlier choices are made bearing in mind their consequences for later choices. Our assumption that size is endogenous is not an assumption about timing of choices. Instead, our intent

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[2] As an example, according to our model, the increased importance of incentive-based compensation, documented in Frydman and Saks (2010), is consistent with a positive demand shock, improved technology, and enhanced managerial hedging.

is to include the familiar Coasian idea that managerial talent and firm size are intimately connected.

Our base model assumes free entry of homogeneous firms. We show that our results are enhanced when firms differ ex ante in, for example, productivity. Intuitively, more capable managers are matched with firms that have higher productivity, exert greater managerial effort, and grow their firms much larger. As a result, convexity of firm size and all components of CEO compensation also increase. Allowing for differences in productivity across firms also results in heterogeneity in firm profits, which enables us to generate new predictions on the responses of the level, slope, and convexity of firm profits to various changes in fundamentals. This extension can also be employed empirically to disentangle the impacts of CEO talent and firm heterogeneity on CEO compensation (e.g., Tervio 2008).

Testing our model's implications is beyond the scope of this article. However, there is encouraging evidence that our model may prove useful empirically. First, Frydman and Saks (2010) show that CEO pay has long (i.e., their dataset begins in 1936) shown nontrivial sensitivity to firm performance, and that CEO pay has recently become more closely correlated with firm size. While there may be various reasons why CEO pay and firm size might become more closely associated, we show that if one assumes growing product demand (due to, for example, economic growth or globalization), our model implies that CEO pay and firm size will become more closely connected, specifically in the sense of an increasing elasticity of pay with respect to size. To explore this idea, we analyze the Fama-French twelve industries and find that the six industries in which pay and size have become more closely associated are also those for which sales are growing more quickly. Thus, our explanation for the increased pay-size association is that in more rapidly growing industries, firms respond by complementing CEO ability with size and optimally chosen pay, which appears in the data as growing elasticity of CEO pay with respect to firm size.

Moreover, using the ExecuComp data on the retail industry during 2003, we examine the relations between CEO pay-for-performance sensitivity (a proxy for profit share) and *each of* firm size, CEO salary and bonus, and CEO total compensation measures.[3] These empirical relations show extreme behavior, which is similar to the functional forms generated by our model. Finally, also for retail, the model produces moments of firm size and CEO compensation that are comparable to those in the data.

The balance of the article is organized as follows. The next section reviews related literature. Section 3 presents the model and its equilibrium. Section 4 provides empirical predictions based on comparative static propositions. Section 5 describes our data analysis and empirical results, and Section 6 concludes. Appendix A discusses the role of moral hazard in our model.

[3] The idea of restricting the analysis this way is that the model assumes a single, competitive, homogenous product industry. Omitting Walmart, as we do, retail reasonably meets the assumptions.

Appendix B contains proofs and derives the elasticity of CEO pay with respect to firm size when capital inputs are required, which is necessary for the empirical analysis. Appendix C includes figures and tables.

## 1. Related Literature

The growing theoretical literature on the level and dispersion of managerial compensation has its roots in seminal papers by Lucas (1978) and Rosen (1981), who investigate the implications of managerial talent for the size distribution of firms. In equilibrium, compensation for the most talented managers is much more than their less talented peers because the marginal product at the large firms they choose to manage is so high. Interest in this idea has been revitalized recently, yielding numerous papers investigating various refinements of the basic problem. For example, with economies of scale and heterogeneous agents, Antras, Garicano, and Rossi-Hansberg (2006) show that in equilibrium, more skilled agents specialize in problem solving (as managers) while less skilled agents specialize in production. This hierarchical organization and recent advances in communication technology lead to greater cross-sectional differences and increases in pay. All these papers omit managerial effort and so do not analyze the composition of managerial compensation (i.e., salaries vs. incentive-based pay), which is the focal point of the current debate on managerial compensation. The moral hazard problem included in our model magnifies the impact of managerial ability, increasing skewness of the distributions of firm size, and managerial compensation. The complementarity between effort and ability implies that more able managers exert higher effort, which, in turn, translates into much higher salaries, total incentive compensation, and total compensation.

Gabaix and Landier (2008) emphasize the size-talent complementarity—managerial talent is matched to an exogenous firm size distribution. In equilibrium, CEO pay increases one-for-one with firm size, consistent with the roughly sixfold increase in both CEO pay and market capitalization of large corporations in the United States between 1980 and 2003. Tervio (2008) obtains similar qualitative results under more general assumptions about the distribution of managerial talent. Edmans, Gabaix, and Landier (2009) extend Gabaix and Landier's work to include effort and incentives, again given a fixed distribution of firms. High effort is assumed to be optimal for all managers regardless of their ability. Finally, Gayle and Miller (2009) stress the importance of incentive pay, and argue that moral hazard in large and complex firms is the main force driving the recent trends in CEO compensation. These four papers leave open the questions of where the distribution of firm size comes from, and why it evolves as it does, including firm entry and exit. Thus, they cannot make predictions about how changes in the economic environment impact the distribution of firm size along with CEO pay and effort. Our article contributes to this stream of literature by endogenizing firm size, which allows us to make separate predictions about how firm size and CEO compensation respond to variation in economic fundamentals, such as the supply of talent and demand for products.

Other papers emphasize the effect of market for managers on executive compensation. Murphy and Zabojnik (2004a,b) examine how intensified competition for talent (driven by the growing importance of general managerial ability in comparison to firm-specific human capital) explains the observed increase in CEO compensation, especially for the highest-ability CEOs. Murphy and Zabojnik's model, similar to ours, assumes free entry of firms and a competitive output market; their empirical implications are qualitative, and they do not study incentive compensation. Himmelberg and Hubbard (2000) and Oyer (2004) link CEO compensation and firm valuation to economic shocks, stressing the role of limited supply of managerial talent. Their models, like ours, predict that positive shocks lead firms to expand, and talented CEOs to be paid substantially more than their less talented peers. But our explicit solutions enable us to derive many more predictions, such as the impact of demand shocks or greater technological uncertainty on the number of firms, firm size and profits, overall executive compensation, and incentive pay.

The just-described literature assumes competitive markets. Aggarwal and Samwick (1999) and Raith (2003) study the effect of product market competition on managerial incentives and executive compensation. Neither paper derives implications for the distribution of managerial pay or talent. Falato and Kadyrzhanova (2007) focus on the interaction between industry structures and CEO compensation. They do not model the effect of managerial talent, nor do they provide closed-form solutions.

In a related and primarily empirical literature, Milbourn (2003) and Baker and Hall (2004) assume exogenous firm size, and argue that much of the cross-sectional variation in incentive pay can be attributed to differences in CEO productivity. Kaplan and Rauh (2009) document that the growth in CEO pay in non-financial firms from 1994 to 2004 is similar to growth in pay for similarly talented Wall Street executives, corporate lawyers, professional athletes, and celebrities. They argue that the evidence is most consistent with theories of Superstars, "skill-biased" technological change, economies of scale, and their interaction. Without clear theoretical guidance, observationally equivalent evidence can be interpreted as either CEO entrenchment or the functioning of the managerial labor market. For example, Bebchuk and Grinstein (2006) document that the expansion of firm size is associated with increases in subsequent CEO compensation and argue that the evidence is consistent with a theory of CEO entrenchment. Actually, this evidence is very consistent with our model, in which firm size is determined in equilibrium, and highly talented managers both run large firms and get paid substantially more. The precise nature of the impact of talent will vary from industry to industry, but our model suggests that the same fundamental economic forces will generally operate.

## 2. Model

The simplest version of our model generalizes a perfectly competitive product market with free entry. Demand is conventional, but the supply side comprises firms that choose managers from a heterogeneous talent pool. Managerial effort is assumed unobservable, so each firm chooses incentives to influence the manager's effort choice. The cooperating resources available to the manager are also a choice, and determine the size of the firm and affect the manager's incentives and effort. Thus, while firms have identical productivity, their choices result in heterogeneity in, for example, output. Later (in Section 4.2), we allow firms to have different productivity. The formal specification follows.

### 2.1 Firms and agents

There are two kinds of players, *firms* and *agents*, and a fixed continuum of each. The firms are all identical, but the agents are heterogeneous. Specifically, each agent has a fixed and known ability level, $a \in [0, \infty)$; heterogeneity in ability is described by the atomless measure $\mu$.

Each firm may produce output in a competitive market where the unit price (given, as far as the firm is concerned) is $p$. Nonparticipation yields payoff normalized to zero. The firm hires a single manager—effectively, the definition of a firm is the entity that one manager can oversee—and chooses how many (formally, a continuum) identical projects to operate. Running these projects requires the manager to exert effort. Specifically, if the firm hires a manager of ability $a$, and operates $n$ projects, *each* of which receives effort $e$ from its manager, total output is given by

$$n(\sqrt{ae} + \varepsilon), \tag{1}$$

where $\varepsilon \sim N(0, \sigma^2)$ is a firm-specific random shock common to all projects within a firm, but differing across firms. The key feature of (1) is that $n$, $a$, and $e$ are complements. One can interpret $n$ either as "scale," i.e., each project is literally identical and produces the same output, or as "the number of projects," in which case projects involve different activities, but a symmetry assumption is made to simplify the analysis. In subsequent discussions, we also refer to $n$ as *firm size*. Empirically, there is no obvious counterpart of $n$; below, we examine various measures, such as employment and assets.

We assume that the manager is compensated by salary $s_0$ and profit share $s_1$.[4] Each project requires one worker, who earns a wage of $w$. Then, for any $a, n, e, s_0$, and $s_1$, assuming the firm is risk neutral, its expected payoff is

[4] The contract we study is the optimal affine contract. As Mirrlees (1974) showed, there is generally a "forcing contract" that performs better. However, the forcing contract yields grossly counterfactual predictions, and so something in the model must change if it is to be empirically relevant. The modification we choose is to restrict the space of contracts to affine. Alternatively, we can reinterpret the model (i.e., the mathematics are identical), assuming that (*i*) effort is exerted continuously over time; (*ii*) at each instant, the manager observes output; and (*iii*) the manager can adjust effort at each point in time. Under these assumptions, Holmström and Milgrom's (1987) theorem 7 applies, so the optimal contract is affine.

$$(1 - s_1)n(p\sqrt{ae} - w) - s_0,$$

i.e., the firm's share of expected total operating profits from projects, less the manager's salary.

Each agent can elect to be either a worker or a manager, and ability matters only for managing. For simplicity, a worker's effort is normalized to zero. Thus, a manager of ability $a$, employed by a firm with $n$ projects, compensated by salary $s_0$ and profit share $s_1$, earns income net of effort costs equal to

$$s_0 + s_1 n\left[p(\sqrt{ae} + \varepsilon) - w\right] - \frac{1}{2}\xi ne,$$

where $\xi$ parameterizes the unit cost of managerial effort. Observe that, given managerial ability and effort, a greater share of profits, more projects, and a higher product price all expose the manager to more income risk.

We assume agents' utility has constant absolute risk aversion, with parameter $\gamma, \gamma > 0$. Given the normality of the shocks to firm output, a manager of ability $a$ has expected utility

$$-\exp\left\{-\gamma\left[s_0 + s_1 n(p\sqrt{ae} - w) - \frac{1}{2}\xi ne - \frac{1}{2}\gamma s_1^2 n^2 p^2 \sigma^2\right]\right\},$$

in which case the agent's choice of whether to manage or work hinges on a comparison of $w$ with the *certainty equivalent*:

$$s_0 + s_1 n\left(p\sqrt{ae} - w\right) - \frac{1}{2}\xi ne - \frac{1}{2}\gamma s_1^2 n^2 p^2 \sigma^2. \tag{2}$$

The information and timing assumptions are as follows. Each agent's ability, $a$, is common knowledge. Firms decide whether to operate. Each operating firm chooses, supposing its manager is of ability $a$, the number of projects it will operate, $n(a)$, and what salary, $s_0(a)$, and profit share, $s_1(a)$, it will offer. Given a salary of $s_0(a)$, a profit share of $s_1(a)$, and the number of projects $n(a)$, an agent of ability $a$ decides whether to be a worker or manager, including, if the latter is chosen, how much effort *per* project to expend; the manager's effort choice is his/her private information. Stated this way, the firm assumes a given manager's talent, then chooses how many projects to operate and the parameters of the manager's compensation. Exactly which of the firms ultimately employs which manager is determined in equilibrium; the homogeneous firm case is easiest to analyze because, in equilibrium, firms are indifferent about ability. Some readers, especially when firms are allowed to be heterogeneous, find it more intuitive to cast the problem as follows: Each firm chooses the number of projects to operate, then how able a manager to hire, then how to compensate the manager. Formally, this setup is identical to the one we analyze, provided the firm is aware that its choice of projects has consequences for its ability decision, and that ability and the number of projects affect the compensation choice it will make.

### 2.2 Equilibrium

The model in which the workers' wage, $w$, is exogenous is most straightforward, so we concentrate on that model and discuss the endogenous wage version later. Given $w$, the definition of equilibrium follows. In the definition, $M$ is the set of agents choosing to be managers, $c^*(a)$ is the equilibrium certainty equivalent for an agent of ability $a$ if the agent elects to be a manager, and consumer behavior is described by a linear demand function, $\alpha - \beta p$, where $\alpha > 0$ and $\beta > 0$.

**Definition 1.** An *exogenous wage equilibrium* is a set $M \subset [0, \infty)$, a price $p^*$, and functions $c^*(a)$, $e^*(a, s_1)$, $s_0^*(a)$, $s_1^*(a)$, and $n^*(a)$, satisfying:

1. *Agents optimize managerial effort*: for any $a$ and $s_1$,

$$e^*(a, s_1) = \arg\max_{e \geq 0} \left\{ s_1 p^* \sqrt{ae} - \frac{1}{2}\xi e \right\}; \tag{3}$$

2. *Agents optimize whether to work or manage*:

$$M = \{a \in [0, \infty) \mid w \leq c^*(a)\};$$

3. *Firms optimize incentives and number of projects*: for all $a$,

$$(s_0^*(a), s_1^*(a), n^*(a)) = \arg\max_{s_0, s_1, n} \left\{ (1 - s_1) n \left[ p^* \sqrt{ae^*(a, s_1)} - w \right] - s_0 \right\} \tag{4}$$

subject to

$$c^*(a) \leq s_0 + s_1 n \left[ p^* \sqrt{ae^*(a, s_1)} - w \right] - \frac{1}{2}\xi n e^*(a, s_1) - \frac{1}{2}\gamma s_1^2 n^2 p^{*2} \sigma^2;$$

4. *All surplus goes to agents*: for all $a$,

$$c^*(a) = n^*(a) \left[ p^* \sqrt{ae^*(a, s_1^*(a))} - w \right] - \frac{1}{2}\xi n^*(a) e^*(a, s_1^*(a))) - \frac{1}{2}\gamma s_1^{*2}(a) n^{*2}(a) p^{*2} \sigma^2;$$

and

5. *The product market clears*: $p^*$ satisfies

$$\alpha - \beta p^* = \int_M n^*(a) \sqrt{ae^*(a, s_1^*(a))} \mu(da).$$

Several aspects of Definition 1 require comment. First, each firm's output is stochastic, whereas industry output and equilibrium price are not.[5] Next, in #1, a manager choosing effort *per project* to maximize (2) will make a choice that is independent of both salary and number of projects. Thus, the manager's optimal effort choice, $e^*$, as defined by (3), has just $a$ and $s_1$ as arguments. In #3, a firm can consider any salary, profit share, and number of projects. But, in equilibrium, it must offer a combination that will attract a manager. Since $c^*(a)$ is required, in #4, to be the equilibrium certainty equivalent for managers, #3 requires a firm's choice to be at least as attractive as $c^*(a)$. Finally, because firms earn zero profits, in equilibrium, managers receive, via a combination of salary and profit share, all profits from operations. Hence, a manager's equilibrium certainty equivalent is comprised of firm profits from operations, less the costs of effort and a risk premium. In our model, the most capable manager does not run an infinitely large firm due to the combination of profit sharing and risk aversion. More specifically, the risk premium term in the certainty equivalent given in #4 is multiplicative in the number of projects, risk aversion, and output uncertainty. This combination limits the number of projects run by the most talented manager.

The simplicity of our model allows us, *for given* $p^*$, to calculate closed-form solutions for all of $c^*(a), e^*(a, s_1^*(a)), s_0^*(a), s_1^*(a)$, and $n^*(a)$. With these expressions in hand, we can show that the model has a unique equilibrium;[6] specific assumptions about $\mu$ (e.g., Lebesgue measure) also allow a closed-form solution for $p^*$. The solution process follows.

Solving the first-order condition for the optimization in (3) gives

$$e^*(a, s_1) = a\left(\frac{s_1 p^*}{\xi}\right)^2. \tag{5}$$

Next, since the firm gains nothing by offering the manager a salary allowing more utility than the agent's best alternative, the constraint in #3 will be binding. Thus,

$$s_0 = c^*(a) - s_1 n\left[p^*\sqrt{ae^*(a, s_1)} - w\right] + \frac{1}{2}\xi n e^*(a, s_1) + \frac{1}{2}\gamma\,(s_1 n p^*)^2\sigma^2, \tag{6}$$

which can be substituted into (4) to yield firm profits:

5 Early applications of this "deterministic aggregate" setup include Lucas (1980), Diamond and Dybvig (1983), and Prescott and Townsend (1984). For an especially clear exposition of the various approaches, see Feldman and Gilles (1985).

6 Existence of an equilibrium in which firms actually operate in the industry requires some assumptions on parameters. For example, if the wage rate, $w$, is too high, the industry will be unable to attract managers at all, even at a price of $\alpha/\beta$ (the highest price any consumer is willing to pay). A similar comment applies if the industry is too risky or if agents are too risk averse. We can handle all the possibilities with a single parametric assumption: The support of $\mu$ is an interval, and there is $\hat{a}$ in the support such that

$$1 < \frac{\hat{a}}{\xi\gamma\sigma^2}\left(\sqrt{\frac{\hat{a}(\alpha/\beta)^2}{2\xi w}} - 1\right)^2.$$

$$\pi(a) \equiv n(p^* \sqrt{ae^*(a,s_1)} - w) - \frac{1}{2}\xi n e^*(a,s_1) - \frac{1}{2}\gamma (s_1 n p^*)^2 \sigma^2 - c^*(a). \quad (7)$$

Firm profits are total profits from operations, less compensation paid to the manager for taking effort and bearing risk, less other payments to the manager reflecting the market value of ability. Using (5), (7) can be written as

$$\pi(a) = \left(\frac{nas_1 p^{*2}}{\xi} - nw\right) - \frac{nas_1^2 p^{*2}}{2\xi} - \frac{1}{2}\gamma (s_1 n p^*)^2 \sigma^2 - c^*(a),$$

in which case the first-order conditions characterizing (i.e., second-order conditions are satisfied) profit maximizing $n$ and $s_1$ are

$$\frac{\partial \pi(a)}{\partial n} = \frac{as_1 p^{*2}}{\xi} - w - \frac{as_1^2 p^{*2}}{2\xi} - \gamma s_1^2 n p^{*2} \sigma^2 = 0 \quad (8)$$

and

$$\frac{\partial \pi(a)}{\partial s_1} = \frac{nap^{*2}}{\xi} - \frac{nas_1 p^{*2}}{\xi} - \gamma \sigma^2 s_1 n^2 p^{*2} = 0. \quad (9)$$

To interpret (8) and (9), note that expanding the number of projects increases profits from operations directly, but also causes the manager to incur greater total effort costs and to bear more risk, both of which must be compensated. Similarly, increasing $s_1$ expands profit from each project by inducing more effort, but causes the manager to incur greater total effort costs and to bear more risk, requiring greater compensation.

Solving the first-order conditions, given $a$, we obtain[7]

$$n^*(a) = \frac{a}{\xi \gamma \sigma^2}\left(\sqrt{\frac{ap^{*2}}{2\xi w}} - 1\right) \quad (10)$$

and

$$s_1^*(a) = \sqrt{\frac{2\xi w}{ap^{*2}}}. \quad (11)$$

Then, (5) and (11) yield

$$e^*(a, s_1^*(a)) = \frac{2w}{\xi}. \quad (12)$$

[7] $n^*(a) > 0$ is ensured by the choice of the ability of the marginal manager, $\bar{a}$ (defined below). Expression (6), combined with $n^*(a) > 0$, guarantees that, in equilibrium, $0 < s_1^*(a) < 1$.

Because effort per project is independent of $a$, we will simply denote it $e^*$. Next, equilibrium condition #4, with some algebra, gives[8]

$$c^*(a) = \frac{aw}{\xi\gamma\sigma^2}\left(\sqrt{\frac{ap^{*2}}{2\xi w}} - 1\right)^2. \tag{13}$$

Observe that $c^*(a)$ is strictly increasing in $a$. Thus, the set of managers is $M = [\bar{a}, \infty)$, where $\bar{a}$, the ability of the marginal manager, is defined by

$$c^*(\bar{a}) = w. \tag{14}$$

Finally, (6) yields

$$s_0^*(a) = \frac{aw}{\xi\gamma\sigma^2}\left(\sqrt{\frac{ap^{*2}}{2\xi w}} - 1\right)^2\left(2 - \sqrt{\frac{2\xi w}{ap^{*2}}}\right). \tag{15}$$

Given (10)–(15), Theorem 1 (in Appendix A) shows that equilibrium exists and is unique.

Since understanding the structure of executive pay is a key objective, it is helpful to define some other objects that correspond more directly to observed variables. Specifically, *expected total incentive compensation* is given by, for $a \in M$,

$$I^*(a) \equiv s_1^*(a)n^*(a)(p^*\sqrt{ae^*} - w),$$

and *expected total compensation* is

$$T^*(a) \equiv s_0^*(a) + I^*(a).$$

Employing (10)–(12) and (15), we obtain

$$I^*(a) = \frac{aw}{\xi\gamma\sigma^2}\left(1 - \sqrt{\frac{2\xi w}{ap^{*2}}}\right)\left(\sqrt{\frac{2ap^{*2}}{\xi w}} - 1\right) \tag{16}$$

8 Substituting the equilibrium values $n^*(a)$, $s_1^*(a)$, and $e^*(a)$ into #4 gives

$$\begin{aligned} c^*(a) &= \frac{a}{\xi\gamma\sigma^2}\left(\sqrt{\frac{ap^{*2}}{2\xi w}} - 1\right)\left(p^*\sqrt{a\frac{2w}{\xi}} - w - \frac{1}{2}\xi\left(\frac{2w}{\xi}\right) - \frac{1}{2}\gamma\left(\frac{2\xi w}{ap^{*2}}\right)\left(\frac{a}{\xi\gamma\sigma^2}\left(\sqrt{\frac{ap^{*2}}{2\xi w}} - 1\right)\right)p^{*2}\sigma^2\right) \\ &= \frac{a}{\xi\gamma\sigma^2}\left(\sqrt{\frac{ap^{*2}}{2\xi w}} - 1\right)\left(p^*\sqrt{\frac{2aw}{\xi}} - w - w - w\left(\sqrt{\frac{ap^{*2}}{2\xi w}} - 1\right)\right) \\ &= \frac{a}{\xi\gamma\sigma^2}\left(\sqrt{\frac{ap^{*2}}{2\xi w}} - 1\right)\left(p^*\sqrt{\frac{2aw}{\xi}} - w - w\sqrt{\frac{ap^{*2}}{2\xi w}}\right) = \frac{aw}{\xi\gamma\sigma^2}\left(\sqrt{\frac{ap^{*2}}{2\xi w}} - 1\right)^2. \end{aligned}$$

and

$$T^*(a) = \frac{aw}{\xi \gamma \sigma^2} \left( \sqrt{\frac{ap^{*2}}{2\xi w}} - 1 \right) \left( \sqrt{\frac{2ap^{*2}}{\xi w}} - 1 \right). \tag{17}$$

It also proves useful to have some notation for *total* managerial effort:

$$E^*(a) \equiv e^* n^*(a). \tag{18}$$

## 3. Predictions

We model a competitive market in which the supply side is comprised of firms that tailor both the scale of their operations and managerial incentives to the ability of their managers. There are two kinds of implications. First, the closed-form expressions derived in the previous section generate predictions for cross-sectional distributions of firm size, managerial incentives, salary, and total compensation. (Naturally, "aggregate variables," for example, the product price and the ability of the marginal manager, are common to all firms and fixed as we look across firms.) These predictions are empirically relevant for the following reasons. If ability is measurable, for example, as illustrated in Kaplan, Klebanov, and Sorensen (2010), the results may be tested directly. If ability is not measurable, one could impose some structure on the distribution of ability to obtain testable implications; we explore this approach in Section 5.3. Finally, ability induces observable correlation in, for example, firm size and profit share. As we discuss in Section 5.2, the equilibrium expressions place restrictions on this induced correlation, which can be tested without relying on either proxies for ability or assumptions on its distribution.[9]

Part of Proposition 1 confirms that our model has some features in common with the Superstars literature. Specifically, firm size and total managerial compensation are increasing and convex in ability. This is unsurprising because our model is effectively a Superstars model combined with agency. More interestingly, as we discuss in more detail below and in Appendix A, moral hazard does not temper the Superstars effect—in fact, the implied optimal contracting

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[9] Consider, for example, the equilibrium number of projects, $n^*(a)$, and profit share, $s_1^*(a)$. For any $s_1$, let the ability of the manager whose equilibrium profit share is $s_1$ be $a(s_1) \equiv s_1^{*-1}(s_1)$; $a(s_1)$ is a decreasing and convex function of $s_1$, (as follows from Proposition 1 below), i.e., $a'(s_1) < 0$ and $a''(s_1) > 0$. Then, define the number of projects at a firm whose manager's profit share is $s_1$ by $n(s_1) \equiv n^*(a(s_1))$. It follows that

$$n'(s_1) = n^{*\prime}(a)a'(s_1) < 0$$

and

$$n''(s_1) = n^{*\prime\prime}(a)(a'(s_1))^2 + n^{*\prime}(a)a''(s_1) > 0,$$

i.e., $n(s_1)$ is a decreasing and convex function of $s_1$ (the inequalities follow from positivity of first- and second-order derivatives of $n^*(a)$ shown in Proposition 1). As we show in Section 4.2, our specific functional forms generate an $n(s_1)$ whose extreme shape is suggestive of the empirical relation between firm size and managerial profit share.

problem enhances it by increasing convexity. Moreover, the monotonicity and convexity results are also applicable to each element of compensation (i.e., salary, incentive compensation, and profit share).

The second collection of results (Propositions 2–6) describes how equilibrium variables are influenced by changes in the model's fundamentals—demand parameters, the workers' wage (in the exogenous wage model), managers' risk aversion and effort cost, production variability, and the ability measure. Changes in parameters not only directly affect the just-mentioned cross-sectional relationships between output, incentives, effort, and ability, but also alter aggregates (e.g., the price of output), which has further implications for the cross-sectional relationships. Why are these results relevant? Our model is not dynamic. Nevertheless, comparative statics results are routinely applied to explore differences across markets or countries at a point in time, as well as to explain variations over time.[10] In our basic model, the workers' wage is exogenous, and firms differ only because they employ managers of different abilities. In Sections 4.1 and 4.2, we study how endogeneity of the workers' wage and differences in firm-specific characteristics affect and expand the two sets of results.

Proposition 1 summarizes the qualitative cross-sectional properties of equilibrium entities.

**Proposition 1.** Higher-ability managers:

1. Are employed by firms choosing more projects;
2. Exert more total effort;
3. Earn higher salaries and more total incentive compensation, although via a smaller share of profits; and
4. Have greater utility (i.e., a greater certainty equivalent).

   Formally, $n^*(a), s_0^*(a), I^*(a), T^*(a), s_1^*(a), E^*(a),$ and $c^*(a)$ are monotone and strictly convex functions of $a$; all but $s_1^*(a)$ are increasing in $a$.

The endogeneity of both firm size (number of projects) and incentive compensation is central for the results in Proposition 1. From (9), for a fixed number of projects, a firm contemplating a manager of higher ability would optimally offer that manager a larger profit share. However, when the number of projects is a matter of choice, a higher-ability manager increases the left-hand side of both (8) and (9), suggesting that firms hiring more able managers might choose to be larger and to provide managers with stronger incentives via larger profit shares (i.e., the number of projects and *manager's* profit share

[10] See, for example, Gabaix and Landier (2008), who argue that exogenous change in firm size explains the recent empirical trends in CEO compensation.

are complements). An offset to this complementarity is that more projects also expand the risk facing the manager. In equilibrium,[11]

$$\frac{\partial^2 \pi(a)}{\partial n \partial s_1} < 0,$$

implying that number of projects and *firm*'s profit share are complements. Thus, as Proposition 1 states, equilibrium number of projects and the manager's profit share vary in *opposite* directions in the cross-section. This negative equilibrium relation provides a non-entrenchment explanation for the low pay-for-performance-sensitivity for CEOs in large firms.

Ability and effort per project are also complements in (1), creating incentive for a firm whose manager is more able to choose greater effort per project. However, this incentive is partially offset by the fact that more projects, together with more effort per project, would raise the total level of managerial effort dramatically. Taking both effects into account, a firm whose manager is of greater ability induces greater total effort as a result of more projects but, in this specific model, the same effort per project as any other firm-manager pair.

Further, a firm whose manager is more able offers the manager more total (salary plus incentive-based) compensation. There are three reasons. First, a more able manager generates more output per project than a less able manager even if the firm makes exactly the same choice of projects and incentives. Second, a firm employing a more able manager chooses a larger number of projects, which further increases firm output and profits. In our competitive model, competition among firms delivers any such extra profits to the firm's manager. Third, although the firm having a more able manager employs a smaller profit share to offset the greater risk implied by more projects, the offset is incomplete. In equilibrium, the firm optimally provides a more able manager with incentives to increase total effort. Thus, the firm must compensate the manager for both more risk-bearing and more effort. Altogether, total pay increases via a higher salary and greater total incentive compensation, albeit by a smaller profit share. Finally, despite bearing more risk and working harder, the higher-ability manager's greater salary and total incentive pay more than compensate, implying greater utility.

Proposition 1 states that number of projects, components of compensation, and total managerial effort are monotonic functions of ability; they are also convex in ability. To highlight the intuition, focus on how the number of projects

11 That is,

$$\frac{\partial^2 \pi(a)}{\partial n \partial s_1} = \frac{a p^{*2}}{\xi} - \frac{a s_1 p^{*2}}{\xi} - 2\gamma s_1 n p^{*2} \sigma^2.$$

Using $\frac{\partial \pi(a)}{\partial n} = 0$ to solve for $2\gamma s_1 n p^{*2} \sigma^2$, and simplifying, yields

$$\frac{\partial^2 \pi(a)}{\partial n \partial s_1} = -\frac{2}{s_1}\left(\frac{a s_1 p^{*2}}{2\xi} - w\right).$$

Substituting for $s_1^*(a)$ from (11) and using $s_1^*(a) < 1$ shows that the bracketed expression is positive.

varies across firms. Suppose first that the manager's profit share is fixed and independent of ability. In this case, a higher-ability manager increases the value of an incremental project, but the manager also works much harder and bears more risk. From (8), for fixed $s_1$, the optimal choice of the number of projects is linear in ability (thus, absent effort, our assumed production function produces no Superstars effect). But when the profit share can be adjusted, the firm whose manager is of higher ability can choose more projects, but can also employ a smaller profit share both to increase effort less and to impose less risk on the manager, thereby allowing the firm to increase the number of projects even more. This effect is stronger for a more able manager, because the overall risk the more able manager bears is much greater. Thus, the more able the manager, the more sensitive the firm's choice of number of projects to managerial ability (i.e., number of projects is a convex function of ability due to the equilibrium adjustment of incentives). Similar logic explains why salary, total effort, etc., are convex functions of ability.

Our second collection of results shows how the model's equilibrium responds to changes in parameters. The model has a variety of parameters: intercept ($\alpha$) and slope ($\beta$) of product demand; workers' wage ($w$); agents' risk aversion ($\gamma$) and unit cost of effort ($\xi$); variance of the firm-specific shock ($\sigma^2$); and ability measure ($\mu$). Propositions 2–6 summarize the results when $w$ is exogenous; Section 4.1 discusses the analogous results when $w$ is endogenous.

Changes in demand parameters $\alpha$ and/or $\beta$ can represent a variety of changes in the economic environment. For example, advertising, economic growth, or a change in customer demographics may cause demanders to purchase more of the product or service at any given price, which we represent by an increase in $\alpha$. On the other hand, substitute goods or services becoming more differentiated, or the introduction of new complementary products, reduces customer price sensitivity, which we would represent by a decline in $\beta$.

**Proposition 2.** If demand increases, either through an increase in $\alpha$ or a decline in $\beta$,

1. More agents become managers ($\bar{a}$ declines) and the product price increases ($p^*$ increases);
2. Managers of given ability, $a$, are employed by firms choosing more projects ($n^*(a)$ increases), earn higher salaries ($s_0^*(a)$ increases), have more total compensation and total incentive compensation ($T^*(a)$ and $I^*(a)$ increase) via a smaller share of profits ($s_1^*(a)$ declines), exert more total effort ($E^*(a)$ increases), and have greater certainty equivalent ($c^*(a)$ increases); and
3. The slope and convexity (i.e., first- and second-order derivatives with respect to $a$) of $n^*(a)$, $s_0^*(a)$, $T^*(a)$, $I^*(a)$, $E^*(a)$, and $c^*(a)$ all increase. The slope of $s_1^*(a)$ also increases (toward zero), but its convexity decreases.

The impact of increasing demand on firm choices is clear because the way demand increase affects firm optimization is purely via a higher price, $p^*$ (i.e., neither $\alpha$ nor $\beta$ enters the firm's problem directly).

When demand increases, the implied excess demand is met in part by entry of new firms employing less talented managers who otherwise would have chosen to be workers, and in part by an increase in product price. The higher product price motivates firms to choose more projects. This adjustment imposes greater risk on managers, for which firms compensate in part by reducing the managers' share of profits, albeit not to the degree that total incentive compensation falls. More projects also translates into more total effort, although effort per project does not change. To compensate for increases in both effort and risk, the managers' compensation must rise. Overall, managerial talent becomes scarcer, and managers enjoy greater utility. The increased slope and convexity of the variables of interest (except profit share) follow from the profit opportunities implied by a higher price. That is, as described in Proposition 1, a firm whose manager is more able is generally also more sensitive to variation in ability, and an increased price augments this sensitivity. Thus, for example, when demand increases, the differences in firm size and managerial compensation across firms increase, the largest firm grows much more than the median firm in the industry, and its manager receives much greater increases in salary, total incentive compensation, and total compensation in comparison to the manager of the median firm.

Proposition 2 also implies that equilibrium compensation responds to some things that are not under the managers' control (e.g., economic conditions). There are two reasons why this occurs. First, managers receive all gross profits of their firms in equilibrium due to firms competing for talent. As a result, managers enjoy some of the benefit from improved economic conditions, and also feel some of the pain of worsened conditions. Second, changing economic conditions imply changing returns to motivating the managers, and the optimal adjustment depends on ability. For example, high oil prices create a considerable profit opportunity for oil companies, and we expect to see incentives to act aggressively when these opportunities are present. Altogether, according to the model, there is no mystery in the observation that managerial compensation rewards—or, more accurately, "is a function of"—entities beyond the managers' control.

Consider next the impact of a change in ability measure, $\mu$, to another measure, $\hat{\mu}$. (Note that $\mu$ and $\hat{\mu}$ are measures, not distributions, and so need not integrate to one.) We are able to consider a wide variety of possibilities. For example, if managers all effectively become more able due to the discovery of superior but widely available management methods, $\hat{\mu}([0,\infty)) = \mu([0,\infty))$, and for all $a \in (0,\infty)$, $\hat{\mu}([a,\infty)) \geq \mu([a,\infty))$. Or, if heterogeneity in managerial talent grows, $\hat{\mu}$ can be constructed from $\mu$ via a series of "mean preserving spreads."

**Proposition 3.** Suppose the ability measure changes from $\mu$ to $\hat{\mu}$. If

$$\int_{\bar{a}_\mu}^{\infty} n_\mu^*(a)\sqrt{ae_\mu^*(a, s_{1,\mu}^*(a))}\,\mu(da) < \int_{\bar{a}_\mu}^{\infty} n_\mu^*(a)\sqrt{ae_\mu^*(a, s_{1,\mu}^*(a))}\,\hat{\mu}(da), \tag{19}$$

where $\bar{a}_\mu$, $n_\mu^*(a)$, $e_\mu^*(a, s_{1,\mu}^*(a))$, and $s_{1,\mu}^*(a)$ are the equilibrium values under measure $\mu$, then the implications of $\mu$ becoming $\hat{\mu}$ are qualitatively identical to a decline in demand via lower $\alpha$ or greater $\beta$ (the opposite of conclusions 1–3 of Proposition 2). If (19) is strictly reversed, then conclusions 1–3 of Proposition 2 hold. If (19) is an equality, then firms whose managers are of given ability make the same choices under both measures.

The intuition for the results in Proposition 3 is quite straightforward. Condition (19) stipulates that if firms having managers of any given ability did not adjust their choices when $\mu$ became $\hat{\mu}$, then industry output (i.e., the integral in (19)) would rise, and so the product price would fall. Thus, the impact on firms of $\mu$ becoming $\hat{\mu}$ is qualitatively the same as the industry facing a decline in demand, in which case the results in Proposition 2 apply immediately. To see how condition (19) might be satisfied, note that, using (12), condition (19) is equivalent to

$$\int_{\bar{a}_\mu}^{\infty} n_\mu^*(a)\sqrt{a}\,\mu(da) < \int_{\bar{a}_\mu}^{\infty} n_\mu^*(a)\sqrt{a}\,\hat{\mu}(da).$$

Using (10), it follows that $n_\mu^*(a)\sqrt{a}$ is an *increasing and strictly convex* function of $a$. Thus, both the above-mentioned examples of how $\mu$ might become $\hat{\mu}$ result in (19) being satisfied, and so, for example, a larger profit share for managers of given ability. In particular, an increase in the heterogeneity of managerial talent increases profit shares generally and augments their heterogeneity.

An increase in the workers' wage arises from general labor market conditions, such as changing workers' tastes for the type of work in the industry. The next proposition describes the impact of such changes on equilibrium outcomes. A similar result can be derived for an increase in the price of any factor employed in firm projects.

**Proposition 4.** If the workers' wage ($w$) increases,

1. Fewer agents become managers ($\bar{a}$ increases) and the product price increases ($p^*$ increases);
2. Managers of given ability, $a$, are employed by firms choosing fewer projects ($n^*(a)$ declines), which provides incentives through a larger share of profits ($s_1^*(a)$ increases); effort per project also increases ($e^*$ increases); and

3. Salary ($s_0^*(a)$), total compensation and total incentive compensation ($T^*(a)$ and $I^*(a)$), total effort ($E^*(a)$), and manager's certainty equivalent ($c^*(a)$), as well as their slope and convexity, may increase or decrease depending on parameter values. The less price sensitive the demand (i.e., the smaller is $\beta$), the greater the tendency for all the ambiguous effects above to be positive, the slope of profit share $s_1^*(a)$ to increase (toward zero), and the convexity of $s_1^*(a)$ to decrease.

An increase in the workers' wage affects firms in two ways. First, there is the direct effect on project costs, motivating firms to reduce the number of projects. However, since all firms face the cost increase, there is upward pressure on the product price, impacting firm incentives in the direction opposite to that of the wage increase. Thus, the overall impact on firm choices depends on the relative importance of these opposing forces.

Firms employing the lowest ability (i.e., $\bar{a}$) managers generate just enough profits from operations to pay their managers the equivalent of the wage, $w$. Thus, when $w$ increases, these firms, as well as some others employing managers with ability just above $\bar{a}$, are no longer sufficiently profitable to attract managers, and must exit. Firms that continue to operate, *given price*, economize on workers by reducing the number of projects. Moreover, since fewer projects means less risk and effort for the manager, the firm can offer a somewhat greater profit share and demand more effort on each project. The net effect of all these changes, in concert with fewer operating firms, is less output. The implied excess demand is met in part through an increase in price, which partly offsets both the tendency of firms to exit and the actions firms take to economize on workers. It is clear that the wage increase improves workers' utility, and utilities of managers whose ability is marginally above the threshold that existed before the wage increase. That is, before the wage increase, their certainty equivalent was barely above the old wage; after the change, it is at least as high as the higher new wage. However, whether higher-ability managers work harder and get paid more depends on how price adjusts. When demand is less price sensitive ($\beta$ is low), excess demand is accommodated primarily through a price increase, generally limiting how firms reduce the number of projects, and giving managers incentives to work harder. This works in the direction of making managers better off in various ways, such as higher salary and overall greater incentive pay, and generally increasing cross-sectional dispersion in these variables. The opposite occurs when demand is more price sensitive.

Propositions 2 and 4 are not dynamic, but they are suggestive of numerous interesting differences in the cross-section of firms at different points in the business cycle, as well as some differences across business cycles. For example, early in a recovery period, the impact of the business cycle on firms will be similar to that of a demand increase, with the effects described in Proposition 2 (e.g., decreasing managerial share of profit). Later in the business cycle, as wages grow, the predicted impact will be more in line with Proposition 4 (i.e., a reversal in the declining trend of managerial share of profit). In addition, according to Proposition 4, the way the recovery impacts (e.g., total compensation) depends on the specifics of the industry (i.e., price sensitivity). Thus, we would expect changes in wages to have effects on managerial compensation that are very different in the oil industry, where there is very limited scope for product substitution, in comparison to the automobile industry, in which there is abundant opportunity for intertemporal substitution. Finally, because modern recessions display longer lags between demand and labor market recovery (Andolfatto and MacDonald 2007, and the references therein), Propositions 2 and 4 imply, for example, a slower decline and subsequent increase in managers' profit shares as the modern business cycle unfolds.

The results explored thus far are driven by the interplay of managerial ability and firm choices of the number of projects and structure of incentives. Managers' attitudes toward risk (described by $\gamma$), and the risks inherent in the production (described by $\sigma^2$), are key parameters that will affect how ability, number of projects, and incentives interact. Attitudes toward risk change for a wide variety of reasons. For example, improvements in hedging or developments in insurance markets may allow managers to reduce other sources of risk, for example from investments outside the firm, thereby making incremental risk due to incentives less painful. Risk preferences may change over the business cycle. Likewise, inherent production risk varies across industries, countries, and time. In (10)–(17), the agent's risk aversion parameter, $\gamma$, and the parameter describing how much risk is inherent in production, $\sigma^2$, enter together in a form that describes the overall importance of risk to the risk-averse agents (i.e., $\gamma\sigma^2$). Thus, the next proposition focuses on this entity.

**Proposition 5.** If $\gamma\sigma^2$ increases,

1. The product price increases ($p^*$ increases), but the ability level of the marginal manager, $\bar{a}$, may increase or decrease. The less price sensitive the demand, the greater the tendency for $\bar{a}$ to decrease;
2. Managers of given ability have a smaller share of profits ($s_1^*(a)$ declines);
3. The number of projects and total managerial effort generally declines ($n^*(a)$ and $E^*(a)$ decline), but can increase for smaller firms; and
4. For managers of given ability, $a$, the level of salary ($s_0^*(a)$), total compensation and total incentive compensation ($T^*(a)$ and $I^*(a)$), and manager's certainty equivalent ($c^*(a)$), as well as their slope and convexity, may increase or decrease depending on parameter values. The less price sensitive the demand, the greater the tendency for all the ambiguous effects above to be positive, the slope of profit share $s_1^*(a)$ to increase (toward zero), and the convexity of $s_1^*(a)$ to decrease.

Similar to an increase in the workers' wage, an increase in $\gamma\sigma^2$ affects firms both directly and indirectly, through output price. The direct impact is the greater compensation required by managers for the greater disutility of inherent risk. Since high-ability managers optimally face much more risk in comparison to the less able, the direct impact of greater $\gamma\sigma^2$ is more significant for firms whose managers are more able. The price increase arises as follows. Increase in $\gamma\sigma^2$ makes managing too risky for low-ability managers, implying exit of their firms. Firms that remain in the market optimally impose less risk on managers by reducing both the number of projects and the managers' profit shares. As a result, effort per project remains unchanged. Exit and reduction in the number of projects combine to reduce supply, creating excess demand for the product, which is met in part through a higher product price. The price increase ameliorates firms' tendency to reduce the number of projects, and may even cause the number of projects to increase for smaller firms where managers face little risk. But it also motivates firms to shrink their managers' share of profits further in order to reduce the increased risk generated by the price increase. As a result, profit share becomes less sensitive to managerial ability. When demand is price insensitive (low $\beta$), so that much of the excess demand is met via an increase in price, firms become more profitable, which translates into both greater pay and utility for managers, as well as entry by firms ($\bar{a}$ declines).

Even though the model's prediction that the equilibrium profit share ($s_1^*(a)$) decreases as $\gamma\sigma^2$ increases seems natural, it is not as straightforward as in a simple agency model. When $\gamma\sigma^2$ increases, firms may reduce the total risk exposure of the manager in a number of ways, such as reducing the number of projects ($n^*(a)$). Given that $n^*(a)$ and $s_1^*(a)$ tend to move in opposite directions, the equilibrium adjustment of $n^*(a)$ might, in principle, actually increase $s_1^*(a)$. Our proof shows that this does not occur in equilibrium.

The managers' cost of effort is parameterized by $\xi$. An increase in $\xi$ has many interpretations. One is simply that for some reason, for example, the availability of superior recreational activities, or twitter, managers find work more onerous. Alternatively, new rules or regulations, for example, Sarbanes-Oxley, makes it time consuming and more difficult to deliver any given level of productive effort.

**Proposition 6.** If $\xi$ increases,

1. The product price increases ($p^*$ increases), and the ability level of the marginal manager, $\bar{a}$, may increase or decrease. The less price sensitive the demand, the greater the tendency for $\bar{a}$ to decrease.
2. Effort per project falls ($e^*$ declines);
3. For managers of given ability, $a$, the number of projects and total managerial effort generally decline ($n^*(a)$ and $E^*(a)$ decline), but can increase for smaller firms; and

4. Salary ($s_0^*(a)$), total compensation and total incentive compensation ($T^*(a)$ and $I^*(a)$), and manager's certainty equivalent ($c^*(a)$), as well as their slope and convexity, may increase or decrease depending on parameter values. The less price sensitive the demand, the greater the tendency for all the ambiguous effects above to be positive, profit share ($s_1^*(a)$) and its convexity to decrease, and its slope to increase (toward zero).

An increase in $\xi$ operates much like a factor price increase, albeit subject to the offsetting effects of a higher product price. Specifically, when $\xi$ rises, *at a given price*, the firm optimally reduces the number of projects and effort per project; thus, total effort also falls. Since reducing the number of projects also reduces the risk the manager must bear, the firm optimally gives the manager greater motivation by increasing the manager's profit share. Being a manager becomes generally less attractive, causing exit by the smallest firms, who employ the least able managers. Since firm exit and reduced number of projects for each firm reduce supply, some upward pressure on the product price is created. When demand is less price responsive, excess demand is met primarily through price increasing, which works further to decrease managers' profit share and to make firms more profitable, leading to greater pay and utility for managers, as well as to entry by firms (i.e., $\bar{a}$ declines).

### 3.1 Endogenous wage

Thus far we have assumed the workers' wage to be exogenous. That is, the alternative to being a manager is to employ one's human capital either as a worker in the industry or elsewhere in the economy, but human capital is not industry specific. In this subsection, we explore the opposite case, i.e., agents' human capital is entirely industry specific.

The primary impact of this change is as follows. When human capital is not industry specific, there is free entry and exit of both firms *and* workers, in which case shocks to the economic environment ultimately impact only managers, possibly including a change in their number. When human capital is industry specific, workers also feel some of the impact of external shocks, which alters the comparative statics results. While Propositions 2, 5, and 6 have close counterparts in the endogenous wage setting (see below), there is no obvious analogue to Proposition 3; we do offer some similar results in Proposition 7. Obviously, there is no analogue to Proposition 4, which studies the effect of external shocks to the workers' wage.

We begin by modifying the definition of an equilibrium.

**Definition 2.** *An endogenous wage equilibrium* is a set $M \subset [0, \infty)$, a price $p^*$, a wage $w^*$, and functions $c^*(a)$, $e^*(a, s_1)$, $s_0^*(a)$, $s_1^*(a)$, and $n^*(a)$, satisfying conditions 1–5 of Definition 1, and

6. *Every agent is either a manager or a worker in the industry*:

$$\mu(M) + \int_M n^*(a)\mu(da) = \mu([0,\infty)). \tag{20}$$

The endogeneity of the workers' wage does not alter the derivation of the optimal number of projects, managerial compensation, effort, and certainty equivalent. Thus, Proposition 1 continues to hold.

Proposition 2 is much simpler when $w$ is endogenous.[12] An increase in demand translates into a higher product price ($p^*$ increases) and more supply via managers working harder on each project ($e^*$ increases). Managers become better off ($c^*(a)$ increases) through higher salary, total incentive compensation, and total compensation ($s_0^*(a)$, $I^*(a)$, and $T^*(a)$ all rise), although managers provide more total effort ($E^*(a)$ rises). Firms do not have to increase their manager's profit share to motivate more work; indeed, the higher product price raises the marginal value of managers' effort sufficiently ($s_1^*(a)$ does not change). Whereas the higher product price gives firms incentives to expand the number of projects, this effect is offset by the required workers being more expensive (so $n^*(a)$ does not change). Moreover, no firms enter or exit ($\bar{a}$ does not change). Finally, the slope and convexity of $s_0^*(a)$, $I^*(a)$, $T^*(a)$, and $c^*(a)$ all increase when demand increases.

Proposition 5 is virtually unchanged when $w$ is endogenous. The primary difference is that increasing $\gamma\sigma^2$, since it causes price to rise, also leads to an increase in the wage rate as managers are drawn into the industry by firms entering ($\bar{a}$ declining does not depend on the sensitivity of demand to price when $w$ is endogenous) and, necessarily, the number of workers falls. The increasing wage causes firms partially to substitute managers' effort for the number of projects, because this allows firms to hire fewer workers. Thus, $e^*$ increases, whereas $e^*$ was independent of $\gamma\sigma^2$ when the wage was exogenous.

Proposition 6 is also less ambiguous. That is, when $\xi$ increases, the managers' profit share and the ability of the marginal manager both decline (these effects do not depend on the sensitivity of demand to price when $w$ is endogenous). Moreover, with a fixed $w$, an increase in $\xi$ leads to managers' decreasing their per-project effort. When $w$ is endogenous, increasing $\xi$ leads to an increase in the wage that may lead to, especially when demand is less sensitive to price, effort per project to increase.

When the wage is endogenous, a change in the ability measure, $\mu$, may affect both the labor market (e.g., through supply of workers) and product market (through changes in the managers' abilities). As we show in the following proposition, it is the imbalance in the labor market that helps us identify what happens to the equilibrium outcomes.

[12] Proofs of the endogenous-wage version of Propositions 2, 5, and 6 are available from the authors upon request.

**Proposition 7.** Suppose the ability measure changes from $\mu$ to $\hat{\mu}$. If the change in the measure results in excess supply of workers,

$$\int_{\bar{a}_{\mu}}^{\infty} n_{\mu}^{*}(a)\hat{\mu}(da) < \hat{\mu}([0, \bar{a}_{\mu}]), \tag{21}$$

where $\bar{a}_{\mu}$ and $n_{\mu}^{*}(a)$ are the equilibrium values under measure $\mu$, then

1. The ability of the marginal manager declines ($\bar{a}$ decreases);
2. Managers of given ability, $a$, are employed by firms choosing more projects ($n^{*}(a)$ increases), which provides incentives through a smaller share of profits ($s_{1}^{*}(a)$ decreases); and
3. Salary ($s_{0}^{*}(a)$), total compensation and total incentive compensation ($T^{*}(a)$ and $I^{*}(a)$), total effort ($E^{*}(a)$), manager's certainty equivalent ($c^{*}(a)$), their slope and convexity, workers' wage ($w^{*}$), and output price ($p^{*}$) may increase or decrease depending on parameter values. The less price sensitive the demand, the greater the tendency for all the ambiguous effects above to be positive.

If the change in measure results in excess demand for labor (or equivalently, (21) is violated), then results 1–3 reverse.

Condition (21) says that, absent changes in product price or wage, demand for workers would fall short of supply of workers when $\hat{\mu}$ replaces $\mu$. As we show in the proof, this change invariably results in $p^{*}$ rising relative to $w^{*}$. However, the level of either may rise or fall, because, for example, $\hat{\mu}$ might have many more agents of ability less than $\bar{a}_{\mu}$, but also more agents of ability above $\bar{a}_{\mu}$. In this case, Condition (21) can be satisfied, but the industry has so many more agents on the supply side that both $p^{*}$ and $w^{*}$ fall.

There are various ways in which the supply of workers may become excessive. For example, regulatory changes may reduce the supply of managerial talent. In the financial services industry, as a result of the repeal of the Glass-Steagall Act, it became important for bank CEOs to have skills in both commercial and investment banking businesses, which effectively disqualified many managers from running large universal banks even though they are still qualified as division managers ("workers" in our model). In another example, globalization has increased the supply of workers, but has had little impact on the supply of managerial talent in many U.S. industries.

Proposition 7, combined with Proposition 2, suggests that globalization may be an important driver of the recent trends in managerial compensation. Globalization has increased not only the supply of workers, but also output demand. According to the propositions, these increases lead to new firm entry, and increases in the level and dispersion of firm size and all components of managerial compensation (given that product demand is not too price sensitive). Moreover, our model suggests that these observed patterns may reverse when the supply of managerial talent increases substantially as globalization progresses.

### 3.2 Heterogeneous firms

The model analyzed above assumes that firms are homogeneous. Thus, firms differ in, for example, size, purely due to differences in ability of the managers they hire. The homogeneity assumption simplifies the analysis while still, as we show in Section 4, allowing a surprisingly good correspondence with the data. However, the model can be extended to accommodate firm heterogeneity, thus capturing inter-firm productivity differences due to technology, location, equipment, patents, access to capital, other talents in the firm, or political connections; Maksimovic and Phillips (2002) employ total factor productivity to measure firm and plant productivity. Our extension has exogenously specified differences across firms, but retains the endogenously determined number of projects. Thus, it can be interpreted as allowing for firm size to be partly exogenously determined by "productivity," and partly driven by managerial ability. Thus, the extension includes as special cases models similar to those with exogenously heterogenous firms (e.g., Gabaix and Landier 2008; Tervio 2008; Edmans, Gabaix, and Landier 2009) and those with ex-ante identical firms and endogenously determined firm sizes (e.g., Lucas 1978; Rosen 1981; and our base model).

Allowing for firm heterogeneity generates even greater convexity of firm size, incentive compensation, and effort, given intuitive and fairly mild restrictions on the distribution of the exogenous firm characteristic; with similar but slightly stronger conditions, total compensation is also more convex when firms are heterogeneous. Essentially, because managerial ability and firm productivity are complementary, and, in equilibrium, more talented managers match with firms having greater productivity, the differences in productivity amplify the complementarity among the number of projects, managerial talent, and managerial effort. Firm heterogeneity also results in positive firm profits, which also vary across firms. This yields new predictions about the impact of changes in economic fundamentals on firm profits, while allowing all comparative statics on other equilibrium entities for the base model (Propositions 2–6) to carry over.[13]

The analysis also offers new empirical implications. For example, Tervio (2008) estimates the effect of managerial ability on compensation by calculating the difference between the observed compensation earned by the CEO of a large firm, and the hypothetical compensation that the CEO of a small firm would earn if employed by this large firm. However, our model suggests

[13] Intuitively one may suspect that the convexity results in the base model are due to the fact that the manager enjoys all profit from heterogeneity in talent. When firms are heterogeneous, firms and managers split the return to heterogeneity in both ability and productivity. As we show, under mild conditions, this split yields equilibrium profit and components of compensation that are not only convex in ability, but also more convex in comparison to the base model (firm profit in the base model was zero for all $a$).

that, had the large firm employed the CEO of the small firm, its size would decrease, and CEO effort would also decline. Both effects would reduce the imputed hypothetical compensation, and thus magnify the difference in compensation attributable to the difference in managerial ability. This notion questions the claim in Tervio (2008) that only a small fraction of variation in CEO pay is attributable to CEO ability. Although not a direct test of our model, the empirical analysis of Bertrand and Schoar (2003) shows that firm policies and characteristics indeed change significantly when CEOs are replaced.

If a firm having productivity $k$ hires a manager with ability $a$, who supplies per-project effort $e$, and there are $n$ projects, total firm output is $n(\sqrt{kae}+\varepsilon)$, where $\varepsilon \sim N(0,\sigma^2)$ is a firm-specific random shock. We will refer to a firm characterized by $k$ as "firm $k$." Let $\nu$ be the atomless measure of $k$, with $\nu([0,\infty))$ being sufficiently large to ensure that there is a firm for every agent who chooses to be a manager. Let $k^*(a)$ be the function that describes the equilibrium firm-manager matching. We can state the equilibrium matching condition as

$$\text{for all } m \subset M,\ \mu(m) = \nu(k^*(m)).$$

For firm $k$ run by a manager with ability $a$, the optimization problem is similar to that for the base model, and yields the same optimal effort per project $e^* = 2w/\xi$. The number of projects and managerial profit share are, respectively,

$$n^*(a) = \frac{ka}{\xi\gamma\sigma^2}\left(\sqrt{\frac{kap^{*2}}{2\xi w}}-1\right), \text{ and}$$

$$s_1^*(a) = \sqrt{\frac{2\xi w}{kap^{*2}}}.$$

Firm heterogeneity implies (except for the least productive firm that hires the marginal manager) positive firm profits. The equilibrium profit of firm $k$ that hires manager $a$ is given by[14]

$$\pi_k(a) = \frac{kaw}{\xi\gamma\sigma^2}\left(\sqrt{\frac{kap^{*2}}{2\xi w}}-1\right)^2 - c^*(a). \tag{22}$$

The equilibrium match between firm $k$ and manager $a$, $k^*(a)$, maximizes firm profit $\pi_k(a)$. As in standard matching models (e.g., Gabaix and Landier

14 The above expression is obtained as follows. First, we express $s_0^*(a)$ using $e^*$, $n^*(a)$, $s_1^*(a)$, and the reservation certainty equivalent $c^*(a)$ (to be determined) given in the manager's participation constraint (#3 in the equilibrium definition, assuming it holds with equality). Next, substitute for the equilibrium values of $e^*$, $n^*(a)$, and $s_1^*(a)$ into the profit definition $\pi_k(a) = (1-s_1^*(a))n^*(a)(p^*\sqrt{k^*(a)ae^*}-w) - s_0^*(a)$.

2008; Tervio 2008), more talented managers are matched with more productive firms. The first-order condition describing a type $k$ firm's optimal choice of manager ability is[15]

$$(\pi_k(a))'_{a|k=k^*(a)} = \left(\frac{kaw}{\xi\gamma\sigma^2}\left(\sqrt{\frac{kap^{*2}}{2\xi w}}-1\right)^2\right)'_{a|k=k^*(a)} - \left(c^*(a)\right)' = 0,$$

implying

$$c^*(a) = F + \int_0^a \left[\frac{k^*(b)w}{\xi\gamma\sigma^2}\left(\sqrt{\frac{k^*(b)bp^{*2}}{2\xi w}}-1\right)\left(\sqrt{\frac{2k^*(b)bp^{*2}}{\xi w}}-1\right)\right]db, \tag{23}$$

where $F$ is independent of $a$, and is determined by the marginal firm/manager condition: The firm that has the lowest productivity in the product market hires the least able manager, $\bar{k} = k^*(\bar{a})$, and thus earns zero profit, $\pi_{\bar{k}}(\bar{a}) = 0$. The certainty equivalent of the least able manager equals the workers' wage, i.e., $c^*(\bar{a}) = w$. Substituting for firm profit given in (22),

$$c^*(\bar{a}) \equiv \frac{\bar{k}\bar{a}w}{\xi\gamma\sigma^2}\left(\sqrt{\frac{\bar{k}\bar{a}p^{*2}}{2\xi w}}-1\right)^2 = w.$$

Thus, the expression for $F$ becomes

$$F = w - \int_0^{\bar{a}} \left[\frac{k^*(b)w}{\xi\gamma\sigma^2}\left(\sqrt{\frac{k^*(b)bp^{*2}}{2\xi w}}-1\right)\left(\sqrt{\frac{2k^*(b)bp^{*2}}{\xi w}}-1\right)\right]db. \tag{24}$$

Substituting (24) into (23), we obtain the certainty equivalent for the manager:

$$c^*(a) = w + \int_{\bar{a}}^a \left[\frac{k^*(b)w}{\xi\gamma\sigma^2}\left(\sqrt{\frac{k^*(b)bp^{*2}}{2\xi w}}-1\right)\left(\sqrt{\frac{2k^*(b)bp^{*2}}{\xi w}}-1\right)\right]db. \tag{25}$$

Finally, because higher-ability managers work for firms with higher productivity, we can determine $k^*(a)$ from the firm-manager equilibrium matching condition, restated as $\mu([a,\infty)) = \nu([k^*(a),\infty))$.

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[15] The condition can also be read as saying that the additional increase in salary that the higher-ability manager demands is bigger than the improvement in profit (net of effort and risk costs) that the manager can generate for the firm. Thus, a manager with ability above $a$ will not switch to firm $k = k^*(a)$.

With heterogeneous firms, the equilibrium expressions for $n^*(a)$, $s_1^*(a)$, $I^*(a)$, and $E^*(a)$ can be obtained from the equilibrium expressions in our base model—Equations (10), (11), (16), and (18)—by replacing $a$ with $k^*(a)a$. The equilibrium solutions for the remaining variables are

$$I^*(a) = \frac{k^*(a)aw}{\xi\gamma\sigma^2}\left(\sqrt{\frac{2k^*(a)ap^{*2}}{\xi w}}-1\right)\left(1-\sqrt{\frac{2\xi w}{k^*(a)ap^{*2}}}\right),$$

$$s_0^*(a) = \frac{k^*(a)aw}{\xi\gamma\sigma^2}\left(\sqrt{\frac{k^*(a)ap^{*2}}{2\xi w}}-1\right)^2\left(1-\sqrt{\frac{2\xi w}{k^*(a)ap^{*2}}}\right)+c^*(a),\ \text{and}$$

$$T^*(a) = \frac{k^*(a)aw}{\xi\gamma\sigma^2}\left(\sqrt{\frac{k^*(a)ap^{*2}}{2\xi w}}-1\right)\sqrt{\frac{k^*(a)ap^{*2}}{2\xi w}}+c^*(a),$$

where $c^*(a)$ is given by (25). As in our base model, all of the equilibrium solutions are monotone and convex in $k^*(a)a$, and all but $s_1^*(a)$ are increasing in $k^*(a)a$.

How does the presence of ex-ante firm heterogeneity affect convexity of, for example, total incentive compensation as a function of managerial ability? To simplify notation, define $\lambda(a) \equiv k^*(a)a$ to be "productivity adjusted" ability. The expression for total incentive compensation with firm heterogeneity is simply the corresponding expression without heterogeneity, evaluated at $\lambda(a)$ instead of $a$. Thus, define $I_\lambda(a) \equiv I^*(\lambda(a))$ to be the total incentive compensation for a manager of ability $a$ in the model allowing firm heterogeneity, where $I^*$ is the expression for total incentive compensation absent firm heterogeneity. Below we refer to $n_\lambda(a)$, etc., defined analogously. Since the equilibrium expressions for salary and total compensation are different in the model with heterogeneity, in what follows, we denote them as $s_{0\lambda}(a)$ and $T_\lambda(a)$.

There are several ways to compare convexity across the model with firm heterogeneity and the model without. The one we explore is a comparison of $I_\lambda''(a)$ to $I^{*''}(\hat{a})|_{\hat{a}=\lambda(a)}$. These two terms can be interpreted as convexities of total incentive compensation in two distinct "experiments." In the first, we consider a manager of given ability, and explore convexity when matching managerial ability to firm productivity is allowed. In the second, we explore convexity in ability when the manager has *locally fixed* firm productivity.[16] This way of examining convexity emphasizes one of two key differences between the models with and without firm heterogeneity (i.e., better managers attract more capable firms). (The other key difference is that the model

16 More precisely, for each $a$, there exists a $\hat{k}$ such that $\hat{k}+a \equiv \lambda(a)$.

with firm heterogeneity effectively has more inputs. This must be addressed somehow, and the above-described method is one way to do so.)[17]

**Proposition 8.** If for any a, $\lambda'(a) \geq 1$ and $\lambda''(a) \geq 0$, then $n''_\lambda(a) \geq n^{*\prime\prime}(\lambda(a))$, $s''_{1\lambda}(a) \geq s_1^{*\prime\prime}(\lambda(a))$, $I''_\lambda(a) \geq I^{*\prime\prime}(\lambda(a))$, $T''_\lambda(a) \geq T^{*\prime\prime}(\lambda(a))$, and $E''_\lambda(a) \geq E^{*\prime\prime}(\lambda(a))$. If, additionally, $\lambda''(a) \geq \frac{\lambda'^2(a)}{\lambda(a)}$, then $s''_{0\lambda}(a) \geq s_0^{*\prime\prime}(\lambda(a))$, and $T''_\lambda(a) \geq T^{*\prime\prime}(\lambda(a))$.

The intuition for the sufficient conditions is straightforward. Take $I_\lambda(a)$ as an example. If productivity-adjusted ability varies little in response to a change in ability, then total incentive compensation may not be very sensitive to ability itself; $\lambda' > 1$ means that productivity-adjusted ability varies at least one-for-one with ability. Further, if productivity-adjusted ability becomes less sensitive to ability as ability rises ($\lambda'' < 0$), then total incentive compensation may not be increasingly sensitive to ability as ability rises. To see this in more economic terms, suppose that the distribution of firm heterogeneity is quite left skewed. Then, for lower-ability managers, there is a lot of scope for matching of productivity to managers, and pay will be very sensitive to ability. But matching will be much less impactful for high-ability managers, and so pay will still be increasing and convex in productivity-adjusted ability, but not nearly as sensitive as to ability itself.

In this model, firm profit ($\pi_k^*(a)$), given by (22) and (25), is positive and increasing in $a$. If $(k^*(a))''$ is not too negative, firm profit is convex in $a$.[18]

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[17] Another way to compare convexity is to compare $I''_\lambda(a)$ to $I^{*\prime\prime}(a)$. The fact that the firm heterogeneity model has more factors must be addressed here, too.

[18] The first- and second-order derivatives are

$$\pi_k^{*\prime}(a) = \left(\frac{k^*(a)aw}{\xi\gamma\sigma^2}\left(\sqrt{\frac{k^*(a)ap^{*2}}{2\xi w}} - 1\right)^2\right)'_a - \left(c^*(a)\right)'$$

$$= \frac{aw}{\xi\gamma\sigma^2}\left(\sqrt{\frac{k^*(a)ap^{*2}}{2\xi w}} - 1\right)\left(\sqrt{\frac{2k^*(a)ap^{*2}}{\xi w}} - 1\right)(k^*(a))' > 0,$$

and

$$\pi_k^{*\prime\prime}(a) = \left(\frac{aw}{\xi\gamma\sigma^2}\left(\sqrt{\frac{k^*(a)ap^{*2}}{2\xi w}} - 1\right)\left(\sqrt{\frac{2k^*(a)ap^{*2}}{\xi w}} - 1\right)(k^*(a))'\right)'_a$$

$$= \frac{w}{\xi\gamma\sigma^2}\frac{(k^*(a))'a + k^*(a)}{k^*(a)}\left(\sqrt{\frac{2k^*(a)ap^{*2}}{\xi w}}\right)\left(\sqrt{\frac{k^*(a)ap^{*2}}{2\xi w}} - \frac{3}{4}\right)(k^*(a))'$$

$$+ \frac{w}{\xi\gamma\sigma^2}\left(\sqrt{\frac{k^*(a)ap^{*2}}{2\xi w}} - 1\right)\left(\sqrt{\frac{2k^*(a)ap^{*2}}{\xi w}} - 1\right)(k^*(a))'$$

$$+ \frac{aw}{\xi\gamma\sigma^2}\left(\sqrt{\frac{k^*(a)ap^{*2}}{2\xi w}} - 1\right)\left(\sqrt{\frac{2k^*(a)ap^{*2}}{\xi w}} - 1\right)(k^*(a))''.$$

When product demand increases, the level, slope, and convexity of firm profit all increase. For shocks to other economic fundamentals, the responses of firm profit depend on parameters in much the same way as total managerial compensation responds to these shocks (recall Propositions 2–6).

## 4. Data

The primary goal of our article is to study the interaction of agency and heterogeneous agents and how this affects the distribution of firm size and managerial compensation. A thorough structural exploration of all empirical implications set out above is far beyond the scope of this article. However, it is both interesting and important to provide some evidence that our model may prove useful empirically. To do so, we study data on CEO compensation and firm characteristics. The CEO compensation data are from the ExecuComp database, covering 1993 to 2007, and include the following variables: *Salary* ($s_0^*$, including salary and bonus payment), *Total compensation* ($T^*$),[19] and *Incentive pay* ($s_1^*$, pay-for-performance sensitivity of estimated CEO wealth, as calculated in Core and Guay 1999). Firm size measures ($n^*$) include *Number of employees*, *Sales*, and *Total assets*, provided in the Compustat database. We also include year indicators, and industry indicators based on Fama-French twelve industry groups (e.g., Bergstresser and Philippon 2006).

### 4.1 Strengthened link between CEO compensation and firm size

In an intriguing empirical paper, Frydman and Saks (2010) document some important facts about CEO compensation. First, the much-studied sensitivity of CEO pay to firm performance is not just a modern phenomenon. In fact, equity holdings and, later, stock options gave CEO pay sensitivity that was "not inconsequentially small" for most of their sample period: 1936 to 2005. Second, the use of performance-sensitive compensation, such as stock and stock options, has grown since the mid-1950s. Third, the cross-sectional association between CEO pay and firm size has increased over time.

Our model has nothing specific to say about the first of these facts, except that, given the model, it would be quite surprising if CEO pay were ever not associated with performance. The second and third facts are inherently dynamic ones, while our model is not. Nevertheless, it is interesting to ask whether repetitions of our static model, allowing parameters to change and equilibrium to adjust, can generate patterns similar to those observed.

The second phenomenon can be generated by our model using changes in $\gamma\sigma^2$. According to Proposition 5, a decrease in $\gamma\sigma^2$ (e.g., when managers become less risk averse or production becomes more controllable) increases $s_1^*$.

[19] According to ExecuComp, *Total compensation* is comprised of the following: salary, bonus, other annual compensation, total value of restricted stock granted, total value of stock options granted (using Black-Scholes), long-term incentive payouts, and all other total compensation.

It also increases $I^*$ and $T^*$ if demand is sufficiently price responsive (high $\beta$). In this case, the dollar value and relative importance of incentive compensation both increase. More interestingly, $n^*$ generally increases but could decrease for small firms.[20] These cross-sectional differences of the implications of our model can be used to guide future empirical research.

There are a variety of ways in which the third issue might be explored. One simple way that can be implemented with existing data is as follows. According to the model, the cross-sectional relation between, for example, total compensation and firm size is induced by cross-firm variation in CEO ability. Thus, we can calculate the theoretical cross-sectional elasticity of total compensation with respect to firm size:

$$\frac{1}{T^*(a)}\frac{dT^*(a)}{da} \div \frac{1}{n^*(a)}\frac{dn^*(a)}{da}.$$

Next, we examine how this pay-size elasticity varies as the model's parameters change, and then explore whether the observed trends appear consistent with observed changes in the economic environment.

Many aspects of the economy evolve over time. We focus on demand growth, which is potentially measurable. Some algebra shows that, provided CEO incentive pay is not too small, the elasticity of total compensation with respect to firm size is increasing in demand.[21] Intuitively, increasing demand causes firms to expand, which imposes greater risk on the CEO and consequently requires greater compensation. If $s_1^*$ is too small, the extra risk that growing firm size imposes on the CEO is small, and so the required increase in compensation is minimal, translating to a decreasing cross-sectional elasticity. We assume that $s_1^*$ is sufficiently large; increasing demand leads to an increase in the pay-size elasticity.

While we do not have independent measures of demand by industry, we do have measures of industry growth, such as changes in median firm sales or total industry sales; we present results using the former, but both measures produce similar results. The empirical question is whether industries in which sales grew quickly are also those where the elasticity of CEO pay with respect to size is growing quickly. If so, the model's explanation for the Frydman-Saks third observation is that the increase in industry demand due to economic growth and globalization has caused firms optimally to be larger and have more sensitive pay.

We studied data on the twelve Fama-French industries from 1993 to 2007. For *each industry and year*, we estimate the cross-sectional elasticity of total (direct) compensation ($TDC$) with respect to firm size (total assets ($TA$);

20 This implication is not available in models such as Edmans, Gabaix, and Landier (2009), where exogenous firm size drives compensation, and incentive pay $s_1^*$ decreases with firm size. In such a model, an (exogenous) increase in firm size implies a lower incentive pay $s_1^*$ for every given manager. Moreover, this implication is not available in models that do not separate salary and total incentive compensation.

21 Detailed derivation is provided in Appendix B.

very similar results hold for sales and number of employees) by running the following regression:

$$\ln(TDC) = b_0 + b_1 \ln(TA) + \upsilon, \tag{26}$$

interpreting the estimate of $b_1$ as the pay-size elasticity. Next, we look for intra-industry trends in the elasticity by running the regression *for each industry*:

$$\ln(TDC) = b_0 + b_1 \ln(TA) + b_2 Year \times \ln(TA) + \upsilon,$$

interpreting the estimate of $b_2$ as the elasticity's trend. Seven industries have a positive and statistically significant estimate of $b_2$ (at better than 5%): manufacturing, healthcare, finance, utilities, telecommunications, retail, and energy. The remaining five industries do not show any trend. Are the industries for which the elasticity shows a positive trend also the industries that grew more quickly? Figure 1 displays the time paths of the year-industry-specific elasticities estimated from (26), $b_1$, for these two groups of industries.

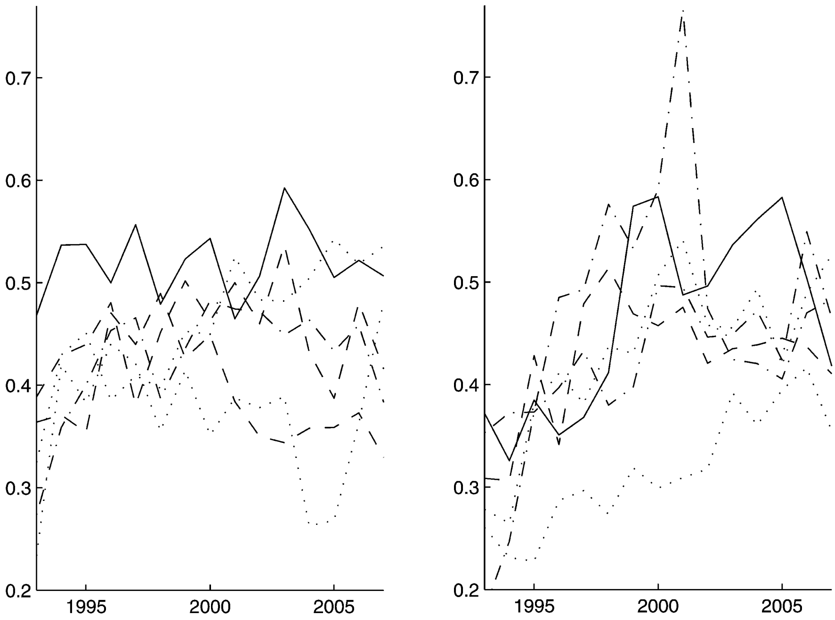


**Figure 1**
**Elasticity of CEO pay with respect to firm size**
This figure depicts the trend in elasticity of CEO pay with respect to firm size, $b_1$. The estimate of $b_1$ is obtained from (26) for each of the twelve Fama-French industries. Industries in the left graph (consumer durables, consumer non-durables, business equipment, chemicals, manufacturing, and other) had low sales growth rates. Industries in the right graph (healthcare, finance, utilities, telecommunications, retail, and energy) had high sales growth rates. The number of observations used in each regression ranges from 20 to 346; the median number of observations is 118.

Consistent with our model, six of the seven industries with a positive trend in pay-size elasticity are also the fastest-growing industries (growth rates ranging from 9.7% to 15.8%), while the seventh, manufacturing, is ranked eighth in terms of growth. In contrast, the industries with no significant trend have growth rates ranging from 5.9% to 8.1%. In addition, the inter-industry correlation between the estimated trend in the elasticity ($b_2$) and the average growth rate in sales is 0.68. Thus, our explanation of the Frydman-Saks finding is that more rapidly growing industries generate stronger motivation for firms to complement CEO ability with firm size and appropriately chosen pay, and that this shows up empirically as a rising pay-size elasticity in those industries.

### 4.2 Comparisons of observed and model-predicted patterns

According to the model, within industry cross-sectional variation in firm size, as well as CEO salary, incentive pay, and total pay, follows from heterogeneity in managerial ability. Thus, the relation between, for example, CEO incentive pay and firm size is also a consequence of this underlying heterogeneity.

We use number of employees as the proxy for number of projects; using the number of segments provided by Compustat generates slightly weaker results. Figure 2 displays scatter diagrams of each of *Number of employees* (proxy for number of projects), *Salary*, and *Total compensation* versus *Incentive pay* (proxy for managerial profit share) for the 128 firms in the retail industry in 2003.[22] Temporarily ignoring the functions graphed in Figure 2, observe that the retail industry has features familiar from the ExecuComp data more generally. That is, the relation between each of the number of employees, salary, and total compensation, and incentive pay, is not only negatively sloped, but also highly nonlinear—the firms displaying high values of the number of employees, salary, and total compensation have very low incentive pay.

To see what the model has to say about these observed relationships, note that (11), together with (10), (15), and (17), can be employed to generate predictions about the relations between $s_1^*$, and each of $n^*$, $s_0^*$, and $T^*$. That is, (11) can be inverted to yield

$$a = \frac{2\xi w}{s_1^{*2} p^{*2}},$$

in which case (10), (15), and (17) give the theoretical values of employment, salary, and total compensation as the following relations to incentive pay:

[22] This industry was chosen because it tolerably satisfies the model's assumptions of free entry and a homogeneous product. On the other hand, retail is unusual in that the largest firm, Walmart, is 2.3, 4.3, and 5.7 times (based on assets, sales, and market capitalization, respectively) larger than the second largest, Target. For this reason, this analysis excludes Walmart.

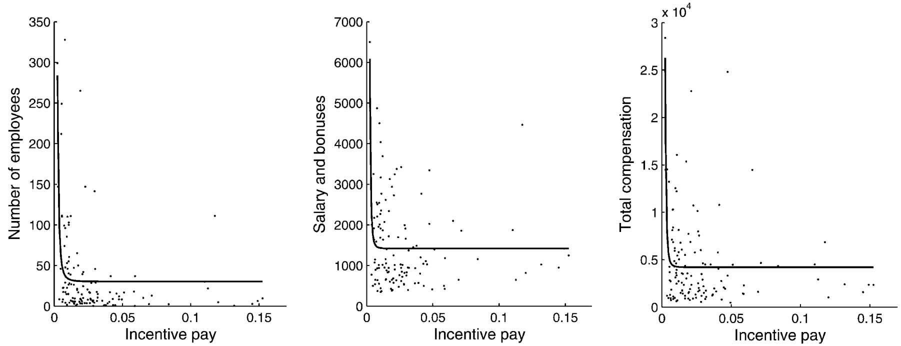


**Figure 2**
**Incentive pay vs. firm size, salary, and total pay**
This figure depicts the observed (scattered points) match of *Number of employees* ($n^*$), CEO *Salary* ($s_0^*$, including salary and bonus payment), and *Total compensation* ($T^*$), with *Incentive pay* ($s_1^*$, pay-for-performance sensitivity as calculated in Core and Guay 1999) for the retail industry in year 2003. The solid green curve in each graph is drawn using the values predicted by our model: $n^* \propto \frac{1-s_1^*}{s_1^{*3}}$, $s_0^* \propto \frac{(1-s_1^*)^2(2-s_1^*)}{s_1^{*4}}$, and $T^* \propto \frac{(1-s_1^*)(2-s_1^*)}{s_1^{*4}}$.

$$n^* \propto \frac{1 - s_1^*}{s_1^{*3}},$$

$$s_0^* \propto \frac{(1 - s_1^*)^2(2 - s_1^*)}{s_1^{*4}}, \text{ and}$$

$$T^* \propto \frac{(1 - s_1^*)(2 - s_1^*)}{s_1^{*4}}.$$

The functions graphed in Figure 2 are these theoretical relations, assuming number of employees and incentive pay measure $n^*$ and $s_1^*$; the factor of proportionality has been estimated from the data by, for example, the sample average value of $n^* \div [(1 - s_1^*)/s_1^{*3}]$. The functions make *no* other use of the data. As shown in the figure, these relations are monotonic and very nonlinear, and correspond with the data in a general way. In particular, the correlation between the observed number of employees and incentive pay is −0.24, whereas the correlation between the number of employees predicted from our theoretical model and incentive pay is −0.20. In addition, the correlation between the observed number of employees and the corresponding predicted value is 0.49. For salary, the correlation between the observed variable and incentive pay is −0.11, whereas the correlation between the value predicted from our model and incentive pay is −0.15. The correlation between the observed salary and the corresponding predicted value is 0.44. For total compensation, the correlation between the observed variable and incentive pay is −0.14, whereas the correlation between the value predicted from our model and incentive pay is −0.15. The correlation between the observed total compensation and the corresponding predicted value is 0.48.

Overall, the empirical relations between incentive pay and measures of firm size and CEO compensation components show quite extreme behavior that is consistent with the behavior generated by optimal incentives, competition among firms, and endogenous firm size.

### 4.3 Moments of employment and components of CEO compensation

In the previous subsection, we showed that without making any particular assumption about the underlying distribution of ability, the model easily generated relations between incentive pay and, for example, firm size that are empirically relevant. We now explore how the model performs when confronted with the more difficult task of generating distributions of firm size and dimensions of CEO pay that are similar to those in the data on retail firms.

The analysis is conducted in three steps. First, as mentioned above, the model abstracts from payments to other factors of production, such as capital, in which case all compensation calculations will be much too high compared to the data. There are a variety of ways to address this issue. The route we follow is very simple. We calculate all the variables of interest from the model, including salary ($s_0^*$), then scale the calculated salaries (by the same factor for all firms) so that the simulated cross-section average ratio of salaries to total compensation agrees with the average in the data, 0.32.[23] Second, we must specify a functional form for the ability distribution. We follow two approaches. One is to assume a zero skewness distribution, i.e., *Uniform*. This approach implies that any skewness in the variables of interest has its origin in the economizing behavior of firms and agents, rather than in the imposed skewness in ability. On the other hand, insofar as it is reasonable to assume that CEOs are generally from the right tail of the population ability distribution, it is likely more descriptively accurate to assume some positive skewness in ability at the outset; we do this by assuming that ability has density proportional to $1/a$. Finally, we must select some values for the model's other parameters, $w$, $\xi$, $\gamma\sigma^2$, $\alpha$, and $\beta$. We assume $w$, the wage of the marginal manager, to be the salary payment of the bottom percentile in the data, \$370,000. Lacking information on the other parameters, we estimate them by minimizing the sum of the squared percentage differences between the first two moments of the observed and the simulated data on incentive pay, the number of employees, salary, and total compensation.

Given the assumed distribution of ability and the estimated parameters, the model implies the cross-sectional distribution of $s_1^*$, $n^*$, $s_0^*$, and $T^*$, whose moments can be compared to the moments of the empirical distribution of incentive pay, the number of employees, salary, and total compensation. Table 1 includes the observed versus simulated moments implied by each of the two ability distributions. There are two notable inconsistencies between the model's predictions and the data. First, the model predicts employment that is on average too small, and that varies too little across firms. Second, while the model predicts mean incentive pay well, the model predicts less variability than is in the data. This is not surprising, because the model ignores CEO stock holdings and options that are granted pre-2003. Otherwise, for both ability distributions, the model produces moments of pay that are in reasonable agreement with those in the data. The distribution with skewness generates slightly better correspondence with the data. A more full-blown empirical analysis would jointly estimate both the shape of the distribution of ability and the other parameters in the model.

## 5. Conclusion

This article studies an agency model embedded in the Superstars framework. There is a novel combination of features—agents differ in their ability, firms choose both the scope of the managers' activities and their incentives, and there

[23] Another approach, which yields very similar results, is to assume the firm to have some fixed cost whose size is calibrated to bring the average salary payment into agreement with the data.

**Table 1**
**Observed versus Simulated Data (Retail Industry 2003)**

Panel A: Uniform Ability Distribution

| | Mean | | Standard Deviation | |
|---|---|---|---|---|
| Variable | Observed | Simulated | Observed | Simulated |
| *Incentive pay* ($s_1^*$) | 0.03 | 0.04 | 0.03 | 0.01 |
| *Number of employees* (000, $n^*$) | 38.61 | 29.17 | 59.56 | 14.84 |
| *Salary* (\$000, $s_0^*$) | 1,514 | 1,870 | 1,103 | 1,210 |
| *Total compensation* (\$000, $T^*$) | 4,650 | 5,845 | 4,815 | 3,751 |

Panel B: Ability Distribution with Positive Skewness

| | Mean | | Standard Deviation | |
|---|---|---|---|---|
| Variable | Observed | Simulated | Observed | Simulated |
| *Incentive pay* ($s_1^*$) | 0.03 | 0.04 | 0.03 | 0.01 |
| *Number of employees*(000, $n^*$) | 38.61 | 22.11 | 59.56 | 16.29 |
| *Salary* (\$000, $s_0^*$) | 1,514 | 1,533 | 1,103 | 1,466 |
| *Total compensation* (\$000, $T^*$) | 4,650 | 4,792 | 4,815 | 4,546 |

*Number of employees* (in thousands) is obtained from the Compustat database. *Salary* and *Total compensation* (in thousands of dollars) are obtained from the ExecuComp database. *Salary* is salary and bonus payment. *Total compensation* includes salary, bonus, other annual compensation, total value of restricted stock granted, total value of stock options granted (using Black-Scholes), long-term incentive payouts, and all other total compensation. *Incentive pay* is the pay-for-performance sensitivity of estimated CEO wealth, as calculated in Core and Guay (1999). It is the sum of stock ownership and stock options (adjusted by estimated delta) scaled by the number of shares outstanding. In both panels, the workers' wage is set to $w = \$370,000$. In Panel A, we assume $a \sim U[0, 1]$, and optimization yields the following values: $\xi = 0.477$, $\sigma = 7.669$, $\alpha = 1,036$, and $\beta = 1.649$. As a result, $\bar{a} = 0.2404$. In Panel B, we assume that ability has a density proportional to $1/a$ on $a \in (0, 10]$, and obtain $\xi = 0.787$, $\sigma = 5.413$, $\alpha = 165.4$, and $\beta = 4.841$. As a result, $\bar{a} = 0.1810$.

is free entry by firms. The outcome is an industry equilibrium in which firms are heterogenous in scope and output. That is, firms hiring more able managers complement higher ability with more projects and stronger incentives, resulting in greater output. Pay has a strong "Superstars" element in the sense that motivating higher-ability managers to accept a job involving more effort and greater risk of managing greater scope requires much greater rewards.

The model is a simple one that makes strong but standard assumptions; this allows us to analyze it completely and arrive at sharp conclusions. We derive a wide variety of empirically testable implications. For example, an increase in demand for the industry's product, perhaps due to a booming economy or opening of foreign economies, increases both the overall level and dispersion of the cross-sectional distribution of all components in CEO compensation.

Some preliminary empirical work suggests that the model may prove quite useful for understanding interesting trends in compensation. For example, empirically, CEO pay and firm size have become more closely associated in cross-section data (see Frydman and Saks 2010). In the model, a closer association follows as the equilibrium response to demand growth. We find that in the twelve Fama-French industries, those in which the association between CEO pay and size has increased are also those in which industry sales growth has been greater.

## Appendix A: The Role of Effort and Incentives

One of the key aspects of the model's predictions and the interaction with the data is that firm size and CEO pay are extremely convex functions of managerial ability. This convexity is in part due to the *Superstars Effect*, but it is much exaggerated by the inclusion of an agency problem. In this section, we study a slightly different version of our model that shows very cleanly how effort and incentives interact with the Superstars idea to exaggerate convexity.

In our model, the assumptions that effort is costly and managers are risk averse, and thus, other things equal, larger firms face more inherent risk, play an important role in determining firm size. With those assumptions, it is not necessary to include any other factor bounding firm size (e.g., organizational inefficiencies leading to decreasing returns to scale). Thus, to begin with a base case where ability matters but agency considerations are absent, our basic model must be adjusted to allow some other factor bounding firm size. A simple specification that accomplishes this is a form of diminishing returns to the number of workers, which, absent effort, leads to a expected firm profits from operations being given by

$$np^*a^\eta - n^2 w, \tag{A1}$$

where $p^*$ is the equilibrium price (whose determination can be ignored at this point because the focus is cross-sectional variation) and $0 < \eta < 1$ parameterizes ability. Under this assumption, the optimal number of projects is given by

$$n^*(a) = \frac{p^*a^\eta}{2w}. \tag{A2}$$

Free entry of firms implies that the manager's total compensation is

$$T^*(a) = p^*n^*(a)a^\eta - (n^*(a))^2 w = \frac{p^{*2}a^{2\eta}}{4w}, \tag{A3}$$

i.e., *linear* in $a^{2\eta}$. Thus, $T^*(a)$ is strictly convex in $a$ provided $\eta > \frac{1}{2}$, i.e., returns to ability do not diminish so quickly that the Superstars effect is wiped out by decreasing returns.[24]

The comparative statics results with respect to demand and workers' wage shocks are similar to that for the main model: A positive demand shock increases firm size and total pay, while a positive wage shock leads to higher output price, smaller firm size, and has an ambiguous impact on total pay (which tends to be more positive when output demand is less elastic). Of course, because there is no effort, there are no implications for effort-related items: split between salary and incentive compensation, strength of incentives $s_1$, effort levels $e^*$ and $E^*$, and all the comparative statics results with respect to risk-aversion $\gamma\sigma^2$ and cost of effort $\xi$.

Allowing for effort, the firm's expected profits from operations become

$$p^*n(ae)^\eta - n^2 w.$$

Proceeding as above, the agent chooses managerial effort $e(a)$ to maximize

$$s_1 p^* n(ae)^\eta - \frac{1}{2}\xi ne.$$

The optimal effort choice satisfies

$$e^*(a) = \left(\frac{2\eta s_1 p^*}{\xi}\right)^{\frac{1}{1-\eta}} a^{\frac{\eta}{1-\eta}}. \tag{A4}$$

24 Compare the expressions (A1), (A2), and (A3) for firm profit, size, and total compensation to those for the base model, where profit is given by $np^*\sqrt{ae} - nw$, firm size by (10), and total compensation by (17).

The firm chooses $n$ and $s_1$ to maximize

$$\pi(a) = p^* n(ae^*(a))^\eta - n^2 w - \frac{1}{2}\xi n e^*(a) - \frac{1}{2}\gamma s_1^2 n^2 p^{*2}\sigma^2 - c^*(a). \tag{A5}$$

Employing the first-order conditions, we obtain below that $n^*(a)$ is linear in $a^{\frac{\eta}{1-\eta}}$ and $s_1^*(a)$ is independent of $a$. The derivations work as follows. Substituting (A4) into (A5), we have

$$\pi(a) = p^* n \left(\frac{2\eta s_1 p^*}{\xi}\right)^{\frac{\eta}{1-\eta}} a^{\frac{\eta}{1-\eta}} - n^2 w - \frac{1}{2}\xi n \left(\frac{2\eta s_1 p^*}{\xi}\right)^{\frac{1}{1-\eta}} a^{\frac{\eta}{1-\eta}} - \frac{1}{2}\gamma s_1^2 n^2 p^{*2}\sigma^2 - c^*(a).$$

The first-order condition with respect to $s_1$ is

$$\frac{\partial \pi(a)}{\partial s_1} = p^* n \frac{\eta}{1-\eta}\left(\frac{2\eta p^*}{\xi}\right)^{\frac{\eta}{1-\eta}} s_1^{\frac{\eta}{1-\eta}-1}(1-s_1)\, a^{\frac{\eta}{1-\eta}} - \gamma s_1 n^2 p^{*2}\sigma^2 = 0,$$

which yields the formula for $n^*(a)$:

$$n^*(a) = \frac{\eta}{1-\eta}\frac{1}{\gamma p^* \sigma^2}\left(\frac{2\eta p^*}{\xi}\right)^{\frac{\eta}{1-\eta}} s_1^{\frac{\eta}{1-\eta}-2}(1-s_1)a^{\frac{\eta}{1-\eta}}. \tag{A6}$$

The first-order condition with respect to $n$ is

$$\frac{\partial \pi(a)}{\partial n} = p^* \left(\frac{2\eta s_1 p^*}{\xi}\right)^{\frac{\eta}{1-\eta}} a^{\frac{\eta}{1-\eta}} - 2nw - \gamma s_1^2 n p^{*2}\sigma^2 - \frac{1}{2}\xi \left(\frac{2\eta s_1 p^*}{\xi}\right)^{\frac{1}{1-\eta}} a^{\frac{\eta}{1-\eta}} = 0. \tag{A7}$$

Substituting (A6) into (A7), each term contains $a^{\frac{\eta}{1-\eta}}$, which can be canceled. As a result, $s_1^*(a)$ is independent of $a$.

The free entry condition implies that total managerial compensation ($T^*(a)$) is equal to expected profit: $p^* n^*(a)(ae^*(a))^\eta - (n^*(a))^2 w$, which is linear in $a^{\frac{2\eta}{1-\eta}}$. Even if returns to ability diminish so quickly that the Superstars effect vanishes (i.e., $\eta \le \frac{1}{2}$), the manager's total compensation is convex in $a$ as long as $\eta \ge \frac{1}{3}$.[25]

The intuition behind the result that the agency problem exaggerates the convexity of firm size and managerial compensation is straightforward. With agency, because number of projects, ability, and effort are all complements, firms that hire more able managers choose to grow larger and grant managers with lower profit shares. The endogenous choice of incentives permits these firms to choose even more projects, translating into much more rapidly increasing profits and managerial compensation.

To improve our illustration of the role of effort in generating greater convexity, we next provide numerical simulations for the two models developed above. We assume $\eta = 0.5$. For consistency with simulations performed in Section 4, we assume a uniform distribution for ability and use wage $w = \$370{,}000$. Both model versions have enough free parameters to allow us to generate the mean of firm size $n^*$ that reflects the average number of employees in our sample of firms analyzed in Section 4: Mean$(n^*) = 38.61$ (in thousands). Given these parameter values, the model without effort has a standard deviation of 12.41 (in thousands) for firm size. Because the model with effort has more parameters, the set of parameters that generates the desired Mean$(n^*)$ is not unique, and thus the standard deviation of $n^*$ is not unique. The standard deviation can get arbitrarily close to $38.61/\sqrt{3}$ (in thousands) from below, which is approximately 50% larger

25 While the calculations are considerably more tedious, very similar conclusions follow if, instead, we assume $\pi(a) = p^* n a^\eta e^{1-\eta} - n^2 w$, where $0 < \eta < 1$.

than the standard deviation produced by the model without effort. The results are similar for total compensation $T^*$.

## Appendix B: Proofs

For many of the following proofs, it is convenient to first rewrite the output market equilibrium condition #5 using expressions (10)–(12), and the indifference condition of the marginal agent (14) using expression (13):

$$H \equiv -\alpha + \beta p + \int_{\bar{a}}^{\infty} \frac{a}{\xi\gamma\sigma^2}\left(\sqrt{\frac{ap^2}{2\xi w}} - 1\right)\sqrt{\frac{2aw}{\xi}}\mu(da) = 0, \tag{A8}$$

$$G \equiv \frac{\bar{a}}{\xi\gamma\sigma^2}\left(\sqrt{\frac{\bar{a}p^2}{2\xi w}} - 1\right)^2 - 1 = 0. \tag{A9}$$

To prove Propositions 2–6, we first show that there exists a unique equilibrium.

**Theorem 1.** There exists a unique equilibrium, as defined by Definition 1.

*Proof.* When the workers' wage is exogenous, we have shown that, given product price $p^*$ and marginal ability level $\bar{a}$, there is a unique solution for all other variables: $n^*$, $s_0^*$, $s_1^*$, and $e^*$. From (A8), note that when $p = 0$ we have $H < 0$, and when $p = +\infty$ we have $H = +\infty$. Because $\frac{\partial H}{\partial p} > 0$, there exists a unique solution for $p^*$ given $\bar{a}$. Because $H$ decreases with $\bar{a}$: $\frac{\partial H}{\partial \bar{a}} < 0$, the output price $p^*$ that solves (A8) is higher in a solution with higher $\bar{a}$. From (A9), note that when $\bar{a} = 0$ we have $G = -1 < 0$, and when $\bar{a} = +\infty$ we have $G = +\infty$. Because $\frac{\partial G}{\partial \bar{a}} > 0$, there exists a unique solution for $\bar{a}$ given $p$. Additionally, because $G$ is also increasing in $p$: $\frac{\partial G}{\partial p} > 0$, the value of $\bar{a}$ that solves (A9) is lower in a solution with higher $p$. Thus, there is a unique pair of $\bar{a}$ and $p^*$ that solve (A8) and (A9), and therefore the equilibrium is unique. ∎

**Proof of Proposition 2.**

*Proof.* Using (A8) and (A9), the derivatives $\frac{dp^*}{d\alpha}$ and $\frac{d\bar{a}}{d\alpha}$ can be found from the following system of equations:[26]

$$\frac{\partial H}{\partial \alpha} + \frac{\partial H}{\partial \bar{a}}\frac{d\bar{a}}{d\alpha} + \frac{\partial H}{\partial p}\frac{dp^*}{d\alpha} = 0$$
$$\frac{\partial G}{\partial \alpha} + \frac{\partial G}{\partial \bar{a}}\frac{d\bar{a}}{d\alpha} + \frac{\partial G}{\partial p}\frac{dp^*}{d\alpha} = 0.$$

Because the denominator is

$$-\det\begin{pmatrix} \frac{\partial H}{\partial \bar{a}} & \frac{\partial H}{\partial p} \\ \frac{\partial G}{\partial \bar{a}} & \frac{\partial G}{\partial p} \end{pmatrix} > 0,$$

the sign of $\frac{dp^*}{d\alpha}$ is the same as the sign of

$$\det\begin{pmatrix} \frac{\partial H}{\partial \bar{a}} & \frac{\partial H}{\partial \alpha} \\ \frac{\partial G}{\partial \bar{a}} & \frac{\partial G}{\partial \alpha} \end{pmatrix} > 0,$$

26 In all equations that follow, the derivatives $\frac{\partial H}{\partial p}$ and $\frac{\partial G}{\partial p}$ are calculated at the equilibrium value $p^*$ of the product price: $\frac{\partial H}{\partial p}\Big|_{p=p^*}$, $\frac{\partial G}{\partial p}\Big|_{p=p^*}$.

and the sign $\frac{d\bar{a}}{d\alpha}$ is the same as the sign of

$$\det\begin{pmatrix} \frac{\partial H}{\partial \alpha} & \frac{\partial H}{\partial p} \\ \frac{\partial G}{\partial \alpha} & \frac{\partial G}{\partial p} \end{pmatrix} < 0.$$

Similarly, the sign of $\frac{dp^*}{d\beta}$ is the same as the sign of

$$\det\begin{pmatrix} \frac{\partial H}{\partial \bar{a}} & \frac{\partial H}{\partial \beta} \\ \frac{\partial G}{\partial \bar{a}} & \frac{\partial G}{\partial \beta} \end{pmatrix} < 0,$$

and the sign $\frac{d\bar{a}}{d\beta}$ is the same as the sign of

$$\det\begin{pmatrix} \frac{\partial H}{\partial \beta} & \frac{\partial H}{\partial p} \\ \frac{\partial G}{\partial \beta} & \frac{\partial G}{\partial p} \end{pmatrix} > 0.$$

The direction of the change in $n^*(a)$, $s_1^*(a)$, $c^*(a)$, $s_0^*(a)$, $I^*(a)$, $T^*(a)$, and $E^*(a)$ follow directly from the equilibrium expressions (10)–(18) because price $p^*$ is increasing in $\alpha$ and decreasing in $\beta$ and everything else in these expressions remains constant.

We next show that the increase in price $p^*$ implies that the first-order derivatives of all these equilibrium entities increase. The second-order derivatives of all but $s_1^*(a)$ increase, while the second-order derivative of $s_1^*(a)$ decreases as demand increases.

The first-order derivatives of (10)–(11), (13), and (15)–(17) with respect to $a$ can be expressed as

$$\frac{\partial n^*(a)}{\partial a} = \frac{1}{\xi\gamma\sigma^2}\left(\frac{3}{2}\sqrt{\frac{ap^{*2}}{2\xi w}} - 1\right)$$

$$\frac{\partial s_1^*(a)}{\partial a} = -\frac{1}{2}\sqrt{\frac{2\xi w}{a^3 p^{*2}}}$$

$$\frac{\partial c^*(a)}{\partial a} = \frac{w}{\xi\gamma\sigma^2}\left(\sqrt{\frac{ap^{*2}}{2\xi w}} - 1\right)\left(\sqrt{\frac{2ap^{*2}}{2\xi w}} - 1\right)$$

$$\frac{\partial s_0^*(a)}{\partial a} = \frac{w}{\xi\gamma\sigma^2}\left(\left(\sqrt{\frac{ap^{*2}}{2\xi w}} - 1\right)^2\left(2 - \sqrt{\frac{\xi w}{2ap^{*2}}}\right) + \left(\sqrt{\frac{ap^{*2}}{2\xi w}} - 1\right)\left(\sqrt{\frac{2ap^{*2}}{\xi w}} - 1\right)\right)$$

$$\frac{\partial I^*(a)}{\partial a} = \frac{w}{\xi\gamma\sigma^2}\left(\left(1 - \sqrt{\frac{2\xi w}{ap^{*2}}}\right)\left(3\sqrt{\frac{ap^{*2}}{2\xi w}} - 1\right) + \left(1 - \sqrt{\frac{\xi w}{2ap^{*2}}}\right)\right)$$

$$\frac{\partial T^*(a)}{\partial a} = \frac{w}{\xi\gamma\sigma^2}\left(\left(\sqrt{\frac{ap^{*2}}{2\xi w}} - 1\right)\left(3\sqrt{\frac{ap^{*2}}{2\xi w}} - 1\right) + \frac{1}{4}\sqrt{\frac{ap^{*2}}{2\xi w}}\left(\sqrt{\frac{ap^{*2}}{2\xi w}} - 2\right)\right).$$

All the first-order derivatives given above are increasing in $p^*$, implying that the slope of each of the variables increases when demand increases.

The second-order derivatives of (10)–(11), (13), and (15)–(17) with respect to $a$ can be expressed as

$$\frac{\partial^2 n^*(a)}{\partial a^2} = \frac{3}{4}\frac{1}{\xi\gamma\sigma^2}\sqrt{\frac{p^{*2}}{2a\xi w}}$$

$$\frac{\partial^2 s_1^*(a)}{\partial a^2} = \frac{3}{4}\sqrt{\frac{2\xi w}{a^5 p^{*2}}}$$

$$\frac{\partial^2 c^*(a)}{\partial a^2} = \frac{w}{\xi\gamma\sigma^2}\sqrt{\frac{p^{*2}}{2a\xi w}}\left(\sqrt{\frac{2ap^{*2}}{\xi w}} - \frac{3}{2}\right)$$

$$\frac{\partial^2 s_0^*(a)}{\partial a^2} = \frac{w}{\xi\gamma\sigma^2}\left(\left(\sqrt{\frac{ap^{*2}}{2\xi w}} - 1\right)\left(3\sqrt{\frac{p^{*2}}{2a\xi w}} - \frac{1}{2}\sqrt{\frac{\xi w}{2a^3 p^{*2}}} - \frac{1}{4a}\right)\right.$$
$$\left. + \frac{1}{2}\sqrt{\frac{p^{*2}}{2a\xi w}}\left(\sqrt{\frac{2ap^{*2}}{\xi w}} - 1\right)\right)$$

$$\frac{\partial^2 I^*(a)}{\partial a^2} = \frac{w}{\xi\gamma\sigma^2}\left(\frac{3}{2}\sqrt{\frac{p^{*2}}{2a\xi w}} - \frac{1}{4}\sqrt{\frac{2\xi w}{a^3 p^{*2}}}\right)$$

$$\frac{\partial^2 T^*(a)}{\partial a^2} = \frac{w}{\xi\gamma\sigma^2}\sqrt{\frac{2p^{*2}}{a\xi w}}\left(\sqrt{\frac{2ap^{*2}}{\xi w}} - \frac{9}{8}\right).$$

The second-order derivatives of all variables above except that for $s_1^*(a)$ are positive and increasing with $p^*$, implying that the curvature of these corresponding variables increases as demand increases. For $s_1^*(a)$, a variable decreasing in ability $a$, the curvature decreases as demand increases. The optimal total effort is $E^*(a) = n^*(a)e^*$. By (12), $e^*$ is independent of $a$, and thus $E^*(a)$ has the same property as $n^*(a)$. ∎

**Proof of Proposition 3**

*Proof.* Rewrite the product market equilibrium condition given in (A8) by substituting for the equilibrium expressions and obtain

$$\alpha - \beta p^* = \int_{\bar{a}_\mu}^{\infty} n_\mu^*(a)\sqrt{ae_\mu^*(a, s_{1,\mu}^*(a))}\mu(da), \tag{A10}$$

where the left-hand side is the demand for and the right-hand side is the supply of product. The marginal agent (with ability $\bar{a}_\mu$) has a certainty equivalent equal to the workers' wage. Dividing both sides of the indifference condition (14) by $w$ (substituting (13) into (14)), we have

$$\frac{\bar{a}_\mu}{\xi\gamma\sigma^2}\left(\sqrt{\frac{\bar{a}_\mu p^{*2}}{2\xi w}} - 1\right) = 1. \tag{A11}$$

Equations (19) and (A10) imply that under the original price $p^*$, there is an excess supply when the ability measure changes from $\mu$ to $\hat{\mu}$. Because, by (A10) and (A11), demand for product decreases with product price and supply of product increases with the price, the equilibrium price $p^*$ should go down to clear the product market. Thus, the shock to the ability measure given by (19) has the same impact on firms' equilibrium choices as a negative product demand shock (that leads to a decline in the equilibrium output price) as described in Proposition 2. ∎

**Proof of Proposition 4**

*Proof.* As in the proof of Proposition 2, the derivatives $\frac{dp^*}{dw}$ and $\frac{d\bar{a}}{dw}$ can be obtained by solving the system of equations

$$\frac{\partial H}{\partial w}+\frac{\partial H}{\partial \bar{a}}\frac{d\bar{a}}{dw}+\frac{\partial H}{\partial p}\frac{dp^*}{dw}=0$$

$$\frac{\partial G}{\partial w}+\frac{\partial G}{\partial \bar{a}}\frac{d\bar{a}}{dw}+\frac{\partial G}{\partial p}\frac{dp^*}{dw}=0.$$

Because the denominator is

$$-\det\begin{pmatrix}\frac{\partial H}{\partial \bar{a}} & \frac{\partial H}{\partial p}\\ \frac{\partial G}{\partial \bar{a}} & \frac{\partial G}{\partial p}\end{pmatrix}>0, \tag{A12}$$

$\frac{dp^*}{dw}$ has the same sign as

$$\det\begin{pmatrix}\frac{\partial H}{\partial \bar{a}} & \frac{\partial H}{\partial w}\\ \frac{\partial G}{\partial \bar{a}} & \frac{\partial G}{\partial w}\end{pmatrix}>0,$$

and $\frac{d\bar{a}}{dw}$ has the same sign as

$$\det\begin{pmatrix}\frac{\partial H}{\partial w} & \frac{\partial H}{\partial p}\\ \frac{\partial G}{\partial w} & \frac{\partial G}{\partial p}\end{pmatrix}. \tag{A13}$$

Substituting for the partial derivatives from (A8) and (A9), we obtain that the above determinant is positive, and thus $\frac{d\bar{a}}{dw}>0$. Because $\bar{a}$ is increasing in $w$, (A9) implies that $p^{*2}/w$ is decreasing in $w$. The positive effect of $w$ on $e^*$ follows from (12); $n^*$ is decreasing in $w$ by (10), and $s_1^*$ is increasing in $w$ by (11).

We show how the ambiguous results depend on the demand slope $\beta$ as follows. First, note that $d\bar{a}/dw\equiv R_1/R_2$, where $R_1$ is the determinant given in (A13) and $R_2$ is the denominator given in (A12), only depends on $\beta$ through $\partial H/\partial p=\beta$. After some algebraic simplifications, we obtain that $d(R_1/R_2)/d\beta$ has the same sign as $\frac{\partial G}{\partial p}\left(\frac{\partial G}{\partial w}\frac{\partial H}{\partial \bar{a}}-\frac{\partial H}{\partial w}\frac{\partial G}{\partial \bar{a}}\right)>0$ because $\frac{\partial G}{\partial p}>0$, $\frac{\partial G}{\partial w}<0$, $\frac{\partial H}{\partial \bar{a}}<0$, $\frac{\partial H}{\partial w}<0$, and $\frac{\partial G}{\partial \bar{a}}>0$ by (A8) and (A9). Thus, a larger $\beta$ implies a larger $d\bar{a}/dw$. By (A9), a larger $\beta$ then implies a more negative $d(p^{*2}/w)/dw$. Because $n^*$, $s_0^*$, $I^*$, and $T^*$ only depend on $\beta$ through $p^{*2}/w$, and they all increase in $p^{*2}/w$, changes of all these variables become more negative when $\beta$ is larger, while the opposite is true for $s_1^*$. The same arguments imply that the changes in slope and convexity of all variables but $s_1$ are more negative when $\beta$ is larger.

The rest of the variables may either increase or decrease, as illustrated by the following numerical examples. Assume $\alpha=1$, $\beta=0.25$, $a\sim U[0,1]$, $\gamma=10$, and $\sigma^2=0.25$. When $\xi=1$, increasing wage $w$ from 0.1 to 0.11 increases $\bar{a}$ from 0.3434 to 0.3609, and decreases the salary $s_0^*(1)$ from 2.0775 to 2.0548 and $s_0^*(0.3609)$ (the new $\bar{a}$) from 0.1953 to 0.1897. When $\xi=0.1$, increasing wage $w$ from 0.1 to 0.11 increases $\bar{a}$ from 0.7229 to 0.7342, and increases $s_0^*(1)$ from 0.4411 to 0.4618 and $s_0^*(0.7342)$ (the new $\bar{a}$) from 0.1456 to 0.1505. ■

**Proof of Proposition 5**

*Proof.* Let $g\equiv\gamma\sigma^2$. Similar to the proof of Proposition 4, we can show that $dp^*/dg>0$, $d\bar{a}/dg>0$, and a larger $\beta$ implies a higher $d\bar{a}/dg$ and a more negative $d(p^{*2}/w)/dg$. Because $n^*$, $s_0^*$, $I^*$, and $T^*$ only depend on $\beta$ through $p^{*2}/w$, and they all increase in $p^{*2}/w$, changes of all these variables become more negative when $\beta$ is larger, while the opposite is true for $s_1^*$.

Effort $e^*$ is independent of $g$ by (12). Incentive pay $s_1^*$ decreases with $g$ by (11) because price $p^*$ increases while everything else does not change. The rest of the variables may either increase or decrease, as illustrated in the following numerical examples.

Assume $\xi = 1$, $w = 0.1$, $\alpha = 1$, $a \sim U[0,1]$, and $\sigma^2 = 0.25$. When $\beta = 0.05$, increasing $\gamma$ from 10 to 11 decreases $\bar{a}$ from 0.1504 to 0.1465, and decreases $n^*(1)$ from 4.8377 to 4.7032 and $n^*(0.1504)$ (the initial $\bar{a}$) from 0.2452 to 0.2408; it increases $s_0^*(1)$ from 11.2547 to 11.7295 and $s_0^*(0.1504)$ from 0.1803 to 0.1925. When $\beta = 0.25$, increasing $\gamma$ from 10 to 11 increases $\bar{a}$ from 0.3434 to 0.347, and decreases $n^*(1)$ from 2.1243 to 1.9913 and $n^*(0.347)$ (the new $\bar{a}$) from 0.3772 to 0.3552; it decreases $s_0^*(1)$ from 2.0775 to 2.0125 and $s_0^*(0.347)$ from 0.1774 to 0.1738.

In the numerical examples above, the number of projects $n^*(a)$ decreases with $\gamma$. This, however, is not always the case. When $\beta = 0.001$, increasing $\gamma$ from 10 to 11 reduces $\bar{a}$ from 0.1069 to 0.1014; it decreases $n^*(1)$ from 6.7416 to 6.7284 while increasing $n^*(0.1069)$ (the initial $\bar{a}$) from 0.2068 to 0.2071. ■

**Proof of Proposition 6**

*Proof.* Similar to the proofs of Propositions 4 and 5; available from authors upon request. ■

**Proof of Proposition 7**

*Proof.* We first rewrite the inequality in the proposition by substituting for the equilibrium expressions and obtain

$$\int_{\bar{a}_\mu}^{\infty} \frac{a}{\xi\gamma\sigma^2}\left(\sqrt{\frac{ap_\mu^{*2}}{2\xi w_\mu^*}} - 1\right)\hat{\mu}(da) < \hat{\mu}([0,\bar{a}_\mu]),$$

where the subscript $\mu$ denotes the equilibrium values under the original measure $\mu$. The left-hand side of the inequality (demand for workers) increases in $p^{*2}/w^*$. The right-hand side (supply of workers) decreases in $p^{*2}/w^*$ because $\bar{a}$ decreases in $p^{*2}/w^*$ according to the participation condition of the marginal manager given in (A11). Thus, in the new equilibrium, $p^{*2}/w^*$ increases to clear the labor market. For any given firm, an increase in $p^{*2}/w^*$ increases firm size. An increase in $p^{*2}/w^*$ also leads to a decline in the marginal manager's ability $\bar{a}$ (i.e., new firms enter).

What happens to the equilibrium wage $w^*$ depends on the demand parameters and on the way ability measure changes, which can be shown as follows. Divide both sides of the output market equilibrium condition given in (A8) by $\sqrt{w^*}$ to obtain that the new equilibrium price and wage satisfy

$$-\frac{\alpha}{w^*} + \beta\sqrt{\frac{p^{*2}}{w^*}} + \int_{\bar{a}_\mu}^{\infty} \frac{a}{\xi\gamma\sigma^2}\left(\sqrt{\frac{ap^{*2}}{2\xi w^*}} - 1\right)\sqrt{\frac{2a}{\xi}}\,\hat{\mu}(da) = 0.$$

In the equation above, all terms but the first $(-\alpha/\sqrt{w^*})$ depend on $p^*$ and $w^*$ only through $p^{*2}/w^*$, they all increase in $p^{*2}/w^*$, and we have shown that $p^{*2}/w^*$ increases. Whether the sum of all terms but the first increases or not, however, depends not only on $p^{*2}/w^*$, but also on the change in ability measure (from $\mu$ to $\hat{\mu}$ on $[\bar{a}_\mu, \infty)$). If the measure remains the same on $[\bar{a}_\mu, \infty)$, then the sum goes up, and we can conclude that workers' wage $w^*$ has to go down to clear the excess supply. However, if the measure drops on $[\bar{a}_\mu, \infty)$, the sum of the last two terms may decrease, which would result in an increase of the equilibrium workers' wage $w^*$. Whether the sum decreases depends on both the magnitude of the drop in the measure and the magnitude of the demand slope $\beta$: The larger $\beta$, the stronger the tendency for the sum to increase.

Whether and by how much $w^*$ declines determines whether managerial compensation components $s_0^*(a)$, $I^*(a)$, $T^*(a)$, and certainty equivalent $c^*(a)$ decline (these variables increase in both $p^{*2}/w^*$ and $w^*$; the increase in $p^{*2}/w^*$ causes these variables to increase; however, the decrease in $w^*$ works in the opposite direction and, if strong enough, may overcome the impact of $p^{*2}/w^*$). ■

**Proof of Proposition 8**

*Proof.* Take, for example, $I_\lambda(a)$. The argument works as follows:

$$I_\lambda''(a) = I^{*\prime\prime}(\lambda(a))\left[\lambda'^2(a) + \frac{I^{*\prime}(\lambda(a))}{I^{*\prime\prime}(\lambda(a))}\lambda''(a)\right].$$

The conditions in Proposition 8 suffice for the factor in square brackets to exceed unity. The argument for $s_{0\lambda}(a)$ and $T_\lambda(a)$ is slightly different, i.e.,

$$T_\lambda''(a) = T^{*\prime\prime}(\lambda(a))\lambda'^2(a) + \frac{w}{\xi\gamma\sigma^2}\left(2\sqrt{\frac{\lambda(a)p^{*2}}{2\xi w}} - \frac{3}{2}\right)\sqrt{\frac{\lambda(a)p^{*2}}{2\xi w}}\left[\lambda''(a) - \frac{\lambda'^2(a)}{\lambda(a)}\right] + c^{*\prime\prime}(a).$$

■

## Elasticity of pay with respect to firm size, with capital input

Assume that in addition to hiring one worker at wage $w$, each project also rents capital at price $r \geq 0$.[27] This affects the equilibrium as follows: In all conditions except the one that determines the agent's choice of whether to be a manager or a worker, $w$ is substituted by $w + r$. Agents with ability $a \geq \bar{a}$ still become managers, and all remaining agents become workers. Because workers earn $w$ (rather than $w + r$), $\bar{a}$ is determined by

$$c^*(\bar{a}) = \frac{\bar{a}(w+r)}{\xi\gamma\sigma^2}\left(\sqrt{\frac{\bar{a}p^{*2}}{2\xi(w+r)}} - 1\right)^2 = w. \tag{A14}$$

The new closed-form expressions can be obtained from the original ones given in (10)–(18) by substituting $w$ with $(w + r)$.

In this extension, we are particularly interested in the impact of changes in demand (increase in $\alpha$ or decrease in $\beta$) on elasticity of total compensation, $T^*(a)$, with respect to firm size, $n^*(a)$. For notational convenience, denote

$$M(a) \equiv \frac{T^*(a)}{n^*(a)} = \sqrt{\frac{2ap^{*2}}{\xi(w+r)}} - 1.$$

Then, elasticity of $T^*(a)$ with respect to $n^*(a)$ can be computed as

$$E_{Tn}(a) \equiv \frac{\left(T^*(a)\right)'}{\left(n^*(a)\right)'}\frac{n^*(a)}{T^*(a)} = \frac{\left(n^*(a)M(a)\right)'}{n^{*\prime}(a)}\frac{n^*(a)}{n^*(a)M(a)}$$
$$= 1 + \frac{\left(\sqrt{\frac{ap^{*2}}{2\xi(w+r)}} - 1\right)\sqrt{\frac{ap^{*2}}{2\xi(w+r)}}}{\left(\left(\sqrt{\frac{ap^{*2}}{2\xi(w+r)}} - 1\right) + \frac{1}{2}\sqrt{\frac{ap^{*2}}{2\xi(w+r)}}\right)\left(\sqrt{\frac{2ap^{*2}}{\xi(w+r)}} - 1\right)}.$$

We now examine whether the elasticity increases in product demand (due to increase in $\alpha$ or decrease in $\beta$) for a given ability $a$, with the remaining model parameters fixed. When demand increases, $p^*$ goes up, and so do $w$ and $r$ if endogenous. These variables enter the above elasticity expressions only as a fraction $p^{*2}/(w + r)$, and neither $\alpha$ nor $\beta$ affect the expression in any

27 The version in the text is a special case in which $r = 0$.

other way. Hence, the effect of an increase in demand on the elasticity can be evaluated by calculating the derivative of the above expression with respect to $p^{*2}/(w+r)$. After some algebraic simplifications, the derivatives can be reduced to

$$(E_{Tn}(a))'_{[p^{*2}/(w+r)]} = \frac{\frac{w+r}{p^{*2}}\sqrt{\frac{ap^{*2}}{2\xi(w+r)}}\left(\sqrt{\frac{ap^{*2}}{2\xi(w+r)}} - \frac{1}{4}\frac{ap^{*2}}{2\xi(w+r)} - \frac{1}{2}\right)}{\left(\left(\frac{3}{2}\sqrt{\frac{ap^{*2}}{2\xi(w+r)}} - 1\right)\left(\sqrt{\frac{2ap^{*2}}{\xi(w+r)}} - 1\right)\right)^2}.$$

In the expression above, the denominator is positive. In the numerator, terms outside the parentheses are also positive. Thus, the sign of the above derivative is the same as the sign of the following expression:

$$\sqrt{\frac{ap^{*2}}{2\xi(w+r)}} - \frac{1}{4}\frac{ap^{*2}}{2\xi(w+r)} - \frac{1}{2},$$

which implies that $(E_{Tn}(a))'_{[p^{*2}/(w+r)]}$ is strictly positive if and only if

$$\sqrt{\frac{ap^{*2}}{2\xi(w+r)}} < 2+\sqrt{2}. \tag{A15}$$

This condition holds when price $p^*$ is sufficiently low relative to $w+r$. When input costs $w$ and $r$ are exogenous, an increase in demand leads to an increase in $p^{*2}/(w+r)$, which increases the elasticity if and only if (A15) holds.

When wage and cost of capital are endogenous, the direction of the change in the elasticity depends on the elasticities of labor and capital supply curves. In particular, if the supply of capital is sufficiently more elastic than the supply of labor so that $w^*/(w^*+r^*)$ increases in demand (which we believe to be the case in practice), the marginal manager condition (A14) combined with the labor market clearance condition (20) imply that $p^{*2}/(w^*+r^*)$ also increases in demand.[28] In this case, the elasticity increases in demand if and only if (A15) holds. The derivation of the elasticity of total compensation with respect to sales is similar.

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[28] This can be shown as follows. First, by (20), labor supply increases in $\bar{a}$, and labor demand decreases in $\bar{a}$ and increases in $p^{*2}/(w^*+r^*)$. Therefore, labor market clearance requires that, in response to a demand shock, $\bar{a}$ and $p^{*2}/(w^*+r^*)$ either both increase or both decrease (if, for example, $\bar{a}$ decreases while $p^{*2}/(w^*+r^*)$ increases, demand for labor increases while supply of labor decreases). The marginal manager condition $\frac{w^*}{w^*+r^*} = \frac{\bar{a}}{\xi\gamma\sigma^2}(\sqrt{\frac{\bar{a}p^{*2}}{2\xi(w^*+r^*)}} - 1)^2$ then implies that $\bar{a}$ and $p^{*2}/(w^*+r^*)$ change in the same direction as $w^*/(w^*+r^*)$ (because $c^*(\bar{a})$ increases in both).

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