Inattention, Forced Exercise, and the Valuation of Executive Stock Options *

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Abstract

We develop a new Black-Scholes type closed-form valuation formula for executive stock options. This formula incorporates four important unique characteristics of these options that distinguish them from standard European options: (i) The presence of the vesting period; (ii) the tendency of executives to exercise portions of their grants right at the end of the vesting period; (iii) the ability of the executives to choose optimally whether to exercise their options or keep them; and (iv) executives may be forced to early exercise their options, possibly due to severe liquidity shocks or due to unexpected departure. We use an extensive executive option data set to calibrate our model. We show that the standard Black-Scholes formula significantly overestimates the value of executive stock options.

Keywords: Executive Stock Options, Valuation.

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I. Introduction

In December 2004, the Financial Accounting Standards Board (FASB) issued Revised Statement No. 123 which changed the procedures regarding the expensing of employee stock options. As of June 15, 2005, public companies are required to report employee stock options as expenses on their financial statements. The statement requires that "The grant-date fair value of employee share options and similar instruments will be estimated using option-pricing models adjusted for the unique characteristics of those instruments (unless observable market prices for the same or similar instruments are available)."

While the statement does not specify what is a "fair value," many firms have been using a version of the Black-Scholes (B-S) pricing formula to evaluate their option grants (e.g., using 7 years as the expiration date in B-S formula). This approach raises several concerns. For example, the B-S formula is designed to price European options, but executive stock options (ESOs) are typically American with vesting requirement.[1] More importantly, the B-S approach does not take into account the unique early exercise characteristics of ESOs observed in practice that is important for a fair valuation. For example, it is widely documented that executives exercise a significant portion of their option grants around the vesting date. In addition, executives may be forced to exercise early (after vesting) due to drastic events such as severe liquidity shock and unexpected departure. Finally, after the options have vested, since it is costly for executives to continuously monitor her option positions, she only considers the exercise decision at discrete times.[2] Compared to the B-S

[1] An interesting case demonstrating the weakness of the B-S formula in evaluating ESOs is of Zions Bancorp. This financial firm has received permission from the SEC to issue securities that closely replicate the cash flows of ESOs and sell these securities in an auction. In preliminary auctions conducted in June 2006 and May 2007 the Zions' own ESOs, the auctions generated prices that are significantly lower than the traditional B-S valuations.

[2] While some of these exercise determinants also apply to standard option holders, it has insignificant impact on the valuation of the standard options as long as some marginal holders (who determine the option price) do not behave in this manner. For ESOs, the number of owners is small and the marginal holders are the executives themselves.

approach, these early exercise features may increase or decrease the option value at grant. Since exercise policy is critical for option valuation, a fair valuation approach for ESOs must take into account these unique early exercise characteristics.

In this paper we attempt to develop a better alternative valuation method that is almost as easy to use as the popular B-S formula and captures the unique exercise characteristics that is critical for option valuation. Specifically, we assume that with a positive probability executives will exercise the option on the vesting date. In addition, executives may be forced to exercise the option early after vesting, possibly due to severe liquidity shock or unexpected departure. Given the stochastic nature of these events, we assume that a forced exercise occurs at the first jump time of a Poisson process. Finally, executives may also choose to optimally early exercise her option.[3] However, due to the cost of continuous attention (e.g., Sims (2003), Huang and Liu (2007)), we assume that she only makes such a decision at the jump times of another independent Poisson process which may represent minor liquidity shocks and/or corporate events such as merger and acquisitions that affect her portfolio risks. Then we use the optimal stopping theory to derive a new, B-S type closed-form valuation formula for executive stock options. Like the B-S formula, this formula only needs the computation of the cumulative distribution function of a standard normal random variable and is therefore easy to implement.[4]

We next use the insider trading data with option grants and exercises information to calibrate our model in order to empirically examine the difference between our valuation and the popular B-S valuation of ESOs. Our data consists of a comprehensive set of option grants between 1986 and 1996 (with the vast majority after 1992), as well as a complete history of exercises of these grants from the vesting date until expi-

[3]The optimal exercise of vested options is the major distinctive feature of our valuation formula. This valuation approach is different from the one taken in Cvitanic, Wiener, and Zapatero (2008), who assume an exogenously specified exercise barrier.

[4]Clearly our model does not explain why executives exercise options in such manners and thus is only a reduced form model. On the other hand, this seems sufficient for our purpose of developing a better valuation method and makes it possible to obtain a closed-form formula that is easy to use.

ration. We match these two data sets and thus are able to track the entire history of each grant, and calibrate the parameters required for our valuation formula. We then use the estimated parameter values to calculate the fair value of all stock-options granted before January 1st 1997 for which we can match grant data with exercise data. We find that compared to our formula, the B-S formula significantly overestimates the fair value of the options (by about 6%-32% for high dividend paying firms and about 34%-42% for low dividend paying firms, depending on the assumptions used for the B-S formula). Our valuation is consistent with the observed pattern that the option price in the Zions' ESO auction market is much lower (sometimes more than 40% lower) than what the B-S formula predicts.

This paper contributes to the growing literature on the valuation of executive stock options. As in our paper, the main theme in this literature is that the fair value to the investors should take into account managerial considerations. Lambert, Larcker, and Verrecchia (1991) were the first to point out that the valuation of options from the manager's point of view is different from that of the investors. Carpenter (1998) offers a dynamic, preference-dependent valuation algorithm that relies on the optimal investment/consumption decisions of a risk averse manager. Bettis, Bizjak, and Lemmon (2005) apply Carpeneter's valuation algorithm to examine the effect of different model assumptions on the valuation of ESOs. Carpenter, Stanton, and Wallace (2007) study the optimal exercise policy for a general utility-maximizing executive holding a non-transferable option. Finally, Cvitanic, Wiener, and Zapatero (2008) offer a closed-form valuation formula that relies on a suboptimal exercise barrier and does not take into account the unique exercise characteristics of ESOs.

The paper proceeds as follows. In Section II we describe our model for option exercises. We derive the closed-form valuation formula in Section III. Section IV describes the data and the matching algorithm, and compares the B-S results to the results using our formula. Section V concludes. All the proofs are in the appendix.

II. The Model

Suppose a call option with a strike price of X that vests at time T_v and expires at T_e is granted to an executive. As an approximation to the fixed expiration date, we assume that the option expires at the first jump time τ_e of a Poisson process N_e with intensity $1/T_e$, i.e., $T_e = \tau_e$.[5] Because of the cost of continuous attention (e.g., Sims (2003), Huang and Liu (2007)), the executive only decides whether to exercise her options or not at discrete times. Because the vesting time is the first time that the executive can exercise her options, we assume that she always pays attention and decides whether to exercise her options at vesting. After the vesting time, she only examines her options at the jump times τ of an independent Poisson process N_1 with with intensity λ. The executive may also be forced to exercise due to severe liquidity shock, drastic changes in financial situation, leaving the firm, and option expiration. We assume a forced exercise shock occurs at the first jump time of an independent stochastic process. To capture the cumulative forced exercise shocks that have occurred since grant, we assume a jump of this stochastic process occurs with probability p at the vesting time T_v. After the vesting time T_v, a forced exercise shock only arrives at the first jump time $\hat{\tau}$ of an independent Poisson process N_2 with intensity $\eta > 1/T_e$.[6]

We assume a complete market with a risk free rate r and thus there exists a unique risk neutral measure. In addition, the jump risks are diversifiable and thus not priced. The stock price S_t follows a Geometric Brownian process:

$$dS_t = (r - \delta)S_t dt + \sigma S_t dB_t, \tag{1}$$

where B_t is a one-dimensional Brownian motion under the risk neutral measure, and r and σ are constants. In addition, the stock pays a continuous dividend yield of

[5] Similar arguments to those in Carr (1998) and Liu and Loewenstein (2002) can show that this is a very close approximation for long horizon options. In addition, using the method in Liu and Loewenstein (2002), we can also obtain an analytical series solution to the executive's optimization problem with a deterministic, finite maturity.

[6] Note that the forced exercise shocks include the expiration shock and thus the Poisson process N_2 is effectively the sum of the Poisson process N_e representing expiration and another independent Poisson process representing other forced exercise shocks.

$\delta \geq 0$.

Let $\tau_i, i = 1, 2, 3, ...$, denote the ith jump time of the Poisson process N_1 and define $\mathcal{S} = \{\tau_1, \tau_2, \tau_3, ...\}$ to be the set of feasible stopping times. After the vesting time T_v, the executive's objective is to choose the optimal stopping time τ to solve

$$\sup_{\tau \in \mathcal{S}} \mathrm{E}[e^{-r(\tau \wedge \hat{\tau})}(S_{\tau \wedge \hat{\tau}} - X)], \tag{2}$$

subject to equation (1).

III. The Solution

As in a standard optimal stopping problem, there exist a continuation region and a stopping region in the space of S. Let $f(S)$ denote the option value at the time of vesting if not exercised at vesting. The Hamilton-Jacobi-Bellman (HJB) equation in the continuation region is

$$\frac{1}{2}\sigma^2 S^2 f_{SS} + (r - \delta)S f_S - rf + \eta((S - X)^+ - f) = 0 \tag{3}$$

and the HJB equation in the stopping region is

$$\frac{1}{2}\sigma^2 S^2 f_{SS} + (r - \delta)S f_S - rf + \eta(S - X - f) + \lambda(S - X - f) = 0. \tag{4}$$

Different from the standard optimal stopping problem, the value function f in our setting is *twice* continuously differentiable across the stopping boundary S^*. Intuitively, in a standard optimal stopping problem, as soon as the relevant random variable enters into the stopping region, then the process is stopped and it can never get back into the continuation region. In contrast, in our model, since the executive can only exercise at the jump times of N_1 or N_2, if a jump has not occurred since the stock price enters into the stopping region, the option will remain unexercised. Therefore, the stock price can move back to the continuation region before she can exercise. In particular, at the boundary between the continuation region and the stopping region, the stock price is equally likely to move into the interior of either region before an exercise. Therefore, the value function becomes twice continuously differentiable in our model.

Define

$$\rho(y) = -\frac{1}{2}\sigma^2 y^2 - (r - \delta - \frac{1}{2}\sigma^2)y + r$$

and

$$d(y) = \frac{\log(S_0/y) + (r - \delta + \frac{1}{2}\sigma^2)T_v}{\sigma\sqrt{T_v}}.$$

The following theorem provides the explicit solution to the executive's problem for dividend paying firms.

Theorem 1 *Suppose $\delta > 0$. It is optimal for the executive to exercise the first time that $\frac{S}{X} \geq k^*$, where $k^* \in (1, \infty)$ is the unique solution to*

$$ak^{*\beta_1} + bk^* + c = 0, \tag{5}$$

where constants a, b, c, and β_1 are as defined in (9)–(12) in Appendix. Define $S^* = k^* X$. The option value at grant is

$$\begin{aligned} & P(S_0) \qquad (6) \\ = \; & p[S_0 e^{-\delta T_v} N(d_1) - X e^{-rT_v} N(d_2)] + (1-p)\Big[S_0 e^{-\delta T_v} N(d_3) \\ & -X e^{-rT_v} N(d_4) + B_1 S_0^{\beta_1} e^{-\rho(\beta_1)T_v}(N(d_5) - N(d_6)) \\ & +B_2 S_0^{\beta_2} e^{-\rho(\beta_2)T_v}(N(d_7) - N(d_8)) + \frac{\eta}{\delta + \eta} S_0 e^{-\delta T_v}(N(d_1) - N(d_3)) \\ & -\frac{\eta}{r+\eta} X e^{-rT_v}(N(d_2) - N(d_4)) + C S_0^{\beta_2} e^{-\rho(\beta_2)T_v} N(d_8)\Big] \end{aligned}$$

where $N(\cdot)$ is the cumulative probability function for a standard normal random variable, constants β_2, B_1, B_2, and C are as defined in (16)–(18) in Appendix, and

$$d_1 = d(X), \quad d_2 = d_1 - \sigma\sqrt{T_v}, \quad d_3 = d(S^*), \quad d_4 = d_3 - \sigma\sqrt{T_v},$$

$$d_5 = -d_4 - \sigma\sqrt{T_v}\beta_1, \;\; d_6 = -d_2 - \sigma\sqrt{T_v}\beta_1, \;\; d_7 = -d_4 - \sigma\sqrt{T_v}\beta_2, \;\; d_8 = -d_2 - \sigma\sqrt{T_v}\beta_2.$$

Proof. See Appendix.

The following theorem provides the explicit solution to the executive's problem for non-dividend paying firms.

Theorem 2 *Suppose $\delta = 0$. Then it is never optimal for the executive to early exercise unless forced to and the option value at grant is*

$$\begin{aligned} P(S_0) &= p[S_0 N(d_1) - X e^{-rT_v} N(d_2)] + (1-p)\Big[B_{10} S_0^{\beta_1} e^{-\rho(\beta_1) T_v} N(-d_6) \\ &\quad + S_0 N(d_1) - \frac{\eta}{r+\eta} X e^{-rT_v} N(d_2) + C_0 S_0^{\beta_2} e^{-\rho(\beta_2) T_v} N(d_8)\Big] \end{aligned} \tag{7}$$

where d_1, d_2, d_6, and d_8 are as defined in Theorem 1 with $\delta = 0$ and constants B_{10} and C_0 are as defined in (21)–(20) in Appendix.

Proof. The proof is almost identical to that of Theorem 1 and thus omitted.

This theorem shows that if the underlying stock does not pay dividend, then the executive only exercises the option when she is forced to and thus the absence of continuous attention is irrelevant for option valuation in this case.

Proposition 1 *As λ increases, the threshold stock price S^* increases.*

Proof. See Appendix.

This proposition implies that when the executive examines her option position more frequently, she chooses a higher threshold stock price for exercising. Intuitively, if it is more costly for the executive to examine her option position and thus she only does it infrequently, whenever she examines it, she is more likely to exercise because of the expected longer time to wait until the next examination time.

To help us estimate the parameters η and λ, we next compute the expected time to exercise. Let the expected time to exercise from the vesting time be

$$g(S) = E[\tau^* \wedge \hat{\tau}].$$

Let S^* be as defined in Theorem 1 and μ be the expected return of the stock. Under the objective measure, the stock price S_t follows a Geometric Brownian process

$$dS_t = \mu S_t dt + \sigma S_t d\hat{B}_t, \tag{8}$$

where $\hat{B}_t$ is a one-dimensional Brownian motion under the objective measure. Then $g(S)$ satisfies

$$\frac{1}{2}\sigma^2 S^2 g''(S) + \mu S g'(S) - \eta g(S) + 1 = 0, \quad \text{for} \quad S < S^*$$

and

$$\frac{1}{2}\sigma^2 S^2 g''(S) + \mu S g'(S) - (\lambda + \eta) g(S) + 1 = 0, \quad \text{for} \quad S \geq S^*.$$

Then we have

Proposition 2 *Suppose $\mu > \frac{1}{2}\sigma^2$. Then*

$$g(S) = \begin{cases} -\frac{\lambda}{\eta(\lambda+\eta)(1-k_+/k_-)} \left(\frac{S}{S^*}\right)^{k_+} + \frac{1}{\eta} & \text{if } S < S^* \\ -\frac{\lambda\, k_+/k_-}{\eta(\lambda+\eta)(1-k_+/k_-)} \left(\frac{S}{S^*}\right)^{k_-} + \frac{1}{\lambda+\eta} & \text{if } S \geq S^*, \end{cases}$$

where

$$k_+ = \frac{-(\mu - \frac{1}{2}\sigma^2) + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\eta\sigma^2}}{\sigma^2} > 0$$

and

$$k_- = \frac{-(\mu - \frac{1}{2}\sigma^2) - \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2(\lambda + \eta)\sigma^2}}{\sigma^2} < 0.$$

PROOF. See Appendix.

This proposition shows that in the continuation region (i.e., $S < k^* X$), the expected time is shorter than $1/\eta$ and in the stopping region (i.e., $S \geq k^* X$), the expected time is longer than $1/(\lambda + \eta)$. Intuitively, conditional on being in the continuation region, the option will be exercised either because a forced exercise shock arrives (with an expected time of $1/\eta$) or because stock price moves into the stopping region and the executive examines her option position. Therefore the expected time is shorter than $1/\eta$. Conditional on being in the stopping region, the option will be exercised either because a forced exercise shock arrives or because she examines her option position (the expected time of either occurs is $1/(\lambda + \eta)$). However, there is a positive probability that before either of these events occur, the stock price moves back to the continuation region. Therefore the expected time is longer than $1/(\lambda+\eta)$.

IV. Data and Calibrations

Stock option grant and exercise data is from the *Thomson Financial Insiders* database. We use the *Table Two* file that contains open market derivatives transactions as well as information on the award, exercise, and expiration of stock options of corporate insiders (officers, directors, affiliates, beneficiary owners, and others such as founder and trustee). Filers must report the type of option involved, number of shares involved, strike price (how much it costs the insider to exercise each option), date on which the options vest, date on which the options expire, and holdings for that particular series of options.

We focus on the options granted to and exercised by firm CEOs.[7] The database started in 1996 and has the earliest transaction in January 1986. We use option grants before January 1, 1997 to have the opportunity to observe the entire life of the options (typically expire 10 years from the date of the grants). We start with 3,266 grants that have available vesting dates, strike prices, and stock prices on the exercise dates in the CRSP database.

The database organizes grants and exercises in separate records.[8] We match an option grant with an option exercise if the two are associated with the same firm and the same executive, are less than one month apart on both the vesting date and expiration date, and are less than 5% apart in the strike price. Using these criteria,

[7]Derivative in {“EMPO”, “ISO”, “OPTNS”, “CALL”, “NONQ”, “DIRO”}, cleansed, shares_adj>=100, (trancode=“A” and acqdisp=“A”) or (trancode=“M” and acqdisp=“D”) or (trancode=“J” and acqdisp=“D”) or (trancode=“H” and acqdisp=“D”) or (trancode=“J” and acqdisp=“A”).

[8]Matching grants with exercises is in general not a trivial task; see discussions in Bettis, Bijzak, and Lemmon (2005).

we get 779 matches, corresponding to 510 unique grants from 300 firms. On average, one grant matches with 1.5 exercises. There are 168 grants that are only partially exercised (the number of options in the grant is greater than the total number of options in the matched exercises). The part of unmatched options is assumed to get expired without exercises. We append the 168 expired portions to the 779 matched exercises, and further extract information on the average stock return, volatility and average dividend yield over the 60 months prior to grants, and information on these variables in the interval between vesting and exits (when options got either exercised or expired), as well as the average risk-free rate over the 60 months prior to grants (using 5-year Treasury Bonds). In the final sample, there are totally 832 grants with all information needed, corresponding to 462 unique grants from 280 firms.

We further divide these 832 grants into two groups in terms of the average dividend yield of the firm during 1991-2006, using 2% as the cut off point. There are 293 grants from 79 high-dividend yield firms and 539 grants from 201 low dividend yield firms. Table 1 summarizes the characteristics of the two groups. The average stock return as well as the volatility of low-dividend yield firms are higher than those of the high-dividend yield firms. Grants from firms with low dividend yields tend to be exercised later than ones from firms with high dividend yields. However, grants from low dividend paying firms have a much higher probability of getting exercised during the vesting window. The value of p for low dividend paying firms is 7.059% and for high dividend paying firms is $p = 0.919\%$.

To structurally estimate the arrival rates of the hard shock and soft shock, η and λ, we use Proposition 2. This proposition gives us a closed form formula for the expected time from the vesting date of the option to exercise or expiration. This provides us with only one equation but two unknowns: η and λ. To get an estimate of these two parameters we divide each one of the two groups into two. Consider first the high dividend paying firms (those with dividend yield above 2%). We divide them into a group of firms with a dividend yield of 3% and above, and those with a dividend yield between 2% and 3%. For each one of these two sub-groups, Proposition 2 specifies a relation between η and λ and the expected time from vesting to exercise/expiration. We equate this expected time to the average time obtained from the data, and thus have two equations with two unknowns. We solve these equations numerically and obtain that $\eta = 0.14$ and $\lambda = 0.58$. Repeating the same process for low dividend paying firms, we split them to two sub-groups based on whether the dividend yield is above or below 1%. We obtain that $\eta = 0.20$ and $\lambda = 0.05$.

Note that these results are quite reasonable. The median time from vesting to expiration in our sample is about 9 years. The value of $\eta = 0.14$ means that hard shocks (including expirations) occur in high dividend paying firms about once every 7.14 years. This means that a non-negligible portion of early exercises result from hard shocks. The value of $\lambda = 0.58$ means that soft shocks occur about once every 1.7 years in high dividend paying firms. By contrast, the value of $\lambda = 0.05$ means that soft shocks are very rare in low dividend paying firms. Apparently, inattention is not very costly to the manager when the firm is paying low dividends, lowering the frequency of monitoring these options. For these firms, the frequency of the hard shocks (about once every 5 years) is responsible for almost all of the early exercises.

We now turn to comparing option values estimated by our approach with the Black-Scholes (B-S) value. When applying the B-S formula one has some flexibility on the value used for the time to maturity. Obviously, the time from grant to expiration (typically 10 years) greatly overestimates this value. We adopt two alternative approaches. One uses the average time between grant and exercise/expiration in the data, and the other simply uses seven years (this naive approach is used in the calculation of B-S values in the *ExecuComp* database). The results are summarized in Table 2.

The B-S value using the average time to exits in the data ($BS(Te)$) is lower than that using the 7-year time to maturity ($BS(7)$) because in our sample the average (and the median) time between grants and exits is shorter than seven years. Our option value is lower than both B-S values, and this is particularly true for the low-dividend yield firms. For the high-dividend yield firms, the median of our option value is 5.9% lower than the median of $BS(Te)$ and is 31.8% lower than the median of $BS(7)$. For the low-dividend yield firms, the median of our option value is 33.7% lower than the median of $BS(Te)$ and is 41.9% lower than the median of $BS(7)$.

V. Conclusions

The recent FASB requirement of expensing stock options at fair value has highlighted the importance of the valuation of these compensation contracts. In practice, a slightly modified version of the standard Black-Scholes formula is widely used for expensing. Several other methods have been proposed by the existing literature. However, all of these valuation methods do not fully take into account the unique exercise characteristics of executive options that are critical for fair valuation. In addition, some of these valuation are preference dependent, which makes it hard to implement. In this paper, we develop a new closed-form valuation formula of the Black-Scholes type that is preference independent and thus easy to use. The implementation of our formula only requires the estimation of three parameters that are implied by the exercise frequency over the lifetime of the options. We use an extensive data set of matched options grants and exercises to structurally estimate the three parameters by examining the entire life of the options from grant to expiration. Using these estimates we find that the B-S formula overestimates the fair value of the options by 6% – 42%.

Our proposed formula incorporates many of the important features that are unique to executive options and is straightforward to implement. We believe that this parsimonious modeling approach captures many of complexities of real life decisions associated with the early exercises of ESOs and makes the valuation a significant step toward being "fair".

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Table 1: Summary Statistics

High dividend firms are firms with average dividend yields greater than 2% during 1991-2006. *"60 mons prior"* means the time interval of 60 months prior to the grant date. *vesting to exit* means the time interval between the vesting date and the exercise date or the expiration date depending on whichever occurs earlier. T(grant to vesting) is the time interval between the grant date and vesting date of the option. T (vesting to expiration) is the time interval between the vesting date and the expiration date. T(vesting to exits) is the time interval between the vesting date and the exercise/expiration date. *Return* (with dividends reinvested) and *volatility* are annual stock returns and volatility derived from the *CRSP* database. *T-Bond rate* (*60 mons prior*) is the average annual yield of 5-year treasury bond in the period 60 months prior to the grant date. *Exercises within the vesting window* indicates exercises of options within a 3-month window after the vesting date.

	High-dividend firms (dividend yield > 0.02)	**Low-dividend firms** (dividend yield ≤ 0.02)
# of matches	293	539
# of unique grants	169	293
# of unique firms	79	201
Mean (median) dividend yield (1991-2006)	0.035 (0.032)	0.005 (0.002)
Mean (median) T(grant to vesting)	1.103(1.000)	1.077(1.000) yrs
Mean (median) T(vesting to expiration)	7.192 (8.151)	7.736 (9.005) yrs
Mean (median) T(vesting to exit)	3.843 (3.043)	4.689 (4.705) yrs
Exercises within the vesting window	0.919%	7.059%
Mean (median) return (60 mons prior)	0.154 (0.146)	0.202 (0.162)
Mean (median) dividend yield (60 mons prior)	0.044 (0.045)	0.008 (0.000)
Mean (median) volatility (60 mons prior)	0.213 (0.197)	0.412 (0.347)
Mean (median) T-Bond rate (60 mons prior)	0.063 (0.062)	0.063 (0.062)
Mean (median) return (vesting to exit)	0.212 (0.154)	0.338 (0.274)
Mean (median) dividend yield (vesting to exit)	0.039 (0.031)	0.007 (0.000)
Mean (median) volatility (vesting to exit)	0.258 (0.246)	0.464 (0.402)

Table 2: Option values: our approach versus B-S formula

High dividend means that the firm has an average dividend yield greater than 2% during the period of 1991-2006. V is the option value given by our valuation formula. In the valuation, we use $\eta = 0.14$, $\lambda = 0.58$, and $p = 0.919\%$ for firms with high dividend yields and $\eta = 0.20$, $\lambda = 0.05$, and $p = 7.059\%$ for firms with low dividend yields. $BS(\text{Te})$ is the B-S value using the average time between grant and exercise/expiration in the data, and $BS(7)$ is the B-S value using seven years as the time to maturity. Values in the table are the relative difference between the option value predicted by our approach and that given by the B-S formula. *Observations* is the number of option grants used in the valuation comparisons.

	$\frac{V-BS(Te)}{BS(Te)}$		$\frac{V-BS(7)}{BS(7)}$	
	High dividend	**Low dividend**	**High dividend**	**Low dividend**
Max	0.941	0.728	0.448	0.698
Min	-0.772	-0.894	-0.730	-0.898
Mean	-0.011	-0.271	-0.286	-0.458
Median	-0.059	-0.337	-0.318	-0.419
Std.	0.379	0.339	0.193	0.215
Observations	168	293	168	293

Appendix

In this Appendix, we collect the proofs for the analytical results.

PROOF OF THEOREM 1. We prove Theorem 1 through a series of lemmas. First, we define several constants to be used below:

$$a = \frac{\eta((r-\delta)\beta_2 - r - \eta)}{(r+\eta)(\delta+\eta)}, \tag{9}$$

$$b = -\frac{\delta(\alpha(\delta+\eta) + \lambda - \beta_2(\delta+\eta+\lambda))}{(\delta+\eta)(\delta+\eta+\lambda)}, \tag{10}$$

$$c = \frac{r((r+\eta)(\alpha-\beta_2) - \lambda\beta_2)}{(r+\eta)(r+\eta+\lambda)}, \tag{11}$$

$$\beta_1 = \frac{-(r-\delta-\frac{1}{2}\sigma^2) - \sqrt{(r-\delta-\frac{1}{2}\sigma^2)^2 + 2(r+\eta)\sigma^2}}{\sigma^2}, \tag{12}$$

$$\beta_2 = \frac{-(r-\delta-\frac{1}{2}\sigma^2) + \sqrt{(r-\delta-\frac{1}{2}\sigma^2)^2 + 2(r+\eta)\sigma^2}}{\sigma^2} \tag{13}$$

and

$$\alpha = \frac{-(r-\delta-\frac{1}{2}\sigma^2) - \sqrt{(r-\delta-\frac{1}{2}\sigma^2)^2 + 2(r+\eta+\lambda)\sigma^2}}{\sigma^2}, \tag{14}$$

$$A = k^{*-\alpha}\left(\frac{\delta}{\delta+\eta+\lambda}k^* - \frac{r}{r+\eta+\lambda}\right)X^{1-\alpha}, \tag{15}$$

$$B_1 = \frac{a}{\beta_1 - \beta_2}X^{1-\beta_1}, \tag{16}$$

$$B_2 = -k^{*-\beta_2}\left(a_1 k^* - b_1\right)X^{1-\beta_2}, \tag{17}$$

where

$$a_1 = \frac{\delta((\alpha-\beta_1)(\delta+\eta) + (1-\beta_1)\lambda)}{(\beta_2-\beta_1)(\delta+\eta)(\delta+\eta+\lambda)}, \quad b_1 = \frac{r((\alpha-\beta_1)(r+\eta) - \lambda\beta_1)}{(\beta_2-\beta_1)(r+\eta)(r+\eta+\lambda)}$$

$$C = B_2 + \frac{\eta(r - r\beta_1 + \beta_1\delta + \eta)}{(\beta_2-\beta_1)(r+\eta)(\delta+\eta)}X^{1-\beta_2}, \tag{18}$$

$$B_{10} = \frac{X^{1-\beta_{10}}(r - r\beta_{20} + \eta)}{(\beta_{20}-\beta_{10})(r+\eta)}, \tag{19}$$

and

$$C_0 = \frac{X^{1-\beta_{20}}(r - r\beta_{10} + \eta)}{(\beta_{20}-\beta_{10})(r+\eta)}, \tag{20}$$

where

$$\beta_{10} = \frac{-(r - \frac{1}{2}\sigma^2) - \sqrt{(r - \frac{1}{2}\sigma^2)^2 + 2(r+\eta)\sigma^2}}{\sigma^2}, \tag{21}$$

$$\beta_{20} = \frac{-(r - \frac{1}{2}\sigma^2) + \sqrt{(r - \frac{1}{2}\sigma^2)^2 + 2(r+\eta)\sigma^2}}{\sigma^2} \tag{22}$$

Lemma 1 *Suppose $\eta > 0$, $\delta \geq 0$, and $\lambda \geq 0$. Then*

(1) $\alpha \leq \beta_1 < 0$, $\beta_2 > 1$, $a_1 \geq 0$, and if $\delta < r$ then

$$\beta_2 < \frac{r+\eta}{r-\delta}; \tag{23}$$

if $\delta > r$ then

$$\alpha > \frac{r+\eta+\lambda}{r-\delta} \tag{24}$$

and

$$\beta_1 > \frac{r+\eta}{r-\delta}; \tag{25}$$

if $\delta > r + \frac{1}{2}\sigma^2$ then

$$\beta_1 > \frac{r+\eta+\frac{1}{2}\sigma^2}{r-\delta+\frac{1}{2}\sigma^2}; \tag{26}$$

(2) $a < 0$, $b \geq 0$, and $c < 0$.

(3) if $\delta = 0$, then the left hand side of equation (5) is always negative; if $\delta > 0$, then equation (5) has a unique solution $k^* \in (1, \infty)$.

(4) $A > 0$, $B_1 > 0$, $B_2 \geq 0$, and $C > 0$.

PROOF. (1) It is obvious that $\alpha \leq \beta_1 < 0$. Since

$$\begin{aligned} \beta_2 - 1 &= \frac{-(r-\delta+\frac{1}{2}\sigma^2) + \sqrt{(r-\delta-\frac{1}{2}\sigma^2)^2 + 2(r+\eta)\sigma^2}}{\sigma^2} \\ &> \frac{-(r-\delta+\frac{1}{2}\sigma^2) + \sqrt{(r-\delta-\frac{1}{2}\sigma^2)^2 + 2(r-\delta)\sigma^2}}{\sigma^2} \\ &= \frac{-(r-\delta+\frac{1}{2}\sigma^2) + \sqrt{(r-\delta+\frac{1}{2}\sigma^2)^2}}{\sigma^2} \\ &= 0. \end{aligned} \tag{27}$$

Therefore $\beta_2 > 1$. If $r > \delta$, we have

$$\begin{aligned} \beta_2 - \frac{r+\eta}{r-\delta} &= \frac{-\left((r-\delta-\frac{1}{2}\sigma^2)+\frac{r+\eta}{r-\delta}\sigma^2\right)+\sqrt{(r-\delta-\frac{1}{2}\sigma^2)^2+2(r+\eta)\sigma^2}}{\sigma^2} \\ &< 0, \end{aligned} \tag{28}$$

because

$$\begin{aligned} &\left((r-\delta-\frac{1}{2}\sigma^2)+\frac{r+\eta}{r-\delta}\sigma^2\right)^2-\left((r-\delta-\frac{1}{2}\sigma^2)^2+2(r+\eta)\sigma^2\right) \\ &= \frac{(r+\eta)(\eta+\delta)}{(r-\delta)^2}\sigma^4 \\ &> 0. \end{aligned} \tag{29}$$

Therefore we have (23) and similar arguments lead to (24), (25), and (26). Next we show that $a_1 \geq 0$. Viewing α as a function of λ, we define

$$h(\lambda) = (\delta+\eta)\alpha(\lambda) + (1-\beta_1)\lambda - (\delta+\eta)\beta_1.$$

It suffices to show $h(\lambda) \geq 0$ for all $\lambda \geq 0$. Obviously $h(0) = 0$.

$$h'(\lambda) = (\delta+\eta)\alpha'(\lambda)+(1-\beta_1) > (\delta+\eta)\alpha'(0)+(1-\beta_1) = \frac{r+\eta+\frac{1}{2}\sigma^2-(r-\delta+\frac{1}{2}\sigma^2)\beta_1}{\sqrt{(r-\delta-\frac{1}{2}\sigma^2)^2+2(r+\eta)\sigma^2}} > 0,$$

where the first inequality follows from the convexity of $\alpha(\lambda)$ and the second inequality follows from the fact that if $\delta \leq r + \frac{1}{2}\sigma^2$, then the numerator is obviously positive and that if $\delta > r + \frac{1}{2}\sigma^2$, then the numerator is also positive by (26).

(2) If $r \leq \delta$, obviously $a < 0$. If $r > \delta$, then (23) implies that $a < 0$. Since $\beta_2 > 1$ and $\alpha < 0$ we have $c < 0$ and

$$b = -\frac{\delta(\alpha(\delta+\eta)+\lambda-\beta_2(\delta+\eta+\lambda))}{(\delta+\eta)(\delta+\eta+\lambda)} = -\frac{\delta((\alpha-\beta_2)(\delta+\eta)+(1-\beta_2)\lambda)}{(\delta+\eta)(\delta+\eta+\lambda)} \geq 0.$$

(3) It is obvious that if $\delta = 0$, then $b = 0$. Since $a < 0$ and $c < 0$, so the left hand side of equation (5) is always negative. In other words, there is no solution to equation (5). Next suppose $\delta > 0$. Define

$$g(z) = az^{\beta_1} + bz + c. \tag{30}$$

Since $\beta_1 < 0$ and $b > 0$, we have that as z approaches infinity, $g(z)$ becomes strictly positive. In addition, it can be verified that

$$g(1) = \frac{((r-\delta)\alpha - (r+\eta+\lambda))(\eta+\lambda)}{(r+\eta+\lambda)(\delta+\eta+\lambda)} < 0, \tag{31}$$

because (i) if $r \geq \delta$ then it is obvious that the numerator is negative (recall that $\alpha < 0$); (ii) if $r < \delta$, then by (24) we have that the numerator is again negative.

Therefore by the continuity of $g(z)$ there must exist a solution to $g(z) = 0$ in the interval $(1, \infty)$. Finally, since

$$g'(z) = a\beta_1 z^{\beta_1 - 1} + b > 0,$$

we have g is strictly and monotonically increasing. Therefore, the solution is unique.

(4) Since $a < 0$ and $\beta_1 < 0 < \beta_2$, we have $B_1 > 0$. In addition, since

$$g\left(\frac{r(\delta+\eta+\lambda)}{\delta(r+\eta+\lambda)}\right) = a\left[\left(\frac{r(\delta+\eta+\lambda)}{\delta(r+\eta+\lambda)}\right)^{\beta_1} + \frac{r\lambda}{\eta(r+\eta+\lambda)}\right] < 0.$$

we have

$$k^* > \frac{r(\delta+\eta+\lambda)}{\delta(r+\eta+\lambda)}, \tag{32}$$

which implies that $A > 0$. If $\lambda = 0$, then $B_2 = 0$. Suppose $\lambda > 0$. By $a < 0$, $a_1 > 0$, and (24), we have

$$g\left(\frac{b_1}{a_1}\right) = a\left(\frac{b_1}{a_1}\right)^{\beta_1} + \frac{\delta r((r-\delta)\alpha - (r+\eta+\lambda))}{a_1(\delta+\eta)(\delta+\eta+\lambda)(r+\eta)(r+\eta+\lambda)} < 0.$$

Therefore $k^* > \frac{b_1}{a_1}$ and $B_2 > 0$. $C > 0$ follows from $B_2 \geq 0$ and (25). □

Define

$$f(S) = \begin{cases} A\,S^{\alpha} + \frac{\lambda+\eta}{\delta+\lambda+\eta}S - \frac{\lambda+\eta}{r+\lambda+\eta}X & \text{if } S \geq S^* \\ B_1 S^{\beta_1} + B_2 S^{\beta_2} + \frac{\eta}{\delta+\eta}S - \frac{\eta}{r+\eta}X & \text{if } X < S < S^* \\ C S^{\beta_2} & \text{if } S \leq X \end{cases}, \tag{33}$$

Lemma 2 *Given the expression for f in (33), we have for any finite $T \geq 0$,*

$$\mathrm{E}\int_0^T |f'(S_t)|^2 S_t^2 dt < \infty$$

and

$$\lim_{t\to\infty} \mathrm{E}[e^{-(r+\eta+\lambda)t} f(S_t)] = 0.$$

PROOF. This follows from the fact that S_t is a geometric Brownian motion with finite moments and $f(S)$ is a polynomial function of S. □

Lemma 3 *Let $S^* = k^*X$. Given the expressions in (33), we have*

1. *$f(S)$ is C^2 except at $S = X$ where it is C^1, strictly positive, strictly increasing, and strictly convex;*
2. *if $S \geq S^*$, then $f(S) \leq S - X$;*
3. *if $S < S^*$, then $f(S) > S - X$.*

PROOF. 1. By plugging in the constants and using equation (5), it can be directly verified that $f(S)$ is C^2 except at $S = X$ where it is C^1. The fact that $f(S) > 0$ follows from $C > 0$ and $f(S)$ is strictly increasing.

2. Define

$$h(S) = f(S) - (S - X).$$

We have $h(S^*) = 0$ by direct verification. For $S \geq S^*$, since

$$h'(S) = A\alpha S^{\alpha-1} - \frac{\delta}{\delta + \eta + \lambda} < 0,$$

we have $h(S) \leq 0$ and thus $f(S) \leq S - X$ for all $S \geq S^*$.

3. Since $f(S)$ is strictly convex, so is $h(S)$. Therefore for $S < S^*$, we have $h'(S) < h'(S^*) < 0$. Since $h(S^*) = 0$, we have $h(S) > 0$ for $S < S^*$, which implies that $f(S) > S - X$ for all $S < S^*$. □

Now we are ready to prove Theorem 1. Define

$$H(S) = \begin{cases} f(S) & S < S^* \\ S - X & S \geq S^*. \end{cases}$$

Then by (3) and (4), we have

$$\frac{1}{2}\sigma^2 S^2 f_{SS} + (r - \delta)Sf_S - (r + \eta + \lambda)f + \eta(S - X)^+ + \lambda H(S) = 0. \tag{34}$$

Define

$$M_t = e^{-(r+\eta+\lambda)t} f(S_t) + \int_0^t \lambda e^{-(r+\eta+\lambda)s} H(S_s) ds.$$

We next show that M_t is a martingale. By Itô's lemma, we have

$$\begin{aligned} M_t &= f(S_0) \\ &+ \int_0^t e^{-(r+\eta+\lambda)s} \Big[\frac{1}{2}\sigma^2 S_s^2 f_{SS} + (r-\delta) S_s f_S - (r+\eta+\lambda) f + \eta(S-X)^+ + \lambda H\Big] ds \\ &+ \int_0^t e^{-(r+\eta+\lambda)s} f_S(S_s) \sigma S_s \, dB_s. \end{aligned}$$

By Lemma 2, the stochastic integral is a martingale. By (34), the second term is equal to zero. Thus M_t is a martingale, which implies that

$$f(S_0) = \mathrm{E}[M_t] = \mathrm{E}[e^{-(r+\eta+\lambda)t} f(S_t)] + \mathrm{E}\left[\int_0^t \lambda e^{-(r+\eta+\lambda)s} H(S_s) ds\right].$$

By Lemma 2, the first term goes to 0 as t goes to infinity. Taking the limit as $t \to \infty$, we then have

$$f(S_0) = \mathrm{E}\left[\int_0^\infty \lambda e^{-(r+\eta+\lambda)s} H(S_s) ds\right]. \tag{35}$$

Thus

$$H(S_0) \geq f(S_0) = \mathrm{E}\left[\int_0^\infty \lambda e^{-(r+\eta+\lambda)s} H(S_s) ds\right] = \mathrm{E}\left[e^{-(r+\eta)\mathcal{T}} H(S_{\mathcal{T}})\right],$$

where $\mathcal{T}$ is an exponential random variable with intensity parameter λ. Combining this with the fact that S_t is a geometric Brownian motion with finite moments and $f(S)$ is a polynomial function of S, one can show that $e^{-(r+\eta)\tau_n} H(S_{\tau_n})$ is a uniformly integrable nonnegative supermartingale for any stopping time τ_n.

Then by optional sampling theorem, we have for any stopping time τ_n,

$$\begin{aligned} \mathrm{E}[e^{-r(\tau_n \wedge \hat{\tau})}(S_{\tau_n \wedge \hat{\tau}} - X)] &= \mathrm{E}[e^{-(r+\eta)\tau_n}(S_{\tau_n} - X)] \\ &\leq \mathrm{E}[e^{-(r+\eta)\tau_n} H(S_{\tau_n})] \\ &\leq \mathrm{E}[e^{-(r+\eta)\tau_1} H(S_{\tau_1})] \\ &= \mathrm{E}\left[\int_0^\infty \lambda e^{-(r+\eta+\lambda)s} H(S_s) ds\right] \\ &= f(S_0), \end{aligned}$$

where the first inequality follows from Lemma 3 and the second inequality follows because of the uniformly integrable supermartingale property. Taking the supremum of all the stopping times τ_n, we then have

$$f(S_0) \geq \sup_{\tau} \mathrm{E}[e^{-r(\tau\wedge\hat{\tau})}(S_{\tau\wedge\hat{\tau}} - X)].$$

Now we show that the above inequality actually holds with equality. This claim can be shown using the strong Markov property of S_t. Let τ^* be the optimal stopping time. Then

$$\begin{aligned}
\mathrm{E}[e^{-r(\tau^*\wedge\hat{\tau})}(S_{\tau^*\wedge\hat{\tau}} - X)] &= \mathrm{E}[e^{-(r+\eta)\tau^*}(S_{\tau^*} - X)] \\
&= \mathrm{E}\left[\int_0^\infty \lambda e^{-\lambda s}\,\mathrm{E}[e^{-(r+\eta)\tau^*}(S_{\tau^*} - X)|\tau_1 = s]ds\right] \\
&= \mathrm{E}\left[\int_0^\infty \lambda e^{-(r+\eta+\lambda)s} H(S_s)ds\right] \\
&= f(S_0).
\end{aligned}$$

Since the probability of severe exercise shock is p at vesting and the executive can always exercise at vesting, the payoff of the option at vesting given a realization z of a standard normal random variable $\tilde{z}$ is equal to

$$\psi(T_v, z) = p(S(T_v, z) - X)^+ + (1-p)Max(S(T_v, z) - X, f(S(T_v, z))),$$

where

$$S(T_v, z) = S_0 e^{(r-\delta-\frac{1}{2}\sigma^2)T_v + \sigma\sqrt{T_v}z}$$

is the stock price at vesting. Let $\phi(z)$ be the standard normal probability distribution function. The present value of $\psi(T_v, z)$ at the grant time is then

$$P(S_0) = \int_{-\infty}^{\infty} e^{-rT_v}\left(p(S(T_v, z) - X)^+ + (1-p)Max(S(T_v, z) - X, f(S(T_v, z)))\right)\phi(z)dz,$$

which yields the formula in the theorem using the expression (33) and Lemma 3. This completes the proof of Theorem 1. □

Proof of Proposition 1. Let $g(z)$ be as defined in (30). As shown before, we have $g'(k^*) > 0$.

$$\begin{aligned}
\frac{\partial b}{\partial \lambda} &= -\frac{\delta}{\delta+\eta+\lambda}\left(\frac{\partial \alpha}{\partial \lambda} + \frac{1-\alpha}{\delta+\eta+\lambda}\right) \\
&= -\frac{\delta}{\delta+\eta+\lambda}\left(\frac{(r-\delta+\frac{1}{2}\sigma^2)^2 + (r-\delta+\frac{1}{2}\sigma^2)y + (\delta+\eta+\lambda)\sigma^2}{(\delta+\eta+\lambda)\sigma^2 y}\right) \quad (36)\\
&\leq 0, \quad (37)
\end{aligned}$$

where

$$y = \sqrt{(r-\delta-\frac{1}{2}\sigma^2)^2 + 2(r+\eta+\lambda)\sigma^2}$$

and the inequality can be verified by direct algebra.

Also,

$$\begin{aligned}
\frac{\partial c}{\partial \lambda} &= \frac{r}{r+\eta+\lambda}\frac{\partial \alpha}{\partial \lambda} - \frac{r\alpha}{(r+\eta+\lambda)^2} \quad (38)\\
&= \frac{r}{r+\eta+\lambda}\left(\frac{\partial \alpha}{\partial \lambda} + \frac{1-\alpha}{\delta+\eta+\lambda}\right) - \frac{r}{r+\eta+\lambda}\left(\frac{1-\alpha}{\delta+\eta+\lambda} + \frac{\alpha}{r+\eta+\lambda}\right) \\
&= \frac{r}{r+\eta+\lambda}\left(\frac{\partial \alpha}{\partial \lambda} + \frac{1-\alpha}{\delta+\eta+\lambda}\right) - \frac{r}{r+\eta+\lambda}\left(\frac{r+\eta+\lambda+\alpha(\delta-r)}{(\delta+\eta+\lambda)(r+\eta+\lambda)}\right) \\
&< \frac{r}{r+\eta+\lambda}\left(\frac{\partial \alpha}{\partial \lambda} + \frac{1-\alpha}{\delta+\eta+\lambda}\right), \quad (39)
\end{aligned}$$

where the inequality follows from (24).

Then by (37) and (32), we have

$$\frac{\partial b}{\partial \lambda}k^* + \frac{\partial c}{\partial \lambda} < \left(-\frac{\delta}{\delta+\eta+\lambda}k^* + \frac{r}{r+\eta+\lambda}\right)\left(\frac{\partial \alpha}{\partial \lambda} + \frac{1-\alpha}{\delta+\eta+\lambda}\right) < 0.$$

Therefore, by implicit function theorem, we have

$$\frac{\partial k^*}{\partial \lambda} = -\frac{\frac{\partial b}{\partial \lambda}k^* + \frac{\partial c}{\partial \lambda}}{g'(k^*)} > 0. \qquad \square$$

Proof of Proposition 2. Since

$$g(S) = \mathrm{E}[\tau^* \wedge \hat{\tau}] = \mathrm{E}\left[\int_0^{\tau^*} s\eta e^{-\eta s} ds + \int_{\tau^*}^{\infty} \tau^* \eta e^{-\eta s} ds\right] = \frac{1}{\eta} - \frac{1}{\eta}\mathrm{E}\left[e^{-\eta\tau^*}\right],$$

similar (but much simpler) arguments to those in the proof of Theorem 1 yield the claim. $\square$