

Empirical Search and Consideration Sets ^{*}

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1 Introduction

“Prices change with varying frequency in all markets, and, unless a market is completely centralized, no one will know all the prices which various sellers (or buyers) quote at any given time. A buyer (or seller) who wishes to ascertain the most favorable price must canvass various sellers (or buyers)—a phenomenon I shall term “search”.” (Stigler 1961, p. 213)

Dating back to the classic work of Stigler (1961), a large literature in economics and marketing documents the presence of substantial price dispersion for similar, even identical goods. For example, looking across 50,000 consumer products, Hitsch et al. (2017) find that, within a 3-digit zip code, the ratio of the 95th to the 5th percentile of prices for the median UPC (brand) is 1.29 (1.43). Substantial price dispersion has been reported in many different product categories including e.g. automobiles (Zettelmeyer et al. 2006), medical devices (Grennan and Swanson 2018), financial products (Duffie et al. 2017, Hortaçsu and Syverson 2004, Ausubel 1991, Allen et al. 2013), and insurance products (Brown and Goolsbee 2002, Honka 2014).

Again dating back to Stigler (1961), the presence and persistence of price dispersion for homogenous goods has often been attributed to search/information costs. Understanding the nature of the search and/or information costs is a crucial step towards quantifying potential losses to consumer and social surplus induced by such frictions, and to assess the impact of potential policy interventions to improve market efficiency and welfare.

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Quantitative analyses of consumer and social welfare rely on empirical estimates of demand and supply parameters and comparing observed market outcomes to counterfactual efficient benchmarks. However, departures from the assumption that consumers have full information pose important methodological challenges to demand (and supply) estimation methods that have been the mainstay of quantitative marketing and economics. Consider, for example, a consumer who is observed to purchase a product for \$5 when the identical product is available for purchase at \$4 somewhere nearby. A naive analysis may conclude that demand curves are upward sloping, or, at the very least, that demand is very inelastic. Similarly, consider the observation that lower income but high-achieving high school seniors do not apply to selective four year colleges despite being admitted at high rates (see Hoxby and Turner 2015). This may be because these seniors do not value a college education or because they are not aware of the financial aid opportunities or their chances of admission. Indeed, as in Stigler (1961), consumers are likely not perfectly informed about both prices and non-price attributes of all products available for purchase in a given market. It is therefore important, from the perspective of achieving an accurate understanding of preferences, to gain a deeper understanding of the choice process, and especially which subsets of products actually enter a consumer’s “consideration” set¹ and how much a consumer knows about the price/non-price attributes.

Understanding how consumers search for products and eventually settle on the product they are observed to purchase is the subject of a large and burgeoning literature in economics and marketing. Our focus in this essay is on the econometric literature that allows for the specification of a search process, leading to the formation of consideration sets, along with a model of preferences. In much of this literature, the specification of the search process is motivated by economic models of consumer search. While this constrains the specification of the search process, economic theory provides a useful guidepost as to how consumers may search under counterfactual scenarios that may be very far out of sample. We will thus start, in Section 2, with a brief survey of popular theoretical models of consumer search that motivate econometric specifications.²

While this chapter is centered around econometric methods and models, many of these methods and models are motivated by substantial findings. For example, in addition to the price dispersion/information cost argument by Stigler (1961), empirical marketing research in the 1980s found that many consumers effectively choose from (or “consider”) a surprisingly small number of alternatives – usually 2 to 5 – before making a purchase decision (see e.g.

¹Throughout this chapter, we use the terms “consideration set,” “search set,” “evoked set,” and “(endogenous) choice set” interchangeably unless stated otherwise.

²Readers interested in much more exhaustive surveys of the theoretical research can refer to Baye et al. (2006) and Anderson and Renault (2018).

Hauser and Wernerfelt 1990, Roberts and Lattin 1991, Shocker et al. 1991). This empirical observation sparked a rich stream of literature in marketing that developed and estimated models that take consumers' consideration into account. We discuss this stream of literature, which was pioneered by Hauser and Wernerfelt (1990) and Roberts and Lattin (1991), in Section 3.1. One of the main findings from this line of research, which has been validated in more recent work, is that advertising and other promotional activities create very little true consumption utility, but first and foremost affect awareness and consideration.

We then turn to the early work in economics in Section 3.2, which was primarily motivated by Stigler (1961)'s price dispersion observation and search/information cost argument. With the proliferation of the Internet around the turn of the century and increased availability of data, researchers worked on quantifying the amount of price dispersion in online markets as well as quantifying consumer search costs. Some of the rather surprising results of these efforts were that that amount of price dispersion remained substantial even in online markets, i.e. prices did not seem to follow the Law of One Price. Starting with Sorensen (2000), Hortacısu and Syverson (2004), and Hong and Shum (2006), researchers have utilized economic theories of search to rationalize observed price dispersion patterns and to infer search costs and preference parameters. The search cost estimates recovered in these papers appeared relatively large at first sight. However, subsequent work has confirmed that the costs consumers incur while gathering information remain quite high for a variety of markets.

One of the main shortcomings of the consideration set literature discussed in Section 3.1 was that it mostly used reduced-form modeling approaches. One of the main shortcomings of the early search literature in economics discussed in Section 3.2 was that it modeled consumers as randomly picking which alternatives to search. In Section 4 we turn to the more recent literature that aims to overcome both shortcomings by covering a more general setting in which, following discrete choice additive random utility models popular in demand estimation, products are both vertically and horizontally differentiated. Here an important distinction is made regarding what consumers are searching for: we discuss models in which consumers are searching for prices in Section 4.1 and models in which consumers search for a good product match or fit in Section 4.2. We also discuss approaches that utilize both individual level data and aggregate (market share) data. Papers discussed in this section have in common that they think carefully about the data is needed to identify search costs. They also advance estimation methodologies by developing approaches to handle the curse of dimensionality that appears in the simultaneous search model and the search path dimensionality problem of the sequential search model. However, many of these models are not straightforward to estimate, and more work is need to obtain models that are both realistic and tractable in terms of estimation.

Since the beginning of the search literature, the question of *how* consumers search, i.e. whether consumers search in a simultaneous or sequential fashion, has been heavily debated. Because researchers did not think that the search method was identified using observational data, it was common to make an assumption on the type of search protocol that consumers were using (frequently driven by computational considerations). Starting with De los Santos et al. (2012) and Honka and Chintagunta (2017), researchers have begun empirically testing observable implications of sequential versus simultaneous search and the broader question of the identifiability of the search method utilized by consumers. This also highlighted the importance of expectations: which search method is supported by data patterns can change depending on whether researchers assume that consumers have rational expectations. How consumers search also has implications on the estimated search costs (and thus any subsequent analyses such as welfare calculations): if consumers search simultaneously (sequentially), but the researcher falsely assumes that they search sequentially (simultaneously), search costs will be overestimated (underestimated).

In Section 6, we discuss various extensions and applications of the econometric frameworks discussed in the prior sections. Section 6.1 explores generalizations of the modeling framework when consumers are not perfectly informed regarding the distribution of prices and/or match utilities and learn about these distributions as they search. Section 6.3 discusses how advertising interacts with search and choice. Section 6.4 discusses the very related setting where the ranking and/or spatial placement of different choices on for instance a webpage affect search and eventual choice. Section 6.5 considers an interesting emerging literature on the issue of information provision made available to consumers at different stages of search, e.g. at different layers of a website. Sections 6.6 and 6.7 discuss how the availability of more granular information on consumer behavior such as search duration can improve inference/testing regarding the search method and preferences. This is clearly an important area of growth for the literature as consumer actions online and, increasingly, offline are being monitored closely by firms. Finally, Section 6.8 discusses the important case in which dynamic demand considerations (such as consumer stock-piling) interact with consumer search.

The econometric literature on consumer search and consideration sets is likely to grow much further beyond what is covered here as more and more data on the processes leading to eventual purchase become available for study. We therefore hope our readers will find what is to follow a useful roadmap into what has been done so far, but that they will ultimately agree with us that there are many more interesting questions to answer in this area than has been attempted so far.

2 Theoretical Framework

2.1 Set-up

We start by presenting the general framework of search models.³ In these models, consumers are utility maximizers. Consumers know the values of all product characteristics but one (usually price or match value) prior to searching and have to engage in search to resolve uncertainty about the value of that one product characteristic. Search is costly so consumers only search a subset of all available products.

Formally, we denote consumers by $i = 1, \dots, N$, firms/products by $j = 1, \dots, J$, and time periods by $t = 1, \dots, T$. Consumer i 's search cost (per search) for product j is denoted by c_{ij} and the number of searches consumer i makes is denoted by $k_i = 1, \dots, K$ with $K = |J|$. Firm j 's marginal cost is denoted by r_j . Consumer i 's indirect utility for product j is an independent draw u_{ij} from a distribution F_j with density f_j where u_{ij} is given by

$$u_{ij} = \alpha_j + X_{ij}\beta + \gamma p_{ij} + \epsilon_{ij}. \quad (1)$$

The parameters α_j are the brand intercepts, X_{ij} represents observable product and/or consumer characteristics, p_{ij} is the price, and ϵ_{ij} is the part of the utility not observed by the researcher.⁴ The parameters α_j , β , and γ are the utility parameters. Although the framework is constructed to analyze differentiated goods, it can also capture special cases such as a price search model for homogenous goods with identical consumers and firms. In this case, equation (1) simplifies to $u_{ij} = -p_{ij}$ with $F_j = F$.

We start by going through the set of assumptions that most search models share.

Assumption 1 *Demand is unit-inelastic.*

In other words, each consumer buys at most one unit of the good. Assumption 1 holds for all papers discussed in this chapter.

Assumption 2 *Prior to searching consumers know the (true) utility distribution, but not the specific utility a firm is going to offer on a purchase occasion.*

³The two most common types of search models are price search models and match value search models. In the former, consumer search to resolve uncertainty about prices, while in the latter consumers search to resolve uncertainty about the match value or fit. The set-up of both price and match value search models fits under the general framework presented in this section. The set-up of both types of search models is identical with one exception denoted in footnote 4. However, the estimation approaches differ as discussed in Section 4.

⁴In price search models, ϵ_{ij} is observed by the consumer prior and post search. In match value search models, consumers do not know ϵ_{ij} prior to search, but know its value after searching. In both price and match value search models, the researcher does neither observe ϵ_{ij} prior nor post search.

To put it differently, while consumer i does not know the *specific* utility u_{ij} he would get from buying from firm j (potentially on a specific purchase occasion), he knows the shape and parameters of the utility distribution, i.e. the consumer knows F_j . This assumption is often referred to as the “rational expectations assumption” because it is assumed that consumers are rational and know the *true* distribution from which utilities are being drawn. We relax Assumption 2 in Section 6.1 and discuss papers which have studied consumer search in an environment in which consumers are uncertain about the utility distribution and learn about it while searching.

Assumption 3 *All uncertainty regarding the utility from consuming a specific product is resolved during a single search.*

In other words, consumers learn a product’s utility in a single search. In Section 6.7, we present recent work that relaxes Assumption 3.

Assumption 4 *The first search is free.*

This assumption is made for technical reasons. Two alternative assumptions are sometimes made in the literature: (search costs are sufficiently low so that) all consumers search at least once (e.g. Reinganum 1979) or all consumers use the same search method, but it is optimal for some consumers to search/not to search depending on the level of their search costs (e.g. Janssen et al. 2005).

2.2 Search Method

The consumer search literature has predominantly focused on two search methods: simultaneous and sequential search. In this subsection, we introduce these two search methods.

Simultaneous Search

Simultaneous search – also referred to as non-sequential, fixed sample, or parallel search – is a search method in which consumers commit to searching a fixed set of products (or stores or firms) before they begin searching. Consumers using this method will not stop searching until they have searched all firms in their predetermined search set. Note that – despite its name – simultaneous search does *not* mean that all firms have to be sampled simultaneously. Firms can be sampled one after another. What characterizes simultaneous search is the consumer’s commitment (prior to the beginning of the search process) to search a fixed set of firms.

In the most general case, consumer i 's search problem consists of picking a subset of firms S_i that maximizes the expected maximum utility to consumer i from searching that subset of firms net of search costs, i.e.,

$$S_i = \arg \max_S \left[E \left[\max_{j \in S} \{u_{ij}\} \right] - \sum_{j \in S} c_{ij} \right], \quad (2)$$

where E denotes the expectation operator. Unfortunately, a simple solution for how a consumer should optimally pick the firms to be included in his search set S_i does not exist for the simultaneous search model. In general, it will not be optimal to search the firms randomly, so the question is: *which firms should the consumer search?* This is referred to as *ordered* or *directed* search in the literature.⁵ When searching simultaneously, to pick the optimal set of products to be searched, the consumer has to enumerate all combinatorially possible search sets (varying by their size and composition) and calculate the corresponding expected gains of search while taking the cost of sampling all products in the search set into account, i.e., calculate the expected maximum utility minus the cost of searching for every search set (as in equation 2).

The following example illustrates the problem. Suppose there are four companies A, B, C, and D in the market. Then the consumer has to choose among the following search sets: A, B, C, D, AB, AC, AD, BC, BD, CD, ABC, ABD, ACD, BCD, and ABCD. The difficulty with this approach is that the number of possible search sets grows exponentially with the number of firms $|J|$, i.e. if there are $|J|$ firms in the market, the consumer chooses among $2^{|J|} - 1$ search sets.⁶ This exponential growth in the number of search sets is referred to as the *curse of dimensionality* of the simultaneous search model.

One avenue to deal with the curse of dimensionality is to only estimate the simultaneous search model for markets with relatively few products (see e.g. Mehta et al. 2003). Another avenue to overcome the curse of dimensionality is to make an additional assumption which allows one to derive a simple strategy for how the consumer should optimally choose his search set. The following two assumptions have been used in the literature:

1. Assumption of *first-order stochastic dominance*: Vishwanath (1992) showed that, for a simultaneous search model with first-order stochastic dominance among the utility distributions, the rules derived by Weitzman (1979) constitute optimal consumer search and purchase behavior.⁷

⁵See Armstrong (2017) for an overview of the theoretical literature on ordered search.

⁶The researcher can reduce the number of search sets the consumer is choosing from by dropping all search sets that do not include the consumer's chosen (purchased) option (see Mehta et al. 2003).

⁷See next page for a detailed discussion of the rules derived by Weitzman (1979).

2. Assumption of *second-order stochastic dominance*:⁸ Chade and Smith (2005) showed that, for a simultaneous search model with second-order stochastic dominance among the utility distributions, it is optimal for the consumer to

- (a) rank firms in a decreasing order of their expected utilities,
- (b) pick the optimal number of searches conditional on the ranking according to the expected utilities, and
- (c) purchase from the firm with the highest utility among those searched.

The assumptions of first- or second-order stochastic dominance are typically implemented by assuming that the means or variances, respectively, of the price distributions are identical (see e.g. Honka 2014). While adding an additional assumption restricts the flexibility of a model, making this additional assumption allows researchers to apply the simultaneous search model to markets with a large number of products. Furthermore, the appropriateness of these assumptions is empirically testable, i.e. using price data, researchers can test the hypothesis of identical means or variances across products.

In the special case of homogenous goods with identical firms, the search problem reduces to choosing the optimal *number* of products to search and the dimensionality problem disappears.⁹ The simultaneous search model for homogenous goods was initially proposed by Stigler (1961). In Stigler’s model, the consumer has to decide which set of firms to search. Since firms are identical, i.e., $F_j = F$, in the setting he analyzes, it is optimal for the consumer to *randomly* pick the firms to be searched. Therefore, the only objective of the consumer is to determine the optimal number of firms to search. Since goods are homogenous in Stigler’s model, the utility function in equation (1) simplifies to $u_{ij} = -p_{ij}$. The consumer’s objective is to minimize his cost of acquiring the good, i.e. to minimize the sum of the expected price paid and his search costs. Formally, a consumer’s objective function can be written as

$$\min_k \underbrace{\int_p^{\bar{p}} kp (1 - F(p))^{k-1} f(p) dp}_{\text{expected min. price for } k \text{ searches}} + \underbrace{(k - 1)c}_{\text{search cost}}, \quad (3)$$

where $F(p)$ is price distribution with a minimum price \underline{p} and maximum price \bar{p} . The intuition behind the expression for the expected minimum price in equation (3) is as follows: the

⁸Additionally, to apply the theory developed by Chade and Smith (2005), the researcher also have to assume that search costs are not company-specific.

⁹This discussion and results hold when search costs are identical across consumers and when search costs are heterogenous across consumers. The discussion and results do not hold when there is heterogeneity in search costs across firms.

probability that all price draws are greater than p is given by $\Pr(p_1 > p, \dots, p_k > p) = (1 - F(p))^k$. This implies that the cdf of the minimum draw is $1 - (1 - F(p))^k$ and the pdf of the minimum draw is $k(1 - F(p))^{k-1}f(p)$. It can be shown that there is a unique optimal number of searches k^* that minimizes equation (3) (see e.g. Hong and Shum 2006). This optimal number of searches k^* is the size of the consumer's search set.

Sequential Search

A main drawback of the simultaneous search method is that it assumes that the consumer will continue searching even after getting a high utility realization early during the search process. For example, consider a simultaneous search in which a consumer commits to searching three firms and in the first search he gets quoted the maximum utility \bar{u} . Because of Assumption 2, the consumer knows that this is the highest possible utility so it would not be optimal to continue searching. To address this drawback, the sequential search method has been developed. When searching sequentially, consumers determine, after each utility realization, whether to continue searching or to stop.

Before we discuss the sequential search model in detail, we have to add another technical assumption to the list of assumptions laid out in Section 2.1:

Assumption 5 *Consumers have perfect recall.*

In other words, once a consumer has searched a firm, he remembers the utility offered by this firm going forward. This assumption is equivalent to assuming that a consumer can costlessly revisit stores already searched. In Section 6.7, we present recent work that relaxes Assumption 5.

The sequential search problem in its most general form has been analyzed by Weitzman (1979). The problem of searching for the best outcome from a set of options that are independently distributed can be stated as the following dynamic programming problem

$$W(\tilde{u}_i, \bar{S}_i) = \max \left\{ \tilde{u}_i, \max_{j \in \bar{S}_i} \left\{ -c_{ij} + F_j(\tilde{u}_i)W(\tilde{u}_i, \bar{S}_i - \{j\}) + \int_{\tilde{u}_i}^{\infty} W(u, \bar{S}_i - \{j\})f_j(u)du \right\} \right\},$$

where \tilde{u}_i is consumer i 's highest utility sampled so far and \bar{S}_i is the set of firms consumer i has not searched yet. Weitzman (1979) shows that the solution to this problem can be stated in terms of J static optimization problems. Specifically, for each product j , consumer i derives a reservation utility z_{ij} . This reservation utility z_{ij} equates the benefit and cost of searching product j , i.e.,

$$c_{ij} = \int_{z_{ij}}^{\infty} (u_{ij} - z_{ij}) f_j(u) du.$$

This consumer- and product-specific reservation utility z_{ij} can then be used to determine the order in which products should be searched as well as when to stop searching. Specifically, Weitzman (1979) shows that it is optimal for a consumer to follow three rules:

1. search companies in a decreasing order of their reservation utilities (“selection rule”),
2. stop searching when the maximum utility among the searched firms is higher than the largest reservation utility among the not-yet-searched firms (“stopping rule”), and
3. purchase from the firm with the highest utility among those searched (“choice rule”).

It is important to note that the consumer will *not* always purchase from the firm searched last. What follows is an example that shows when this can happen: in Table 1, we show the reservation utilities and utilities (which the consumer only knows after searching) for three firms. Given Weitzman (1979)’s selection rule, the consumer searches the firms in a decreasing order of their reservation utilities. The consumer first searches firm A and learns that the utility is 11. Using the stopping rule, the consumer determines that the maximum utility among the searched firms (11) is smaller than the largest reservation utility among the not-yet-searched firms (12) and thus decides to continue searching. In the second search, the consumer searches firm B and learns that the utility is 7. Using the stopping rule, the consumer determines that the maximum utility among the searched firms (11 from firm A) is higher than the largest reservation utility among the not-yet-searched firms (10 for firm C) and thus decides to stop searching. The consumer then purchases from the firm with the highest utility among those searched – firm A. Note that firm A is the firm the consumer searched in his first and not in his second (and last) search.

Table 1: Example

Option	Reservation Utilities (z_{ij})	Utilities (u_{ij})
A	14	11
B	12	7
C	10	9

In the special case of homogenous products and identical firms (i.e., $u_{ij} = -p_{ij}$ and $F_j = F$), just like for the simultaneous search model, the sequential search model greatly

simplifies.¹⁰ Because firms are identical, the consumer *randomly* picks a firm to search. As in the more general case, the consumer needs to solve an optimal stopping problem, i.e. solve the problem of balancing the benefit of further search with the cost of searching. Following McCall (1970), the first-order condition for the optimal stopping problem is given by

$$\underbrace{c}_{\text{marginal cost}} = \underbrace{\int_p^z (z - p) f(p) dp}_{\text{marginal benefit}} \quad (4)$$

where z is the lowest price found in the search so far. According to equation (4), a consumer is indifferent between continuing to search and stopping the search when the marginal cost of an additional search equals the marginal benefit of performing an additional search given the lowest price found so far. A consumer thus searches as long as the marginal benefit from searching is greater than the marginal cost of searching. The marginal benefit in equation (4) is the expected savings from an additional search given the lowest price found so far.

Equation (4) implies that there is a unique price z^* for which the marginal cost of searching equals the marginal benefit of searching. This unique price z^* is the beforementioned reservation price. Note that z^* is a function of consumer search cost c . We can now describe the consumer's decision rule: if the consumer gets a price draw above his reservation price, i.e. $p > z^*$, he continues to search. If he gets a price draw below his reservation price, i.e. $p \leq z^*$, he stops searching and purchases.

Note that in the case that all firms are identical (homogenous good) and consumer search cost are identical across all firms, a consumer has a *single* (constant) reservation price z^* (for all firms). The consumer stops searching after receiving the first price below his reservation price and makes a purchase. Thus the consumer *always* purchases from the firm searched last in such a setting.

Discussion

A question that often comes up is whether one search method is “better” than the other, i.e. whether consumers should always search simultaneously or should always search sequentially. While in many settings searching sequentially is better for consumers because they can stop searching at any point of time if they get a good draw early on, Morgan and Manning (1985) showed that simultaneous search can be better than sequential search when a consumer is close to a deadline, i.e. needs to gather information quickly. Chade and Smith (2006)

¹⁰This discussion and results hold when search costs are identical across consumers and when search costs are heterogenous across consumers. The discussion and results do not hold when there is heterogeneity in search costs across firms.

and Kircher (2009) further found that simultaneous search is better when the other side of the market might reject the individual (e.g. students searching for higher education by submitting college applications).

It is important to note that neither simultaneous nor sequential search by itself is the best search method for consumers. Morgan and Manning (1985) show that a combination of both simultaneous and sequential search (at the various stages of search) dominates either pure simultaneous or pure sequential search. In an experimental study, Harrison and Morgan (1990) make a direct comparison between such a hybrid search strategy and either simultaneous or sequential search strategies and find that their experimental subjects use the least restrictive search strategy if they are allowed to do so.

3 Early Empirical Literature

3.1 Consideration Set Literature

The marketing literature has long recognized that consumers may not consider all products in a purchase situation. Consideration has been viewed as one among commonly three stages (awareness, consideration, and choice/purchase) in a consumer’s purchase process.¹¹ However, the marketing literature has varied in the approaches used to view and model consideration.

We structure our discussion of this stream of literature in a chronological order and review three groups of papers from the early 90’s, the late 90’s and 00’s, and more recent work from the 10’s.¹² This chronological structure is not driven by time itself, but rather by papers written around the same time sharing common themes. For example, the papers from the early 90’s are rooted in the economic search literature, while the papers from late 90’s and 00’s employ more descriptive, statistical models. The last group of papers from the ‘10s contains a more diverse set of papers that, for example, uses experimentation in support of statistical modeling or studies under which circumstances unobserved consideration sets can be identified.

Before we dive into the details, it is important to note *why* consideration matters. When consumers have limited information, i.e. only consider a subset of all available products for purchase, and this limited information is not accounted for in the model and estimation, it will lead to biased preference estimates. Since preference estimates are used to calculate (price) elasticities, make recommendations on the employment of marketing mix elements,

¹¹These three stages are also sometimes referred to as the “purchase funnel.”

¹²Roberts and Lattin (1997) provide an overview of the marketing literature on consideration sets between 1990 and 1997.

draw conclusions of the competitiveness of a market, etc., biased preference estimates might result in the wrong conclusions. This is a point that has been consistently made throughout this stream of literature.

Early 1990's

This group of papers contains work by Hauser and Wernerfelt (1990) and Roberts and Lattin (1991). Both papers base their approaches on the consumer search literature.¹³ Hauser and Wernerfelt (1990) propose that consumers search sequentially to add (differentiated) products to their consideration sets.¹⁴ Through search, consumers resolve uncertainty about the net product utility, i.e. utility minus price. The authors then discuss aggregate, market-level implications of their model such as order-of-entry penalties and competitive promotion intensity. Hauser and Wernerfelt (1990) also provide an overview table with mean or median consideration set sizes from previously published studies and the Assessor database for a variety of product categories. They find most consideration sets to include 3 to 5 brands.

Roberts and Lattin (1991) develop a simultaneous search model in which consumers consider a brand as long as the utility of that brand is above an individual-specific utility threshold. To make the model estimable, the authors include a mis-specification error. They calibrate their model using survey data from the ready-to-eat cereal market containing information on consumers' considerations and purchases. Roberts and Lattin (1991) find a median consideration set size of 14.¹⁵ Lastly, the authors compare the predictive ability of their two-stage model to several benchmark models.

Late 1990's and 2000's

This group of papers contains a body of work that focuses on more descriptive, statistical models. The main characteristics can be summarized as follows: first, consideration is neither viewed nor modeled as driven by uncertainty about a specific product attribute, e.g. price or match value. And second, there is limited empirical consensus on the drivers of consideration. Commonly, consideration is modeled as a function of marketing mix variables (advertising, display, feature, and, to a lesser extent, price). For example, Allenby and Ginter (1995)

¹³Ratchford (1980) develops a simultaneous search model for differentiated goods in which consumers have uncertainty about prices and other product attributes and estimates the gains to searching using data on four household appliances. However, search is not explicitly connected to consideration in this paper.

¹⁴Hauser and Wernerfelt (1990) also propose that, conditional on consideration, consumers pick a smaller subset of products that they evaluate for purchase. This evaluation is costly to consumers and consumers form the smaller subset for purchase evaluation using a simultaneous search step.

¹⁵Roberts and Lattin (1991) explain the larger consideration set sizes by clarifying that they equate consideration with awareness and that aided awareness was used to elicit the considered brands.

and Zhang (2006) estimate the effects of feature and display on consideration; Andrews and Srinivasan (1995) model consideration as a function of loyalty, advertising, feature, display, and price; Bronnenberg and Vanhonacker (1996) model consideration (saliency) as a function of promotion, shelf space, and recency (among others); Ching et al. (2009) model consideration as a function of price; and Goeree (2008) and Terui et al. (2011) model consideration as a function of advertising.

In the following, we discuss three aspects of this group of papers: modeling, decision rules, and data together with empirical results. Two approaches to modeling consideration emerge: one in which the probability of a consideration *set* is modeled (e.g. Andrews and Srinivasan 1995, Chiang et al. 1999) and a second one in which the probability of a specific product being considered is modeled (e.g. Siddarth et al. 1995, Bronnenberg and Vanhonacker 1996, Goeree 2008). The papers also vary in terms of the specific model being estimated ranging from a heteroscedastic logit model (Allenby and Ginter 1995) over dogit models (e.g. Siddarth et al. 1995) and (utility) threshold models (e.g. Siddarth et al. 1995, Andrews and Srinivasan 1995) to aggregate random coefficients logit demand model based on Berry et al. (1995) (e.g. Goeree 2008).^{16,17}

Most of the consideration set papers published during this time period assume that consumers use compensatory decision rules, i.e. a product can “compensate” a very poor value in one attribute with a very good value in another attribute. However, a smaller set of papers models consideration using non-compensatory rules, i.e. a product attribute has to meet a certain criterion for the product to be considered and/or chosen (see also Aribarg et al. 2018). Non-compensatory rules are often proposed based bounded rationality arguments, often in the form of decision heuristics. For example, Fader and McAlister (1990) develop an elimination-by-aspects model in which consumers screen brands depending on whether these brands are on promotion. Gilbride and Allenby (2004) propose a model that can accommodate several screening rules for consideration: conjunctive, disjunctive, and compensatory. While Fader and McAlister (1990) find that their elimination-by-aspects and a compensatory model fit the data similarly (but result in different preference estimates), Gilbride and Allenby (2004) report that their conjunctive model fits the data best. In general, identification of compensatory and non-compensatory decision rules with non-experimental data is very difficult (see also Aribarg et al. 2018).

¹⁶In a dogit model, a consumer probabilistically chooses from either the full set of alternatives or from a considered set of alternatives.

¹⁷Threshold models such as in Siddarth et al. (1995) and Andrews and Srinivasan (1995) assume that a consumer’s *utility* for a product has to be above a certain value for the product to be in the consideration set. In contrast, models using non-compensatory rules assume that one or more *attributes* of a product have to have a value above or below a certain threshold for the product to be in the consideration set.

Due to data restrictions, all the models suggested in the beforementioned papers are estimated using choice data alone (usually supermarket scanner panel data). Therefore identification of consideration comes from functional form, i.e. non-linearities of the model and modeling approaches are assessed based on model fit criteria. Few of the beforementioned papers report predicted consideration set sizes. Two exceptions are Siddarth et al. (1995) and Bronnenberg and Vanhonacker (1996): the former paper reports that the average predicted consideration set includes 4.2 brands, while the latter predicts that the average consideration set for loyal and not-loyal customers includes 1.5 and 2.8 brands, respectively. And lastly, most papers find that marketing mix variables rather affect consideration than purchase. For example, Terui et al. (2011) find advertising to affect consideration, but not purchase.

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This group contains a more diverse set of papers that, for example, uses experimentation in support of statistical modeling, studies under which circumstances unobserved consideration sets can be identified or investigates preference heterogeneity estimates.

Van Nierop et al. (2010) estimate a consideration and purchase model in which advertising affects the formation of consideration sets, but does not affect preferences. The authors combine scanner panel data with experimental data to show that consideration sets can be reliably recovered using choice data only and that feature and display affect consideration in their empirical context.

And lastly, as discussed at the beginning of this section, not accounting for consumers' limited information results in biased preference estimates (e.g. Bronnenberg and Vanhonacker 1996, Chiang et al. 1999, Goeree 2008, De los Santos et al. 2012, Honka 2014). Two papers, Chiang et al. (1999) and Dong et al. (2018), put a special focus on preference heterogeneity estimates under full and limited information. Both papers find that the amount of preference heterogeneity is overestimated if consumers' limited information is not accounted for.

Identification of Unobserved Consideration Sets

An important identification question in the consideration set literature as well as the search literature is whether changes in demand originate from shifts in consideration or shifts in utility. In many empirical settings the researcher does not have access to data on consideration, or may not have access to an instrument that can be excluded from utility or consideration, which makes it important to know to what extent a consideration set model can still be separately identified from a full information model. Abaluck and Adams (2018)

show that consideration set probabilities are identified from asymmetries of cross-derivatives of choice probabilities with respect to attributes of competing products. This means that, for identification, it is not necessary to use data on consideration sets or to assume that there are characteristics that affect consideration set probabilities but do not appear in the utility function.

In a model in which consumers have full information, consumers will consider all available options. The full consideration assumption implies that there is a symmetry in cross derivatives with respect to one or more characteristic of the product: a consumer will be equally responsive to a change in the price of product A as to a similar change in the price of product B . However, under certain assumptions it can be shown that this symmetry breaks down when a change in the characteristic of the product changes the consideration set probability of that product. Abaluck and Adams (2018) provide formal identification results for two classes of consideration set models: the “Default-Specific Consideration” (DSC) model and the “Alternative-Specific Consideration” (ASC) model. The DSC model fits into a rational inattention framework and assumes that the probability of considering other options than a default option only depends on the characteristics of the default option. The ASC model assumes that the probability of considering an option only depends on the characteristics of that good.

In most theoretical search models, the probability of considering an option depends on the characteristics of *all* goods, which means that conventional search models do not fit in either the DSC or ASC framework. Even though this implies that their formal identification results do not apply directly to search models, Abaluck and Adams (2018) do suggest that cross-derivative asymmetries remain a source of identifying power for consideration probabilities in more complicated consideration set models in which the consideration probability for one good depends on the consideration probability for another good. Whether this indeed implies that conventional search models can be identified using asymmetric demand responses only is not formally shown, however, and remains an important area for future research.

In a related paper, Crawford et al. (2018) show how to estimate preferences that are consistent with unobserved, heterogeneous consumer choice sets using the idea of sufficient sets. These sufficient sets are subsets of the unobserved choice sets and can be operationalized as products purchased by the consumer in the past or products contemporaneously purchased by other similar consumers. Kawaguchi et al. (2018) focus on advertising effectiveness and show how to use variation in product availability as a source of identification in consideration set models.

3.2 Consumer Search Literature

The early empirical literature in economics initially focused on documenting price dispersion as well as testing some of the comparative statics results that were derived from theoretical search models. For instance, Sorensen (2000) examines retail prices for prescription drugs and finds substantial price variation, even after controlling for differences among pharmacies. In addition, he finds evidence that prices and price dispersion are lower for prescriptions that are purchased more frequently. This finding is consistent with search theory predictions since the gains from search are higher for frequently purchased products.

Driven by the rise of e-commerce around the turn of the millennium, subsequent work focused on price dispersion for goods sold on the Internet and how online prices compared to prices for products sold in traditional brick and mortar stores. Most of these studies found substantial price dispersion for products sold online, despite the popular belief around the time that online comparison shopping would lead to Bertrand pricing. For instance, Clay et al. (2001) find considerable heterogeneity in pricing strategies for online bookstores and Clemons et al. (2002) report that online travel agents charge substantially different prices, even when given the same customer request.

Starting with Hortaçsu and Syverson (2004) and Hong and Shum (2006), the literature began to move away from a reduced-form focused approach to more structural modeling. The idea was to use the structure of a theoretical search model to back out the search cost distribution from observed market data such as prices and quantities sold. In this section, we discuss both papers as well as several other studies that build on these papers. Hong and Shum (2006) focus on the estimation of homogeneous goods search models, whereas Hortaçsu and Syverson (2004) allow for vertical product differentiation. However, in both papers, consumers search randomly across firms. This model feature distinguishes these two papers from more recent contributions in which consumers search a combination of horizontally and vertically differentiated firms, which makes consumers want to search in an ordered way.

Estimation of Search Costs for Homogeneous Products

Hong and Shum (2006) develop methods to estimate search cost distributions for both simultaneous and sequential search models using only price data. An attractive feature of their simultaneous search model, which is based on Burdett and Judd (1983), is that search costs can be non-parametrically identified using price data only. To identify search costs in their sequential search model, parametric assumptions are needed. Since most of the follow-up literature has focused on simultaneous search, we will now briefly describe the essence of

their estimation method for that search model.

The main idea is to use the equilibrium restrictions of the theoretical search model as well as observed prices to back out the search cost distribution that is consistent with the theoretical model. As in Burdett and Judd (1983), firms are assumed to be homogenous. Price dispersion emerges as a symmetric mixed-strategy Nash equilibrium: firms have an incentive to set lower prices to attract consumers who are searching, but at the same time face an incentive to set higher prices to extract surplus from consumers who are not searching. By playing a mixed strategy in prices according to a distribution $F(p)$, firms can balance these two forces. Given such a price distribution $F(p)$, a firm's profit when setting a price p is given by

$$\Pi(p) = (p - r) \left[\sum_{k=1}^{\infty} q_k (1 - F(p))^{k-1} \right],$$

where q_k is the share of consumers who search k times and r is the firm's unit cost. The mixed strategy equilibrium requires firms to be indifferent between setting any price in the support of the price distribution, which results in the following equilibrium profit condition:

$$(\bar{p} - r)q_1 = (p - r) \left[\sum_{k=1}^{\infty} q_k (1 - F(p))^{k-1} \right], \quad (5)$$

where the expression on the left-hand side of this equation is the profit when setting a price equal to the upper bound \bar{p} of the price distribution $F(p)$. Equation (5) has to hold for any observed price that is consistent with this equilibrium condition, i.e.,

$$(\bar{p} - r)q_1 = (p_i - r) \left[\sum_{k=1}^K q_k (1 - F(p_i))^{k-1} \right], \quad i = 1, \dots, n - 1, \quad (6)$$

where K is the maximum number of firms from which a consumer obtains price quotes and n is the number of price observations. Since equation (6) implies $n - 1$ equations and K unknowns, this system can be solved for the unknowns $\{r, q_1, \dots, q_K\}$ as long as $K < n - 1$. Hong and Shum (2006) develop a Maximum Empirical Likelihood (MEL) approach to do so. To obtain a non-parametric estimate of the search cost distribution, the estimates of the q_k 's can then be combined with estimates of the critical search cost values Δ_i , which are given by

$$\Delta_i = Ep_{1:i} - Ep_{1:i+1}, \quad i = 1, 2, \dots, n - 1,$$

where $p_{1:i}$ is the lowest price out of i draws from the price distribution $F(p)$.

To illustrate their empirical approach, Hong and Shum (2006) use online prices for four economics and statistics textbooks for model estimation. The estimates of their nonsequential search model indicate that roughly half of consumers do not search beyond the initial free price quote.

Moraga-González and Wildenbeest (2008) extend Hong and Shum (2006)'s approach to the case of oligopoly. Besides allowing for a finite number of firms instead of infinitely many firms, the model is similar to the simultaneous search model in Hong and Shum (2006). However, instead of using equation (6) and a MEL approach, they use a maximum likelihood (MLE) procedure. Specifically, Moraga-González and Wildenbeest (2008) solve the first-order condition for the equilibrium price density, which is then used to construct the likelihood function. The density function is given by

$$f(p) = \frac{\sum_{k=1}^N kq_k(1 - F(p))^{k-1}}{(p - r) \sum_{k=1}^N k(k - 1)q_k(1 - F(p))^{k-2}}$$

where N is the number of firms and $F(p)$ solves equation (6) for $K = N$. The log-likelihood function is then

$$LL = \sum_n \log f(p; q_1, \dots, q_N),$$

where the parameters to be estimated are the shares of consumers searching q_k times. Moraga-González and Wildenbeest (2008) estimate the model using online price data for computer memory chips and find that even though a small share of consumers of around ten percent searches quite intensively, the vast majority of consumers does not obtain more than three price quotes. Moreover, estimates of average price-cost margins indicate that market power is substantial, despite having more than twenty stores operating in this market.

Although MEL has some desirable properties such as requiring fewer assumptions regarding the underlying distribution, estimating the model using MEL requires solving a computationally demanding high-dimensional constrained optimization problem, which may fail to converge when the number of search cost parameters is large. Indeed, Moraga-González and Wildenbeest (2008) compare the two approaches in a Monte Carlo study and find the MLE approach to work better in practice, especially with respect to pinning down the consumers who search intensively. Moreover, they find that the MLE procedure outperforms the MEL procedure in terms of fit.

Several papers have extended the Hong-Shum approach. Most of these papers use a MLE approach as in Moraga-González and Wildenbeest (2008). A general finding is that in most of the markets studied, consumers either search very little (at most two times) or search

a lot (close to all firms). This finding has been interpreted as some consumers using price comparison tools, which allows a consumer to get a complete picture of prices without having to visit each retailer individually.

Wildenbeest (2011) adds vertical product differentiation to the framework and derives conditions under which the model can still be estimated using price data only. Specifically, by assuming that consumers have identical preferences towards quality, that input markets are perfectly competitive, and that the quality production function has constant returns to scale, he maps a vertical product differentiation model into a standard homogeneous goods model with firms playing mixed strategies in utilities. The model is estimated using price data for a basket of staple grocery items that are sold across four major supermarket chains in the United Kingdom. The estimates indicate that approximately 39 percent of price variation is explained by search frictions, while the rest is due to quality differences among stores. About 91 percent of consumers search at most two stores, suggesting that there is not a lot of search going on in this market. Moreover, ignoring vertical product differentiation when estimating the model leads to higher search cost estimates.

Moraga-González et al. (2013) focus on the non-parametric identification of search costs in the simultaneous search model and show that the precision of the estimates can be improved by pooling price data from different markets. They propose a semi-nonparametric (SNP) density estimator that uses a flexible polynomial-type parametric function, which makes it possible to combine data from different markets with the same underlying distribution of search costs, but with different valuations, unit costs, and numbers of firms. The estimator is designed to maximize the joint likelihood from all markets, and as such the SNP procedure exploits the data more efficiently than the spline methods that are used in earlier papers (e.g. Hong and Shum 2006, Moraga-González and Wildenbeest 2008). To illustrate the estimation approach, Moraga-González et al. (2013) use a dataset of online prices for ten memory chips. Median search costs are estimated to be around \$5. Search costs are dispersed, with most consumers having high enough search costs to find it optimal to search at most three stores, while a small fraction of consumers searches more than four times.

Blevins and Senney (2019) add dynamics to the model by allowing consumers to be forward looking. In addition to deciding how many times to search in each period, consumers have the option to continue searching in the next period. Per-period search costs can be estimated using the approach in Moraga-González and Wildenbeest (2008) or Wildenbeest (2011), but to estimate the bounds of the population search cost distribution, a specific policy function must be estimated. Blevins and Senney (2019) apply the estimation procedure to the online market for two popular econometrics textbooks and find that median search costs for the dynamic model are much lower than for a static model, which suggests that search

cost estimates are likely to be biased upwards when forward-looking behavior of consumers is ignored.

Sanches et al. (2018) develop a minimum distance approach to estimate search costs, which is easier to implement than previous methods. In addition, they propose a two-step sieve estimator to estimate search costs when data from multiple markets are available. The sieve estimator only involves ordinary least squares estimation and is therefore easier to compute than other approaches that combine data from multiple markets, such as the SNP estimator in Moraga-González et al. (2013). As an illustration of their approach, Sanches et al. (2018) estimate search costs using online odds for English soccer matches as prices and find that search costs have fallen after bookmakers were allowed to advertise more freely as a result of a change in the law.

Nishida and Remer (2018) provide an approach to combine search cost estimates from different geographic markets and show how to incorporate wholesale prices and demographics into the Hong-Shum framework. Specifically, they first non-parametrically estimate market-specific search cost distributions for a large number of geographically isolated gasoline markets using a vertical product differentiation model similar to Wildenbeest (2011). Then they use these estimates to parametrically estimate a search cost distribution that allows them to incorporate demographic information. Nishida and Remer (2018) find significant variation in search costs across the different geographic markets. Moreover, they find a positive relation between the estimated distribution of search costs and the income distribution.

Zhang et al. (2018) use a MEL approach, as in Hong and Shum (2006), and show how to incorporate sales data into the estimation of both the simultaneous and the sequential search model. They show that including sales data results in estimates that are less sensitive to assumptions about the maximum number of searches consumers can conduct. Moreover, the sequential search model can be estimated non-parametrically when both price and sales data are used. The model is estimated using price and transaction data for a chemical product in a business-to-business environment. Findings show that the sequential search model provides a better fit.

Estimation of Search Costs for Vertically Differentiated Products

Hortaçsu and Syverson (2004) extend the methodology of Hong and Shum (2006) to the case where products are allowed to be vertically differentiated and a sequential search protocol is followed. Vertical differentiation takes the form of an index based on observable product attributes and an unobservable attribute, where the index weightings, along with search cost parameters, can be estimated. Unlike Hong and Shum (2006), price data alone is not sufficient to identify model parameters; quantity and market share information, along with

data on product characteristics are also necessary. Like Hong and Shum (2006), nonparametric identification results are obtained for the underlying search cost distribution and the quality index for each product (which, in a manner similar to Berry (1994) and Berry et al. (1995), can be projected onto observable product characteristics with a suitable instrumental variable for the unobserved product attribute). While the model allows for specification of preference heterogeneity across different consumer segments, horizontal differentiation in the form of additive random utility shocks is not considered; the model rationalizes nonzero market shares for dominated products through search costs.

Empirically, Hortaçsu and Syverson (2004) study the S&P 500 index fund market, where substantial dispersion in fees is documented. While this may be surprising given that all index funds have the goal of replicating the return characteristics of the S&P 500 index, some return deviations across funds may exist, along with non-financial drivers of differentiation. Thus, the model allows for vertical differentiation between funds with non-trivial market shares for dominated products arising from costly sequential search.

The utility from investing in fund j is a linear function of fund characteristics:

$$u_j = X_j\beta - p_j + \xi_j, \quad (7)$$

where X_j are fund characteristics other than price p_j and an unobservable component ξ_j . The coefficient on the price term is normalized to -1 , so utilities are expressed in terms of basis points in fees (one hundredth of a percentage point). Thus one can think of u_j as specifying fund utility per dollar of assets the investor holds in it.

Search costs are heterogeneous in the investor population and follow distribution $G(c)$. As in Carlson and McAfee (1983) investors search with replacement and are allowed to revisit previously researched funds. Defining investors' belief about the distribution of funds' indirect utilities as $H(u)$, the optimal search rule for an investor with search cost c_i is given by the reservation utility rule

$$c_i \leq \int_{u^*}^{\bar{u}} (u - u^*) dH(u),$$

where \bar{u} is the upper bound of $H(u)$, and u^* is the indirect utility of the highest-utility fund searched up to that point.

Assuming that investors observe the empirical cumulative distribution function of funds' utilities, $u_1 < \dots < u_N$, the expression for $H(u)$ becomes

$$H(u) = \frac{1}{N} \sum_{j=1}^N \mathbb{I}[u_j \leq u].$$

The optimal search rule yields critical cut-off points in the search distribution given by

$$c_j = \sum_{k=j}^N \rho_k (u_k - u_j), \quad (8)$$

where ρ_k is the probability that fund k is sampled on each search and c_j is the lowest possible search cost of any investor who purchases fund j in equilibrium.

Funds' market shares can be written in terms of the search cost cdf by using the search-cost cutoffs from equation (8). Only investors with very high search costs ($c > c_1$) purchase the lowest-utility fund, u_1 ; all others continue to search. However, not all investors with $c > c_1$ purchase the fund; only those ones who happen to draw fund 1 first, which happens with probability ρ_1 . Thus the market share of the lowest-utility fund is given by

$$q_1 = \rho_1(1 - G(c_1)) = \rho_1 \left(1 - G \left(\sum_{k=1}^N \rho_k (u_k - u_1) \right) \right). \quad (9)$$

Analogous calculations produce a generalized market share equation for funds 2 to N :

$$q_j = \rho_j \left[1 + \frac{\rho_1 G(c_1)}{1 - \rho_1} + \sum_{k=2}^{j-1} \frac{\rho_k G(c_k)}{(1 - \rho_1 - \dots - \rho_{k-1})(1 - \rho_1 - \dots - \rho_k)} - \frac{G(c_j)}{(1 - \rho_1 - \dots - \rho_{j-1})} \right]. \quad (10)$$

These equations form a system of linear equations linking market shares to cutoffs in the search cost distribution. Equation (10) maps observed market shares to the cdf of the search cost distribution evaluated at the critical values. Given the sampling probabilities ρ_j , all $G(c_j)$ can be calculated directly from market shares. Solving the linear system (10) to recover $G(c_1), \dots, G(c_{N-1})$ and using the fact that $G(c_N) = 0$ (equation (8) implies $c_N = 0$ and search costs cannot be negative) gives all critical values of the cdf. If the sampling probabilities are unknown and must be estimated, the probabilities as well as the search cost distribution can be parameterized as $\rho(\omega_1)$ and $G(c; \omega_2)$, respectively. Given ω_1 and ω_2 of small enough dimension, observed market shares can be used to estimate these parameters.

While market share data can be mapped into the cdf of the search cost distribution, market shares do not generally identify the level of the critical search cost values c_1, \dots, c_N ,

but only their relative positions in the distribution. However, shares do identify search cost levels in the special but often-analyzed case of homogeneous (in all attributes but price) products with unit demands; i.e., when $u_j = u' - p_j$, where u' is the common indirect utility delivered by the funds. In this case, equation (8) implies

$$c_j = \sum_{k=j}^N \rho_k (u' - p_k - (u' - p_j)) = \sum_{k=j}^N \rho_k (p_j - p_k). \quad (11)$$

Now, given sampling probabilities (either known or parametrically estimated), c_1, \dots, c_{N-1} can be calculated directly from observed fund prices using equation (11).

In the more general case where products also differ in other attributes than price, information on fund companies' optimal pricing decisions is required to identify cutoff search cost values. To do this, a supply side model has to be specified. Hortaçsu and Syverson (2004) assume that the F funds choose prices to maximize current static profits. Let S be the total size of the market, p_j and mc_j be the price and (constant) marginal costs for fund j , and q_j be fund j 's market share given the price and characteristics of all sector funds. Then the profits of fund j are given by

$$\Pi_k = S q_j(p, X) (p_j - mc_j).$$

Profit maximization implies the standard first-order condition for p_j :

$$q_j(p, X) + (p_j - mc_j) \frac{\partial q_j(p, X)}{\partial p_j} = 0. \quad (12)$$

The elasticities $\partial q/\partial p$ faced by the fund are determined in part by the derivatives of the share equations (10). These derivatives are:

$$\begin{aligned} \frac{\partial q_j}{\partial p_j} = & -\frac{\rho_1 \rho_j^2 g(c_1)}{1 - \rho_1} - \frac{\rho_2 \rho_j^2 g(c_2)}{(1 - \rho_1)(1 - \rho_1 - \rho_2)} - \sum_{k=3}^{j-1} \frac{\rho_k \rho_j^2 g(c_k)}{(1 - \rho_1 - \dots - \rho_{k-1})(1 - \rho_1 - \dots - \rho_k)} \\ & - \frac{\rho_j \left(\sum_{k=j+1}^N \rho_k \right) g(c_j)}{(1 - \rho_1 - \dots - \rho_{j-1})}. \end{aligned} \quad (13)$$

The pdf of the search cost distribution (evaluated at the cutoff points) enters the derivatives of the market share equations with respect to price (see equation 13). Under Bertrand-Nash competition, the first order conditions for prices (equation 12) imply:

$$\frac{\partial q_j(p)}{\partial p_j} = -\frac{q_j(p)}{p_j - mc_j}. \quad (14)$$

Given knowledge of marginal costs mc_j , we can compute $\partial q_j/\partial p_j$ using the first-order condition in equation (14). From equation (13), these derivatives form a linear system of $N - 1$ equations that can be used to recover the values of the search cost density function $g(c)$ at the critical values c_1, \dots, c_{N-1} . If marginal costs are not known, they can be parameterized along with the search cost distribution and estimated from the price and market share data.

Once both the values of the search cost cdf and pdf (evaluated at the cutoff search costs) have been identified, the level of these cutoff search costs c_j in the general case of heterogeneous products can be identified. By definition, the difference between the cdf evaluated at two points is the integral of the pdf over that span of search costs. This difference can be approximated using the trapezoid method, i.e.,

$$G(c_{j-1}) - G(c_j) = 0.5[g(c_{j-1}) + g(c_j)](c_{j-1} - c_j).$$

This equation is inverted to express the differences between critical search cost values in terms of the cdf and pdf evaluated at those points, i.e.

$$c_{j-1} - c_j = \frac{2[G(c_{j-1}) - G(c_j)]}{g(c_{j-1}) + g(c_j)}. \quad (15)$$

Given the critical values of $G(c)$ and $g(c)$ obtained from the data above, one can recover the c_j , and from these trace out the search cost distribution.¹⁸ In non-parametric specifications, a normalization is required: the demand elasticity equations do not identify $g(c_N)$, so a value must be chosen for the density at zero-search costs (recall that $c_N = 0$).

Finally, the critical values of the search cost distribution can be used to estimate the indirect utility function (equation(7)). The implied indirect utilities of the funds u_j are derived from the cutoff search costs via the linear system in equation (8) above.¹⁹ One can then regress the sum of these values and the respective fund's price (because of the imposed unit price coefficient) on the observable characteristics of the fund to recover β , the weights of the characteristics in the indirect utility function. One must be careful, however, as the unobservable components ξ are likely to be correlated with price, which would result in

¹⁸Any monotonically increasing function between the identified cutoff points could be consistent with the true distribution; the trapezoid approximation essentially assumes this is linear. The approximated cdf converges to the true function as the number of funds increases.

¹⁹In the current setup, equation (8) implies that $u_1 = 0$, so fund utility levels are expressed relative to the least desirable fund. This normalization results from the assumption that all investors purchase a fund; if there is an outside good that could be purchased without incurring a search cost, one could alternatively normalize the utility of this good to zero.

biased coefficients in ordinary least squares regressions. Therefore, as in Berry (1994) and Berry et al. (1995), one can use instrumental variables for price to avoid this problem.

Estimation of the model using data on S&P 500 index funds between 1995-2000 reveals that product differentiation indeed plays an important role in this market: investors value funds' non-financial characteristics such as fund age, total number of funds in the fund family, and tax exposure. After taking vertical product differentiation into account, fairly small but heterogeneous search costs (the difference between the 25th and 75th percentiles varies between 0.7 to 28 basis points) can rationalize the very substantial price dispersion (the 75th percentile fund charged more than three times the fee charged by the 25th percentile fund). The estimates also suggest that search costs are shifting over time, consistent with the documented influx of high search cost and financially inexperienced mutual fund investors into the market during a period of sustained stock market gains.

Roussanov et al. (2018) utilize the Hortacısu and Syverson (2004) model to analyze the broader market for U.S. equity mutual funds and find that investor search and the marketing efforts of mutual fund managers to influence investor search towards their funds can explain a substantial portion of the empirical relationship between mutual fund performance and mutual fund flows. Using their structural estimates, the authors find that marketing is a very important determinant along with performance, fees, and fund size. In a counterfactual exercise that bans marketing by mutual fund managers, Roussanov et al. (2018) find that capital shifts towards cheaper funds, and that capital is allocated in a manner more closely aligned with (estimated) manager skills.

Econometrically estimated search models have found applications in several other important financial products markets, where products are complex and consumers are relatively inexperienced and/or uninformed about contract details. Allen et al. (2013) estimate search costs that rationalize the dispersion of mortgage rates in Canada. Woodward and Hall (2012) find substantial dispersion in mortgage closing/brokerage costs and, using a model of search with broker competition, estimate large gains (exceeding \$1,000 for most borrowers) from getting one more quote. An important modeling challenge in many financial products markets is the fact that loans and most securities are priced through a process of negotiation. This poses an interesting econometric challenge in that the prices of alternatives that are not chosen by the consumer are not observed in the data. Woodward and Hall (2012), Allen et al. (2018), and Salz (2017) are recent contributions that address this problem by specifying an auction process between lenders/providers in the consumer's consideration set. Given the importance of understanding choice frictions faced by consumers in these markets, which have been under much scrutiny and regulatory action since the 2008 financial crisis, future research in this area is very well warranted.

4 Recent Advances: Search and Consideration Sets

In this section, we discuss recent empirical work which makes an explicit connection between search and consideration, i.e. search is viewed as the process through which consumers form their consideration sets. While this idea might appear intuitive, the two streams of literature on consideration sets (in marketing) and consumer search (in economics) have existed largely separately until recently. We organize this section by consumers' source of uncertainty: In Section 4.1, we discuss papers in which consumers search to resolve uncertainty about prices and in Section 4.2, we discuss papers in which consumers search to resolve uncertainty about the match value or product fit.

4.1 Searching for Prices

We structure our discussion of papers that have modeled consumer search for prices by search method: in Mehta et al. (2003), Honka (2014), and De los Santos et al. (2012), consumers search simultaneously, whereas consumers search sequentially in Honka and Chintagunta (2017).

Mehta et al. (2003)

The goal of Mehta et al. (2003) is to propose a structural model of consideration set formation. The authors view searching for prices as the process that consumers undergo to form consideration sets. Mehta et al. (2003) apply their model to scanner panel data, i.e., data that contains consumer purchases together with marketing activities but does not contain information on consideration sets, from two categories (liquid detergent and ketchup) with four brands in each. The authors find average predicted consideration set sizes to vary between 1.8 and 2.8 across both product categories, pointing to consumers incurring sizable search costs to resolve uncertainty about prices. Further, they find that in-store display and feature ads significantly reduce search costs, while income significantly increases search costs. Lastly, Mehta et al. (2003) report that consumers' price sensitivity is underestimated if consumers' limited information is not taken into account.

In the following, we provide details on the modeling and estimation approach. As mentioned before, Mehta et al. (2003) develop a structural model in which consumers search simultaneously to learn about prices and this search process leads to the formation of consumers' consideration sets. Consumer i 's utility function is given by

$$u_{ijt} = \theta q_{ijt} - p_{ijt}$$

where prices p_{ijt} are assumed to follow a Extreme Value (EV) Type I distribution with $p_j \sim EV(\bar{p}_j, \sigma_{p_j}^2)$, and q_{ijt} being the perceived quality of a product which is observed by both the consumer and the researcher.²⁰ The parameter θ is consumer i 's sensitivity to quality and is estimated. Note that there is no error term in the above utility specification. If an error term were to be included, Mehta et al. (2003) would not be able to separately identify the baseline search cost c_0 from the true quality of brands q_j .

Given the distributional assumption for prices, consumer i 's utility also follows a EV Type I distribution with

$$u_{ijt} \sim EV\left(\theta q_{ijt} - \bar{p}_j, \sigma_{u_j}^2\right)$$

and $\sigma_{p_j} = \sigma_{u_j}$. Mehta et al. (2003) use the choice model approach described in Section 2.2 to model consumers' choices of consideration sets, i.e. consumers calculate the net benefit of every possible consideration set and pick the one that gives them the largest net benefit. The choice model approach is feasible despite the curse of dimensionality of the simultaneous search model because Mehta et al. (2003) apply their model to scanner panel data from two categories and focus on the four main brands in each category.²¹

The expected net benefit of a specific consideration set Υ (determined by its size k and its composition) is given by²²

$$EB_{\Upsilon} = \frac{\sqrt{6}\sigma_u}{\pi} \ln \left(\sum_{l=1}^k \exp \left(\frac{\pi}{\sqrt{6}\sigma_u} (\theta w_{ilt} - \bar{p}_{il}) \right) \right) - \sum_{l=1}^k c_{ilt} \quad (16)$$

with $c_{ilt} = c_0 + W_{ilt}\delta$. The consumer picks the consideration set (determined by its size and composition) that maximizes the net benefit of searching. Once consumer i has searched all products in his consideration set, he has learned about their prices and all uncertainty is resolved. The consumer then picks the product that provides him with the largest utility among the considered ones.

Next, we describe how Mehta et al. (2003) estimate their model. Consumer i 's unconditional purchase probability is the product of consumer i 's consideration and conditional purchase probabilities, i.e.

$$P_{ij} = P_{C_i} P_{ij|C_i}.$$

²⁰The perceived quality is assumed to be updated in a Bayesian fashion after each product purchase. We refer the reader to Mehta et al. (2003) for details on this process.

²¹Mehta et al. (2003) can reduce the number of consideration sets by dropping those that do not include the purchased brand.

²²Since prices and thus utilities (see equation (4.1)) follow an EV Type I distribution, the maximum utility of a set of EV Type I distributions also follows an EV Type I distribution (Johnson et al. 1995).

The probability that consumer i considers consideration set Υ (determined by its size k and its composition) can be written as

$$P(C_i = \Upsilon) = P(EB_{\Upsilon} \geq EB_{\Phi} \quad \forall \Upsilon \neq \Phi).$$

Lastly, given that the qualities q_{ijt} are truncated normal random variables (see Mehta et al. 2003), the conditional choice probabilities are given by probit probabilities.

Honka (2014)

Honka (2014) studies the auto insurance market. Insurance markets can be inefficient for several reasons with adverse selection and moral hazard being the two most extensively studied reasons. Honka (2014) investigates a different source of market inefficiency: market frictions. She focuses on two types of market frictions, namely, search and switching costs and estimates demand for auto insurance in their presence. Honka (2014) uses an individual-level data set in which, in addition to information and purchases, prices, and marketing activities, she also observes consumers' consideration sets. She finds search costs to vary from \$35 to \$170 and switching costs of \$40. Further, she reports that search costs are the main driver of the very high customer retention rate in this industry and their elimination is the main lever to increase consumer welfare.

In the following, we provide details on the modeling and estimation approach: as in Mehta et al. (2003), Honka (2014) also estimates a structural model in which consumers search simultaneously to resolve uncertainty about prices. She models search as the process through which consumers form their consideration sets. Consumer i 's indirect utility for company j is given by

$$u_{ij} = \alpha_{ij} + \beta_i I_{ij,t-1} + \gamma p_{ij} + X_{ij} \rho + W_i \phi + \epsilon_{ij}$$

with α_{ij} being consumer-specific brand intercepts. β_i captures consumer inertia and can be decomposed into $\beta_i = \tilde{\beta} + Z_i \kappa$ with Z_i being observable consumer characteristics. $I_{ij,t-1}$ is a dummy variable indicating whether company j is consumer i 's previous insurer. Note that as observed heterogeneity interacts with $I_{ij,t-1}$, it plays a role in the conditional choice decisions. The parameter γ captures a consumer's price sensitivity and p_{ij} denotes the price charged by company j . Note that - in contrast to the consumer packaged goods industry - in the auto insurance industry, prices depend on consumer characteristics. Prices p_{ij} follow an EV Type I distribution with location parameter η_{ij} and scale parameter μ .²³

²³This means the PDF is given by $f(x) = \mu \exp(-\mu(x - \eta)) \exp(-\exp(-\mu(x - \eta)))$ and CDF is given by $F(x) = \exp(-\exp(-\mu(x - \eta)))$ with location parameter η and scale parameter μ . Mean is $\eta + \frac{e\gamma}{\mu}$ and

Given that consumers know the distributions of prices in the market, they know η_{ij} and μ . X_{ij} is a vector of product- and consumer-specific attributes and W_i contains regional fixed effects, demographic and psychographic factors that are common across j . Although these factors drop out of the conditional choice decision, they may play a role in the search and consideration decisions. And lastly, ϵ_{ij} captures the component of the utility that is observed by the consumer but not the researcher.

Given that Honka (2014) studies a market with 17 companies, the curse of dimensionality of the simultaneous search model has to be overcome (see also Section 2.2). To do so, she assumes first-order stochastic dominance among the price belief distributions and uses the optimal selection strategy for consideration sets suggested by Chade and Smith (2005).²⁴ She assumes a specific form of first-order stochastic dominance, namely, that the price belief distributions have consumer- and company-specific means but the same variance across all companies and tests the appropriateness of this assumption using data on prices.

Note that the consumer makes the decisions of which and how many companies to search at the same time. For expository purposes, we first discuss the consumer's decision of which companies to search followed by the consumer's decision of how many companies to search. Both decisions are jointly estimated. A consumer's decision regarding which companies to search depends on the expected indirect utilities (EIU; Chade and Smith 2005) where the expectation is taken with respect to the characteristic the consumer is searching for - in this case, prices. So consumer i 's EIU is given by

$$E[u_{ij}] = \alpha_{ij} + \beta_i I_{ij,t-1} + \gamma E[p_{ij}] + X_{ij}\rho + W_i\phi + \epsilon_{ij}.$$

Consumer i observes these EIUs for every company in his market (including ϵ_{ij}). To decide which companies to search, consumer i ranks all companies other than his previous insurance provider (because the consumer gets a free renewal offer from the previous insurer) according to their EIUs (Chade and Smith 2005) and then picks the top k companies to search. R_{ik} denotes the set of top k companies consumer i ranks highest according to their EIU. For example, R_{i1} contains the company with the highest expected utility for consumer i , R_{i2} contains the companies with the two highest expected utilities for consumer i , etc.

To decide on the number of companies k a consumer searches, the consumer calculates the net benefit of all possible search sets *given the ranking of EIUs*, i.e. if there are N companies in the market, the consumer can choose among $N - 1$ search sets (one free quote comes from the previous insurer). A consumer's benefit of a searched set is then given by

variance $\frac{\pi^2}{6\mu^2}$ where e_c is Euler constant (Ben-Akiva and Lerman 1985).

²⁴Honka (2014) also assumes that search costs are not company-specific - an assumption that also has to be made to apply the theoretical results developed by Chade and Smith (2005).

the expected *maximum* utility among the searched brands. Given the EV distribution of prices, the maximum utility also has an EV distribution

$$\max_{j \in R_{ik}} u_{ij} \sim EV \left(\frac{1}{b} \ln \sum_{j \in R_{ik}} \exp(ba_{ij}), b \right) \quad (17)$$

with $a_{ij} = \alpha_{ij} + \beta_i I_{ij,t-1} + \gamma \eta_{ij} + X_{ij} \rho + W_i \phi + \epsilon_{ij}$ and $b = \frac{\mu}{\gamma}$. If we further define $\tilde{a}_{R_{ik}} = \frac{1}{b} \ln \sum_{j \in R_{ik}} \exp(ba_{ij})$, then the benefit of a searched set is given by

$$E \left[\max_{j \in R_{ik}} u_{ij} \right] = \tilde{a}_{R_{ik}} + \frac{e_c}{b}$$

where e_c denotes the Euler constant. The consumer picks S_{ik} which maximizes his net benefit of searching denoted by $\Gamma_{i,k+1}$, i.e. the expected maximum utility among the considered companies minus the cost of search, given by

$$\Gamma_{i,k+1} = E \left[\max_{j \in R_{ik} \cup \{j_{I_{ij,t-1}}\}} u_{ij} \right] - kc_i. \quad (18)$$

The consumer picks the number of searches k which maximizes his net benefit of search. If a consumer decides to search k companies, he pays kc_i on search costs and has $k+1$ companies in his consideration set.

Consumers can be heterogenous in both preferences and search costs. Consumer-specific effects in *both* the utility function and search costs are not identified because of the linear relationship between utilities and search costs in equation (18). If we increase, for example, the effect of a demographic factor in the utility function and decrease its effect on search costs by an appropriate amount and the benefit of a consideration set, $\Gamma_{i,k+1}$, would remain the same. In the empirical specification, Honka (2014) therefore controls for observed and unobserved heterogeneity in the utility function and for quoting channels (e.g. agent, insurer website) in search costs.

This concludes the description of how a consumer forms his consideration set. Once a consumer has formed his consideration set and received all price quotes he requested, all price uncertainty is resolved. Both the consumer and the researcher observe prices. The consumer then picks the company with the highest utility among the considered companies with the utilities now including the quoted prices for consumer i by company j .

Next, we describe how Honka (2014) estimates her model. The crucial differences between what the consumer observes and what the researcher observes are as follows:

1. Whereas the consumer knows each company's position in the EIU ranking, the researcher only partially observes the ranking by observing which companies are being

searched and which ones are not being searched.

2. In contrast to the consumer, the researcher does not observe α_{ij} and ϵ_{ij} .

Honka (2014) tackles the first point by pointing out that partially observing the ranking contains information that allows her to estimate the composition of consideration sets. Because the consumer ranks the companies according to their EIU and only searches the highest ranked companies, the researcher knows from observing which companies are searched that the EIUs among all the searched companies have to be larger than the EIUs of the non-searched companies or, to put it differently, that the minimum EIU among the searched companies has to be larger than the maximum EIU among the non-searched companies, i.e.

$$\min_{j \in S_i} (E[u_{ij}]) \geq \max_{j' \notin S_i} (E[u_{ij'}]).$$

As a consumer decides simultaneously which and how many companies to search, the following condition has to hold for any searched set

$$\min_{j \in S_i} (E[u_{ij}]) \geq \max_{j' \notin S_i} (E[u_{ij'}]) \quad \cap \quad \Gamma_{ik} \geq \Gamma_{ik'} \quad \forall k \neq k' \quad (19)$$

i.e. the minimum EIU among the searched brands is larger than the maximum EIU among the non-searched brands *and* the net benefit of the chosen searched set of size k is larger than the net benefit of any other search set of size k' .

And finally, Honka (2014) accounts for the fact that the researcher does not observe α_{ij} and ϵ_{ij} by integrating over their distributions. She assumes that $\alpha \sim MVN(\bar{\alpha}, \Sigma_\alpha)$ where $\bar{\alpha}$ and Σ_α contain parameters to be estimated and $\epsilon_{ij} \sim \text{EV Type I}(0, 1)$. Then the probability that a consumer picks a consideration set Υ is given by

$$P_{i\Upsilon|\alpha,\epsilon} = \Pr \left(\min_{j \in S_i} (E[u_{ij}]) \geq \max_{j' \notin S_i} (E[u_{ij'}]) \quad \cap \quad \Gamma_{i,k+1} \geq \Gamma_{i,k'+1} \quad \forall k \neq k' \right). \quad (20)$$

Note that the quote from the previous insurer directly influences the consumer's choice of the size of a consideration set. A consumer renews his insurance policy with his previous provider if the utility of doing so is larger than the expected net benefit $\Gamma_{i,k+1}$ of any number of searches.

Next, she turns to the purchase decision given consideration. The consumer's choice probability conditional on his consideration set is

$$P_{ij|\Upsilon,\alpha,\epsilon} = \Pr(u_{ij} \geq u_{ij'} \quad \forall j \neq j', \quad j, j' \in C_i) \quad (21)$$

where u_{ij} now contains the quoted prices. Note that there is a selection issue: Given a consumer's search decision, ϵ_{ij} do not follow an EV Type I distribution and the conditional choice probabilities do not have a closed-form expression. The consumer's unconditional choice probability is given by

$$P_{ij|\alpha,\epsilon} = P_{i\Upsilon|\alpha,\epsilon} P_{ij|\Upsilon,\alpha,\epsilon}. \quad (22)$$

In summary, the researcher estimates the price distributions, only partially observes the utility rankings, and does not observe α_{ij} and ϵ_{ij} in the consumer's utility function. Accounting for these issues Honka (2014) derived an estimable model with consideration set probability given by (20) and the conditional and unconditional purchase probabilities given by (21) and (22). Parameters are estimated by maximizing the joint likelihood of consideration and purchase given by

$$L = \prod_{i=1}^N \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left(\prod_{l=1}^L \prod_{j=1}^J P_{i\Upsilon|\alpha,\epsilon}^{\vartheta_{il}} P_{ij|\Upsilon,\alpha,\epsilon}^{\delta_{ij}} \right) f(\alpha) f(\epsilon) d\alpha d\epsilon$$

where ϑ_{il} indicates the chosen consideration set and δ_{ij} the chosen company. Neither the consideration set nor the conditional purchase probability have a closed-form solution. Honka (2014) therefore uses a simulation approach to calculate them. In particular, she simulates from the distributions of α_{ij} and ϵ_{ij} . She uses a kernel-smoothed frequency simulator (McFadden 1989) in the estimation and smooths the probabilities using a multivariate scaled logistic CDF (Gumbel 1961)

$$F(w_1, \dots, w_T; s_1, \dots, s_T) = \frac{1}{1 + \sum_{t=1}^T \exp(-s_t w_t)} \quad \forall t = 1, \dots, T \quad (23)$$

where s_1, \dots, s_T are scaling parameters. McFadden (1989) suggests this kernel-smoothed frequency simulator which satisfies the summing-up condition, i.e. that probabilities sum up to 1, and is asymptotically unbiased.

Discussion

In discussing Mehta et al. (2003) and Honka (2014), we start with pointing out the similarities: both papers estimate structural demand models, view search as the process through which consumers form their consideration sets, and use simultaneous search models. However, there are also multiple differences between the two papers. First, Mehta et al. (2003) only have access to purchase data and thus search cost identification comes from functional form, i.e. the data do not contain any direct search outcome measures (e.g. number of

searches) and search costs are identified based on model non-linearities and fit. In contrast, Honka (2014) observes the sizes of consumers’ consideration sets and thus search costs are identified by variation in consumers’ consideration set sizes in her empirical application. Second, the utility function in Mehta et al. (2003) does not have an error term for identification reasons. The addition of such an error term would preclude Mehta et al. (2003) from separately identifying baseline search cost and the true quality of brands. Third, Mehta et al. (2003) use an exclusion restriction: the authors assume that promotional activities such as in-store display and feature affect consumers’ search costs but not their utilities. In contrast, advertising enters consumers’ utilities (but not their search costs) in Honka (2014). Note that – without exogenous variation – the effects of advertising and promotional activities on both utility and search cost are only identified based on model non-linearities and fit. And lastly, Mehta et al. (2003) and Honka (2014) use different approaches to deal with the curse of dimensionality of the simultaneous search model. While Mehta et al. (2003) use the choice model approach which makes estimation only feasible in categories with a small number of options, Honka (2014) applies the theory developed by Chade and Smith (2005) and develops an estimation approach that is feasible in categories with a large number of options. However, she has to assume second-order stochastic dominance among the price distributions and that search costs do not vary across firms – two assumptions that Mehta et al. (2003) do not have to make.

De los Santos et al. (2012)

De los Santos et al. (2012) develop empirically testable predictions of data patterns that identify whether consumers search simultaneously or sequentially (see also Section 5). They use online browsing and purchase data for books to test these predictions and find evidence that consumers search simultaneously. Next, De los Santos et al. (2012) estimate a simultaneous search model and find average search costs of \$1.35.

In the following, we provide details on the modeling and estimation approach: De los Santos et al. (2012) estimate a model in which consumers search across differentiated online bookstores using simultaneous search. As in Mehta et al. (2003) and Honka (2014), consumers search for prices. Consumer’s i indirect utility of buying product at store j is given by $u_{ij} = \mu_j + X_i\beta_j + \alpha_i p_j + \varepsilon_{ij}$, where μ_j are store fixed effects, X_i are consumer characteristics, p_j is store j ’s price for the product, and ε_{ij} is an EV Type I-distributed utility shock that is observed by the consumer. Consumer search costs c_i are consumer-specific and depend on consumer characteristics. Let m_{iS} denote the expected maximum utility of visiting the stores in S net of search costs, i.e., $m_{iS} = E[\max_{j \in S}\{u_{ij}\}] - k \cdot c_i$. By adding an EV Type I choice-set specific error term ζ_{iS} to m_{iS} , the probability that consumer i finds it optimal to

search a subset of stores S can be written as

$$P_{iS} = \frac{\exp[m_{iS}/\sigma_\zeta]}{\sum_{S' \in S} \exp[m_{iS'}/\sigma_\zeta]}, \quad (24)$$

where σ_ζ is the scale parameter for ζ_{iS} . Conditional on visiting stores in S , the probability of purchasing from store j is then

$$P_{ij|S} = \Pr(u_{ij} > u_{ik} \quad \forall \quad k \neq j \in S).$$

This probability does not have a closed-form solution, because a store with a higher ε draw is more likely to be selected in the choice-set selection stage. The probability of observing consumer i visiting all store in S and buying from store j is found by multiplying the two probabilities, i.e.,

$$P_{ijS} = P_{iS}P_{ij|S}. \quad (25)$$

De los Santos et al. (2012) estimate the model using simulated maximum likelihood. They observe the stores visited by consumers in their data, and use P_{ijS} in equation (25) to construct the log-likelihood function, i.e., the log-likelihood function is

$$LL = \sum_i \log \hat{P}_{ijS},$$

where \hat{P}_{ijS} is the probability that individual i bought at store j from the observed choice set S . To obtain a closed-form expression for $E[\max_{j \in S}\{u_{ij}\}]$, De los Santos et al. (2012) follow Mehta et al. (2003) and Honka (2014) in their assumption that prices follow an EV Type I distribution with known parameters γ_j and σ , i.e.

$$E \left[\max_{j \in S} \{u_{ij}\} \right] = \alpha_i \sigma \log \left(\sum_{j \in S} \exp \left[\frac{\mu_j + X_i \beta_j + \varepsilon_{ij} + \alpha_i \gamma_j}{\alpha_i \sigma} \right] \right).$$

Note that the utility shock ε_{ij} that appears in both probabilities is integrated out using simulation, as part of a simulated maximum likelihood procedure.

Discussion

There are several similarities and differences between De los Santos et al. (2012) and the two previously discussed papers, Mehta et al. (2003) and Honka (2014). First, De los Santos et al. (2012) have more detailed data on consumer search than Mehta et al. (2003) or Honka (2014): while Mehta et al. (2003) do not observe consumer search at all in their data and

Honka (2014) observes consumers’ consideration sets, De los Santos et al. (2012) observe consumers’ consideration sets *and* the sequence of searches. Second, the error structures in the three papers are different: the utility function in Mehta et al. (2003) does not contain a classic error term. While there is an error term in the utility function in Honka (2014) and De los Santos et al. (2012), in De los Santos et al. (2012) there is also a search-set specific error term (see also Moraga-González et al. 2015). As shown in equation (24), the latter error term gives De los Santos et al. (2012) a closed-form solution for the search set probabilities. This closed-form solution makes estimation of the model easier, but may necessitate a discussion of what this search-set specific error term, which is assumed to be independent across (possibly similar) search sets, represents. And lastly, the positioning and contributions of the three papers are different: Mehta et al. (2003) is one of the first structural search models estimated with individual-level data on purchases. The contribution of this paper lies in the model development. Honka (2014) extends the model of Mehta et al. (2003) and develops an estimation approach that is feasible even in markets with a large number of alternatives. De los Santos et al. (2012)’s primary contribution is to show that consumers’ search method can be identified when the sequence of searches (and characteristics of searched products) made by individual consumers is observed in the data (see also Section 5).

Honka and Chintagunta (2017)

Similar to De los Santos et al. (2012), Honka and Chintagunta (2017) are also primarily interested in the question of search method identification. They analytically show that consumers’ search method is identified by patterns of prices in consumers’ consideration sets (see also Section 5). They use the same data as Honka (2014), i.e. cross-sectional data in which consumers’ purchases and consideration sets are observed, to empirically test whether consumers search simultaneously or sequentially in the auto insurance industry. They find the price pattern to be consistent with simultaneous search. Then Honka and Chintagunta (2017) estimate both a simultaneous and a sequential search model and find preference and search cost estimates to be severely biased when the incorrect assumption on consumers’ search method is made.

In the following, we discuss the details of the modeling and estimation approach for the sequential search model: Honka and Chintagunta (2017) develop an estimation approach for situations in which the researcher has access to individual-level data on consumers’ consideration sets (but not the sequence of searches) and purchases. Suppose consumer i ’s utility for company j is given by

$$u_{ij} = \alpha_{ij} + \beta p_{ij} + X_{ij}\gamma + \epsilon_{ij}$$

where ϵ_{ij} are *iid* and observed by the consumer, but not by the researcher. α_{ij} are brand intercepts and p_{ij} are prices which follow a normal distribution with mean μ_{ij}^p .

Even though the sequence of searches is not observed, observing a consumer's consideration set allows the researcher to draw two conclusions based on Weitzman (1979)'s rules: First, the minimum reservation utility among the searched companies has to be larger than the maximum reservation utility among the non-searched companies (based on the selection rule), i.e.

$$\min_{j \in S_i} z_{ij} \geq \max_{j' \notin S_i} z_{ij'} \quad (26)$$

Otherwise, the consumer would have chosen to search a different set of companies. And second, the stopping and choice rules can be combined to the following condition

$$\max_{j \in S_i} u_{ij} \geq u_{ij'}, \max_{j'' \notin S_i} z_{ij''} \quad \forall j' \in S_i \setminus \{j\} \quad (27)$$

i.e. that the maximum utility among the searched companies is larger than any other utility among the considered companies *and* the maximum reservation utility among the non-considered companies.

Equations (26) and (27) are conditions that have to hold based on Weitzman (1979)'s rules for optimal behavior under sequential search and *given the search and purchase outcome* that is observed in the data. At the same time, it must also have been optimal for the consumer not to stop searching and purchase earlier given Weitzman (1979)'s rules. The challenge is that the order in which the consumer collected the price quotes is not observed. The critical realization is that, *given the parameter estimates*, the observed behavior must have a high probability of having been optimal.

To illustrate, suppose a consumer searches three companies. Then the parameter estimates also have to satisfy the conditions under which it would have been optimal for the consumer to continue searching after his first and second search. Formally, in the estimation, given a set of estimates for the unknown parameters, for each consumer i , let us rank all searched companies j according to their reservation utilities \hat{z}_{it} (the “ $\hat{\cdot}$ ” symbol refers to quantities computed at the current set of estimates) where $t = 1, \dots, k$ indicates the rank of a consumer's reservation utility among the searched companies. Note that $t = 1$ ($t = k$) denotes the company with the largest (smallest) reservation utility \hat{z}_{it} . Further rank all utilities of searched companies in the same order as the reservation utilities, i.e. $\hat{u}_{i,t=1}$ denotes the *utility* for the company with the highest *reservation* utility $\hat{z}_{it=1}$. Then *given the current parameter estimates*, the following conditions have to hold

$$\hat{u}_{i,t=1} < \hat{z}_{it=2} \quad \cap \quad \max_{t=1,2} \hat{u}_{it} < \hat{z}_{i,t=3}$$

In other words, although by definition the *reservation utility* of the company with $t = 1$ is larger than that with $t = 2$, the *utility* of the company with $t = 1$ is smaller than the *reservation utility* of the company with $t = 2$ thereby prompting the consumer to do a second search. Similarly, the maximum utility from the (predicted) first and second search has to be smaller than the reservation utility from the (predicted) third search; otherwise the consumer would not have searched a third time. Generally, for a consumer searching $t = 2, \dots, k$ companies, the following set of conditions has to hold

$$\bigcap_{l=2}^k \max_{t<l} \hat{u}_{it} < \hat{z}_{it=l}. \quad (28)$$

To calculate a consumer's reservation utilities, Honka and Chintagunta (2017) follow the approach suggested by Kim et al. (2010). The additional estimation conditions from equation (28) are necessary to correctly recover search costs. These conditions impose restrictions on the utilities and bound the search cost parameter from above. Without these conditions, search cost estimate is biased upwards.

Since in the sequential search model, in contrast to the simultaneous search model, the consideration and conditional purchase stages are not separate, the probability of observing a consumer search a set of companies Υ and purchase from company j under sequential search is

$$P_{ij\Upsilon|\epsilon} = P\left(\min_{j \in S_i} z_{ij} \geq \max_{j' \notin S_i} z_{ij'} \quad \cap \quad \max_{j \in S_i} u_{ij} \geq u_{ij''}, \max_{j' \notin S_i} z_{ij'} \quad \cap \quad \bigcap_{l=2}^k \max_{t<l} \hat{u}_{it} < \hat{z}_{it=l} \quad (29)\right. \\ \left. \forall j'' \in S_i \setminus \{j\}, t = 2, \dots, k\right).$$

Then the loglikelihood of the model is given by

$$L = \prod_{i=1}^N \int_{-\infty}^{+\infty} \prod_{l=1}^L \prod_{j=1}^J P_{ij\Upsilon|\epsilon}^{y_{il}} f(\epsilon) d\epsilon$$

where y_{il} indicates the chosen consideration set and the purchased company. In principle, all rankings of utilities and reservation utilities that satisfy the conditions in equation (29) can be written out and the probability of observing a consumer's search and purchase behavior can be calculated as the sum of the probabilities of all admissible rankings. The challenge with writing out all utility and reservation utility rankings that satisfy the conditions in equation

(29) is that their number and complexity increases very quickly with the number of searches a consumer makes (see also Section 4.2). Since, in the empirical application, consumers are observed to search up to ten times, this approach is not feasible. A second challenge is that, even if all admissible rankings of utilities and reservation utilities are written out, the probability as described in equation (29) does not have a closed-form solution. Honka and Chintagunta (2017) use SMLE to estimate the sequential search model as it allows them to overcome both challenges. SMLE does not solve the combinatorial problem, but it circumvents it by allowing the authors to estimate the probability of observing a consumer search a set of companies Υ and purchase from company j in equation (29) without having to write out all admissible rankings. SMLE is implemented by using a kernel-smoothed frequency simulator (McFadden 1989) and smoothing the probabilities using a multivariate scaled logistic CDF (Gumbel 1961).

4.2 Searching for Match Values

Instead of uncertainty about prices, in differentiated product markets, consumers might have imperfect information regarding some or all product characteristics (other than price). As a result, consumers might search to resolve this type of uncertainty. Starting with Wolinsky (1986), a part of the theoretical literature on search has focused on search for a good match or product fit. In this stream of literature, whether a product is a good fit for a consumer is typically determined by a random utility shock that is IID across consumers and products. This utility shock is assumed to be ex-ante unobserved by firms and consumers and only observed by consumers after having searched and inspected the product.

We discuss two approaches that follow this framework: the first approach was developed in Kim et al. (2010) and Kim et al. (2017) and the second approach was developed in Moraga-González et al. (2018). In all three papers, consumers are assumed to search sequentially and the models can be estimated using aggregate data (aggregate view and sales rank data or market share data).²⁵ The starting point for both approaches is the following indirect utility function

$$u_{ij} = \delta_{ij} + \varepsilon_{ij}, \tag{30}$$

where δ_{ij} is consumer i 's “mean” utility for product j . For simplicity, let us assume that all firms sell a single product. Consumers search sequentially, and therefore consumer i 's reservation utility for alternative j , denoted by z_{ij} , is the utility that makes the consumer

²⁵Although the model in Moraga-González et al. (2018) can be estimated using only aggregate data, their main specification also uses data on store visits from a survey.

indifferent between continuing to search and stopping. It is implicitly defined by

$$\int_{z_{ij}}^{\infty} (u - z_{ij}) dF_{ij}(u) = c_{ij}, \quad (31)$$

where F_{ij} is consumer i 's utility distribution for alternative j .

Before we dive into the details of the two approaches, we first want to discuss a search path dimensionality problem that can occur in the empirical implementation of a sequential search model. As discussed in Section 2.2, the optimal search strategy according to Weitzman (1979) is to visit sellers in a descending order of reservation utilities (selection rule) and to stop searching when the highest observed utility exceeds the reservation utility of the next best option that can be searched (stopping rule). With individual-level data and observed search sequences, there is no search path dimensionality problem: Weitzman (1979)'s selection rule implies that there are only as many potential search sets as there are search alternatives. For example, suppose that there are three firms A, B, and C in the market and that the ranking according to reservation utilities is BCA. Then a consumer "chooses" among three possible search sets: (i) to only search B or (ii) to first search B and then C or (iii) to first search B followed by C and then A.

With individual-level data, observed search sets, but *unobserved* search sequences (such as in Honka and Chintagunta 2017), a "milder" version of the search path dimensionality problem occurs: coming back to the three firm example, a researcher might, for example, observe that a consumer searches all three firms resulting in six possible (but unobserved) search paths: ABC, BAC, ACB, BCA, CAB, and CBA. In this case, the ranking according to the reservation utilities is not observed by the researcher. The search path dimensionality problem is aggravated when consumer search is completely unobserved in individual-level data or in aggregate data. Suppose that, in the three firms example, a researcher observes a purchase of C. This purchase can be the result of the following 11 search paths: C, AC, BC, CA, CB, ABC, BAC, ACB, BCA, CAB, and CBA. More generally, the number of search paths increases factorially, which can be especially problematic if there are many alternatives or sellers to be searched.

Recall that both Kim et al. (2010) and Kim et al. (2017) as well as Moraga-González et al. (2018) can be estimated using aggregate data. Thus all three papers have to address the most severe form of the search path dimensionality problem discussed in the previous paragraph. Kim et al. (2010) start from the property of Weitzman (1979)'s optimal search strategy that at the individual level there are only as many possible search sets as there are search alternatives and use this to derive closed-form expressions for the probability distributions of search sets and a univariate expression for choice probabilities. Moraga-

González et al. (2018) take a different approach and use recent insights from the theoretical literature on search that make it possible to reformulate the sequential search problem as a standard discrete choice problem with a single choice variable that corresponds to choice probabilities. It is important to note that both approaches – the one developed in Kim et al. (2010) and Kim et al. (2017) and the one developed in Moraga-González et al. (2018) – do not make any additional assumptions to address the search path dimensionality problem but only use Weitzman’s optimal search rules. Nevertheless, the two approaches differ substantially in their estimation approach, mostly because they utilize different types of data (sales and view rankings on Amazon versus data on sales of automobiles supplemented with information on dealer visits from a survey). Further, both approaches also differ in the assumptions they make (e.g. on the distribution of error terms) to obtain a demand model that can be estimated using the available data.

Kim et al. (2010) and Kim et al. (2017)

The empirical application in both Kim et al. (2010) and Kim et al. (2017) is based on online consumer search for digital camcorders at Amazon.com. The basic idea is that, when consumers search for digital camcorders on Amazon.com, they get a list of products (“results page”) with information on a few attributes such as price and average user star ratings. However, to learn more about the product, consumers have to perform a “costly” click on the product link, which allows them to learn the product-consumer specific matching value ε_{ij} for the product. This match value ε_{ij} might contain information on size/weight, review content, and additional photos that is not available on the results page.

Both Kim et al. (2010) and Kim et al. (2017) use view rank data for estimation, which give a measure of how closely related products are. The model estimation uses restrictions imposed by the optimal search strategy when searching sequentially for differentiated products, as characterized in Weitzman (1979). Since the data are aggregate and search sequences are unobserved, Kim et al. (2010) face the search path dimensionality problem. Kim et al. (2010) use the property that, at the individual level, consumers search products in a decreasing order of reservation utilities (even though these search paths are unobserved in their data), to derive closed-form expressions for the probability distributions of search sets and deal with the search path dimensionality problem.

Amazon’s view rank data, used in both Kim et al. (2010) and Kim et al. (2017), give information on which other products consumers viewed in the same session and is based on the following commonality index

$$CI_{jk} = \frac{n_{jk}}{\sqrt{n_j} \cdot \sqrt{n_k}},$$

where n_j (n_k) is the number of consumers who viewed product j (product k) and n_{jk} is the number of consumers who viewed both products in the same session. If the commonality index of product j with product k is higher than that with product l , product k will appear higher in the view list for product j .

Kim et al. (2010) show how to map the view rank data into the sequential search model. Specifically, the optimal search procedure is to rank products according to reservation utilities. Therefore, the probability that option k is included is equal to the probability that the first $k - 1$ utility draws are lower than the reservation utility of option k :

$$\begin{aligned} \pi_{ik} &= \Pr\left(\max_{l=1}^{k-1} \delta_{il} + \varepsilon_{il} < z_{ik}\right); \\ &= \prod_{l=1}^{k-1} F_{il}(z_{ik} - \delta_{il}). \end{aligned}$$

Note that search costs enter the model through reservation values z_{ik} , which are implicitly defined in equation (31). The probability π_{ik} that product k is in consumer i 's search set can be used to obtain an estimate of the commonality index. Specifically, the number of consumers who viewed product j can be obtained by taking the sum across individuals of the probability that product j is included in the search set, i.e.,

$$\hat{n}_j = \sum_i \pi_{ij}.$$

Moreover, given that a search set that contains the lower ranked product also includes the higher ranked product and that the search set that contains both has a lower probability, the probability that both products j and k are in the same search set is equal to the sum across individuals of the minimum of the probabilities that product j and product k are in the search set. Therefore, the predicted commonality index is

$$\widehat{CI}_{jk}(\Theta, X) = \frac{\sum_i \min(\pi_{ij}, \pi_{ik})}{\sqrt{\sum_i \pi_{ij}} \sqrt{\sum_i \pi_{ik}}},$$

where Θ are the parameters to be estimated. Assuming that the difference between the true and the estimated commonality index follows a normal distribution, i.e., $CI_{jl} = \widehat{CI}_{jk} + \varepsilon_{jl}^V$, where $\varepsilon_{jl}^V \stackrel{iid}{\sim} N(0, \tau_V^2/2)$, the probability that products j and l are more often viewed together in the same search session than products j and k is given by

$$\Pr(I_{j,kl}^V = 1) = \Pr(CI_{jl} > CI_{jk}) = \Phi\left(\frac{\widehat{CI}_{jl}(\Theta, X) - \widehat{CI}_{jk}(\Theta, X)}{\tau_V}\right), \quad (32)$$

where $\Phi(\cdot)$ is the CDF of the normal distribution. Kim et al. (2010) use this probability in the following nonlinear least-squares estimator

$$\arg \min_{\Theta} \sum_{(j,k,l) \in \mathbf{S}} [\Pr(I_{j,kl}^V = 1) - 1]^2,$$

where $j, k, l = 1, \dots, J$, and \mathbf{S} is the subset of products that are searched together according to the view rank data, i.e., $\mathbf{S} = \{(j, k, l) | I_{j,kl}^V = 1, j \neq k \neq l\}$. This set is potentially quite large—the view rank data contain many different pairs of products, which means that the view rank data give a large number of restrictions that can be used when estimating the model.

Two points on Kim et al. (2010) should be noted. First, they only use view rank data, i.e. data on searches, but no data on purchases to estimate their model. In general, to empirically identify all preference parameters and search costs, data on searches are sufficient. Second, recall that reservation utilities are defined by the following implicit function $c_{ij} = \int_{z_{ij}}^{\infty} (u_{ij} - z_{ij}) f_j(u) du$, which poses an estimation challenge as this implicit function would have to be solved for every individual, every product, and every parameter draw. Kim et al. (2010) show that, under the normality assumption for ε_{ij} , reservation utilities can be written as the sum of the utilities and a function of a second term which can be computed prior to the estimation – rendering the actual model estimation much quicker.²⁶

For digital camcorders on Amazon.com, Kim et al. (2010) predict that search sets contain a mean (median) of 14 (11) products with about 40% of consumers searching fewer than 5 products and the distribution of the number of searches having a long right tail. The authors also find that online competition between many products is very limited because they are not jointly searched by consumers, resulting in cross-price elasticities for many product pairs that are (close to) zero.

While Kim et al. (2010) only use view rank data, i.e. data on searches, to estimate their sequential search model, Kim et al. (2017) also use sales rank data and data on choices conditional on search, in addition to the view rank data, to estimate a sequential search model. Kim et al. (2017) use the probability in equation (32) as part of a maximum likelihood estimation procedure. To derive the likelihood contribution from the sales rank data, Kim

²⁶Elberg et al. (2018) develop two alternative methods to recover reservation utilities: first, under the assumption that ε_{ij} follow a Logistic distribution, there is a closed-form solution for the reservation utilities. And second, for a broad class of continuous distributions, the authors show that reservation utilities can be obtained through fixed-point iteration.

et al. (2017) use Weitzman (1979)'s selection rule, stopping rule, and choice rule to obtain the joint probability that S_K is the ordered set and j is chosen with $S_K = \{1, 2, \dots, K\}$ being a set ordered by decreasing reservation values. For $j < K$, this joint probability is given by²⁷

$$\Pr(j, S_K) = \int_{z_{K+1}-\delta_j}^{z_K-\delta_j} \prod_{l \neq j}^K F_{il}(\delta_{ij} - \delta_{il} + \varepsilon_j) f_{ij}(\varepsilon_{ij}) d\varepsilon_{ij}. \quad (33)$$

The probability that the j th ranked product is chosen is obtained by summing over all search sets that contain product j , i.e.,

$$\begin{aligned} \Pr(j) &= \sum_{K=j}^J \Pr(j, S_K); \\ &= \sum_{K=j}^J \int_{z_{K+1}-\delta_j}^{z_K-\delta_j} \prod_{l \neq j}^K F_{il}(\delta_{ij} - \delta_{il} + \varepsilon_j) f_{ij}(\varepsilon_{ij}) d\varepsilon_{ij}. \end{aligned}$$

It is useful to compare this probability to the full information case, in which, assuming ε_{ij} follows a normal distribution, we get the familiar probit model:

$$\Pr(j) = \int_{-\infty}^{\infty} \prod_{l \neq j}^J F_{il}(\delta_{ij} - \delta_{il} + \varepsilon_j) f_{ij}(\varepsilon_{ij}) d\varepsilon_{ij}.$$

The difference between the two probabilities is due to how the unobserved distribution of ε affects consumers' search decisions: the lower bound in equation (33) reflects termination decisions, whereas the upper bound reflects the decision to continue searching. The utility draw for product j needs to be low enough to continue searching until K , which means that $\delta_j + \varepsilon_j < z_K$ and implies an upper bound of $z_K - \delta_j$. Moreover, stopping at K means that the realized utility for product j exceeds the reservation value for the next alternative to be searched. This means that $\delta_j + \varepsilon_j > z_{K+1}$, which implies a lower bound of $z_{K+1} - \delta_j$ in the integral in equation (33).

Product j 's predicted market share \hat{s}_j is obtained by averaging the buying probabilities $\Pr(j)$ across consumers. The relation between market shares and sales ranks for pairs of products is modeled as follows:

$$I_{jl}^S = \begin{cases} 1 & \text{if } s_j > s_l; \\ 0 & \text{otherwise.} \end{cases}$$

²⁷For $J = K$ the term $(1 - \Phi_j(z_j - \delta_j))\pi_j$ has to be added to equation (33).

Assuming that the difference between the actual and predicted market share has a normally distributed measurement error, i.e., $s_j = \hat{s}_j + \varepsilon_j^S$ with $\varepsilon_j^S \stackrel{iid}{\sim} N(0, \tau_S^2/2)$, we get

$$\Pr(I_{jl}^S = 1 | \Theta, X) = \Phi \left(\frac{\hat{s}_j(\Theta, X) - \hat{s}_l(\Theta, X)}{\tau_S} \right). \quad (34)$$

The data used for estimation also contains information on the top products that were purchased conditional on searching for a specific product. These choices conditional on search correspond to the probability that product j is chosen conditional on searching an option l , i.e.,

$$\Pr(j|l) = \frac{\sum_{K=\max(j,l)}^J \Pr(j, S_K)}{\pi_l},$$

where $K = j$ if j has a lower reservation utility than l and $K = l$ otherwise and π_l is the probability of searching the l th option. Assuming that the difference between the predicted and observed conditional choice share data represents a measurement error, i.e., $s_{j|l} = \hat{s}_{j|l}(\Theta|X) + \varepsilon_{jl}^C$ with $\varepsilon_{jl}^C \stackrel{iid}{\sim} N(0, \tau_C^2)$, we get

$$\Pr(s_{j|l} | \Theta, X) = \Phi \left(\frac{\hat{s}_{j|l}(\Theta, X) - s_{j|l}}{\tau_C} \right). \quad (35)$$

Combining the probabilities in equations (32), (34), and (35) and summing over all relevant products, gives the following log-likelihood function, which is used to estimate the model by maximum likelihood:

$$LL(\Theta|Y, X) = \sum_j \sum_{l \neq j} \sum_{k \neq l} \Pr(I_{j,lk}^V = 1 | \Theta, X) + \sum_j \sum_{l \neq j} \Pr(I_{jl}^S = 1 | \Theta, X) + \sum_l \sum_j \Pr(s_{j|l} | \Theta, X).$$

Kim et al. (2017) estimate their model using view rank data, sales rank data, and data on choices conditional on search. They find mean and median search costs of \$1.30 and \$0.25, respectively, and predict median and mean search set sizes conditional on choice of 17 and 10 products, respectively. Kim et al. (2017) use their results to investigate substitution patterns in the camcorder market and conduct a market structure analysis using the framework of clout and vulnerability.

Moraga-González et al. (2018)

Moraga-González et al. (2018) develop a structural model of demand for car purchases in the Netherlands. The starting point for their search model is an aggregate random coefficients logit demand model in the spirit of Berry et al. (1995). However, whereas Berry et al.

(1995) assume consumers have full information, Moraga-González et al. (2018) assume that consumers have to search to obtain information on ε_{ij} in equation (30).

As in Kim et al. (2017), consumers are assumed to search sequentially. To deal with the aforementioned search path dimensionality problem that arises because of the number of search paths that result in a purchase increases factorially in the number of products, Moraga-González et al. (2018) use insights from Armstrong (2017) and Choi et al. (2018) that make it possible to treat the sequential search problem as a discrete choice problem in which it is not necessary to keep track of which firms are visited by the consumer. Specifically, for every alternative (i.e. dealer) f , define the random variable

$$w_{if} = \min\{z_{if}, u_{if}\},$$

where z_{if} is the reservation utility for alternative f . According to Armstrong (2017) and Choi et al. (2018), the solution to the sequential search problem is equivalent to picking the alternative with the highest w_{if} from all firms. To see that this is indeed optimal, consider the following example. Suppose there are three products, A, B, and C. The reservation and (ex-ante unobserved) realized utilities are 5 and 2 for product A, 10 and 4 for product B, and 3 and 7 for product C, respectively. Using Weitzman’s optimal search rules, the consumer first searches product B because it has the highest reservation utility, but continues searching product A because the realized utility for product B of 4 is smaller than product A’s reservation utility of 5. The consumer then buys product B because the next-best product in terms of reservation utilities, product C, has a reservation utility of 3, which means that the highest observed realized utility of 4 does not justify searching further. Now characterizing the search problem in terms of w avoids having to go through the specific ordering of firms in terms of reservation utilities and immediately reveals which product will be bought: since $w_B = \min\{10, 4\} = 4$ exceeds $w_A = \min\{5, 2\} = 2$ as well as $w_C = \min\{3, 7\} = 3$, product B will be purchased. Note that no additional assumptions have been made to resolve the search path dimensionality problem—all that is used is a re-characterization of Weitzman’s optimal search rules.

To use this alternative characterization of Weitzman’s optimal search rules in practice, the distribution of w_{if} has to be derived—this can be obtained by deriving the CDF of the minimum of two independent random variables:

$$F_{if}^w(x) = 1 - (1 - F_{if}^z(x))(1 - F_{if}(x)),$$

where F_{if} is the utility distribution and F_{if}^z is the distribution of reservation utilities. Since $F_{if}^z(x) = 1 - F_{if}^c(H(x))$ with F_{if}^c being the search cost CDF and $H(x)$ being the gains from

search, this can be written as

$$F_{if}^w(x) = 1 - F_{if}^c(H(x))(1 - F_{if}(x)).$$

The probability that consumer i buys from dealer f is then given by

$$\begin{aligned} P_{if} &= \Pr(w_{ig} < w_{if} \forall g \neq f); \\ &= \int \left(\prod_{g \neq f} F_{ig}^w(w_{if}) \right) f_{if}^w(w_{if}) dw_{if}, \end{aligned}$$

where F_{if}^w and f_{if}^w are the CDF and PDF of w_{if} , respectively. The probability that consumer i buys car j conditional on buying from seller f is given by

$$P_{ij|f} = \frac{\exp(\delta_{ij})}{\sum_{h \in G_f} \exp(\delta_{ih})}.$$

The probability that buyer i buys product j is thus

$$s_{ij} = P_{ij|f} P_{if}.$$

Note that these expressions are not necessarily closed-form. Although one can use numerical methods to directly estimate these expressions, this may slow down model estimation enough to make using it impractical. To speed up the estimation, Moraga-González et al. (2018) provide an alternative approach by working backwards and deriving a search cost distribution that gives a tractable closed-form expression for the buying probabilities. Specifically, a Gumbel distribution for w with location parameter $\delta_{if} - \mu_{if}$, where μ_{if} contains search cost shifters and parameters, can be obtained using the following search cost CDF:

$$F_{if}^c(c) = \frac{1 - \exp(-\exp(-H_0^{-1}(c) - \mu_{if}))}{1 - \exp(-\exp(-H_0^{-1}(c)))},$$

where $H_0(z) = \int_z^\infty (u - z) dF(u)$ represents the (normalized) gains from search at z . Product j 's purchase probability then simplifies to

$$s_{ij} = \frac{\exp(\delta_{ij} - \mu_{if})}{1 + \sum_{k=1}^J \exp(\delta_{ik} - \mu_{ig})}.$$

The closed-form expression for the purchase probabilities makes the model estimation of similar difficulty as most full information discrete choice models of demand. The estimation

of the model closely resembles the estimation in Berry et al. (1995) – the most basic version of the model can be estimated using market shares, prices, product characteristics, as well as a search cost shifter (e.g. distance from the consumer to the car dealer is used in the application). The similarity with the framework in Berry et al. (1995) allows for dealing with price endogeneity: as in Berry et al. (1995), Moraga-González et al. (2018) allow for an unobserved quality component in the utility function, i.e., $\delta_{ij} = \alpha_i p_j + X_j \beta_{ij} + \xi_j$, and allow ξ_j to be correlated with prices.

When the model is estimated using aggregate data, the essence of the estimation method is to match predicted market shares to observed market shares, i.e.,

$$s_j(\xi_j, \theta) - \hat{s}_j = 0 \quad \text{for all products } j,$$

which gives a nonlinear system of equations in ξ . As in Berry et al. (1995), this system can be solved for ξ through contraction mapping. The identification assumption is that the demand unobservables are mean independent of a set of exogenous instruments W , i.e., $E[\xi_j|W] = 0$, so that the model can be estimated using GMM while allowing for price endogeneity, as in Berry et al. (1995).

An important limitation of estimating the model using data on market shares and product characteristics is that variables that enter the search cost specification have to be excluded from the utility function. For instance, distance cannot both affect utility and search costs when only purchase data (either aggregate or individual specific) is used to estimate the model. However, Moraga-González et al. (2018) show that when similar covariates appear in both the utility specification and the search cost specification, in their model it is possible to separate the effects of these common shifters using search data. Search behavior depends on reservation values, which respond differently to changes in utility than to changes in search costs; variation in observed search decisions therefore allows one to separately estimate the effect of common utility shifters and search cost shifters. To exploit this fully, Moraga-González et al. (2018) use moments based on individual purchase and search data for their main specification, which are constructed by matching predicted search probabilities to consumer-specific information on store visits from a survey. The aggregate sales data is then used to obtain the linear part of utility, following the two-step procedure in Berry et al. (2004). Search costs are found to be sizable, which is consistent with the limited search activity observed in this market. Moreover, demand is estimated to be more inelastic in the search model than in a full information setting. The counterfactual results suggest that the price of the average car would be €341 lower in the absence of search frictions.

Other Papers

Here, we discuss several other papers that have also modeled consumer search for a match value. We start by reviewing papers that assume that consumers search sequentially and then discuss papers that assume that consumers search simultaneously.

Koulayev (2014) analyzes search decisions on a hotel comparison site using clickstream data, i.e. individual-level data with observed search sequences. The paper models the decision to click on one of the listed hotels or to move to the next page of search results. Koulayev (2014) derives inequality conditions that are implied by search and click decisions and which are used to derive the likelihood function for the joint click and search behavior. He finds that search costs are relatively large: median search costs are around \$10 per result page. Note that Koulayev's approach leads to multi-dimensional integrals for choice and search probabilities, which is manageable in settings with a small number of search options, but is potentially problematic in settings with larger choice sets.

Jolivet and Turon (2018) derive a set of inequalities from Weitzman (1979)'s characterization of the optimal sequential search procedure and use these inequalities to set identify distributions of demand side parameters. The model is estimated using transaction data for CDs sold at a French e-commerce platform. Findings suggest that positive search costs are needed to explain 22 percent of the transactions and that there is heterogeneity in search costs.

Dong et al. (2018) point out that search costs may lead to persistence in purchase decisions by amplifying preference heterogeneity and show that ignoring search frictions leads to an overestimation of preference heterogeneity. The authors use search and purchase decisions of individual consumers shopping for cosmetics at a large Chinese online store to separately identify preference heterogeneity from search cost heterogeneity. Two drawbacks are that the authors only observe a small proportion of consumers making repeat purchases in their data and that they have to add an error term to search costs to be able to estimate the model. Dong et al. (2018) find that the standard deviations of product intercepts are overestimated by 30 percent if search frictions are ignored, which has implications for targeted marketing strategies such as personalized pricing.

Several studies have used a simultaneous instead of a sequential search framework when modeling consumer search for the match value. An advantage of simultaneous search is that search decisions are made before search outcomes are realized, which typically makes it easier to obtain closed-form expressions for purchase probabilities. However, as discussed in Section 2, the number of choice sets increases exponentially in the number of alternatives that can be searched (curse of dimensionality of the simultaneous search model). Moraga-González et al. (2015) show that tractability can be achieved by adding an EV Type I distributed choice-set

specific error term to the search costs that correspond to a specific subset of firms. Murry and Zhou (2018) use this framework in combination with individual-level transaction data for new cars to quantify how geographical concentration among car dealers affects competition and search behavior. Donna et al. (2018) estimate the welfare effects of intermediation in the Portuguese outdoor advertising industry using a demand model that extends this framework to allow for nested logit preferences. Finally, Ershov (2018) develops a structural model of supply and demand to estimate the effects of search frictions in the mobile app market and uses the search model in Moraga-González et al. (2015) as a demand side model.

5 Testing Between Search Methods

In this section, we discuss the identification of the search method consumers are using. Beyond intellectual curiosity, the search conduct, i.e. search method, matters for consumers' decision-making. It influences which and how many products a consumer searches. More specifically, holding a consumer's preferences and search cost constant, a consumer might end up with a different consideration set depending on whether he searches simultaneously or sequentially.²⁸ In fact, it can be shown that, again holding a consumers' preferences and search cost constant, a consumer who searches sequentially, on average, has a smaller search set compared to when the same consumer searches simultaneously.²⁹

From a researcher's perspective, this implies that estimates of consumer preferences and search cost will be biased under the incorrect assumption on the search method. This bias of consumer preferences and search cost estimates can, in turn, lead to, for example, biased (price) elasticities and different results in counterfactual simulations. For example, Honka and Chintagunta (2017) show that consideration set sizes and purchase market shares of the largest companies are over predicted under when it is wrongfully assumed that consumers search sequentially.

For a long time, it was believed that the search method is not identified in observational data. In a first attempt to test between search methods, Chen et al. (2007) develop nonparametric likelihood ratio model selection tests which allow them to test between simultaneous and sequential search models. Using the Hong and Shum (2006) framework and data on prices, they do not find significant differences between the simultaneous and sequential search models using the usual significance levels in their empirical application.

Next, we discuss two papers that developed tests to distinguish between simultaneous

²⁸If a consumer has different consideration sets under simultaneous and sequential search, he might also end up purchasing a different product.

²⁹This is the case because, under sequential search, a consumer can stop searching when he gets a good draw early on.

and sequential search using individual-specific data on search behavior. In Section 5.1, we discuss De los Santos et al. (2012), who use data on the sequence of searches to discriminate between simultaneous and sequential search, whereas in Section 5.2, we discuss Honka and Chintagunta (2017), who develop a test that does not require the researcher to observe search sequences.

Jindal and Aribarg (2018) apply the key identification insights from De los Santos et al. (2012) and Honka and Chintagunta (2017) to a situation of search with learning (see also Section 6.1). Using their experimental data, the authors find that, under the assumption of rational price expectations, consumers appear to search simultaneously, while the search patterns are consistent with sequential search conditional on consumers' elicited price beliefs. This finding highlights the importance of the rational expectations assumption for search method identification.

5.1 De los Santos et al. (2012)

One of the objectives of De los Santos et al. (2012) is to provide methods to empirically test between sequential and simultaneous search. The authors use data on web browsing and online purchases of a large panel of consumers, which allows them to observe the online stores consumers visited as well as the store they ultimately bought from. Thus De los Santos et al. (2012) observe the sequence of visited stores, which is useful for distinguishing between sequential and simultaneous search.

De los Santos et al. (2012) first investigate the homogenous goods case with a market-wide price distribution. Recall that, under simultaneous search, a consumer samples all alternatives in his search sets before making a purchase decision. In contrast, under sequential search, a consumer stops searching as soon as he gets a price below his reservation price (see also Section 2.2). Since the latter implies that the consumer purchases from the last store he searched, revisits should not be observed when consumers search sequentially, while they may be observed when consumers search simultaneously (a consumer may revisit a previously searched store to make a purchase). Whether or not consumers revisit stores can be directly explored with data on search sequences. De los Santos et al. (2012) find that approximately one-third of consumers revisit stores – a finding that is inconsistent with a model of sequential search for homogenous goods.

Recall that the no-revisit property of the sequential search model for homogenous goods does not necessarily apply to more general versions of the sequential search model, including models in which stores are differentiated. Specifically, if goods are differentiated, the optimal sequential search strategy is to search until a utility level is found that exceeds the reser-

vation utility of the next-best alternative. As pointed out in Section 2, when products are differentiated, reservation utilities are generally declining, so a product that was not good enough early on in the search may pass the bar after a number of different products have been inspected, triggering a revisit to make a purchase. In the following, we show a simple example of how that can happen.

Suppose a consumer wants to purchase one of five products denoted by A, B, C, D, and E. Table 2 shows (realized) utilities u and reservation utilities z for all five alternatives. Note that the alternatives are ranked in a decreasing order of their reservation utilities z . In this example, the consumer first searches A – the alternative with the highest reservation utility. Since the reservation utility of the next-best alternative B is larger than the highest utility realized so far, i.e. $z_B > u_A$, he continues to search. The consumer also decides to continue searching after having sampled options B and C since $z_C > \max\{u_A, u_B\}$ and $z_D > \max\{u_A, u_B, u_C\}$. However, after having searched alternative D, the consumer stops because the reservation utility of option E is smaller than the maximum utility realized so far, i.e., $z_E < \max\{u_A, u_B, u_C, u_D\}$.³⁰ The maximum realized utility among the searched options is offered by alternative B with $u_B = 9$. The consumers therefore revisits and purchases B. Thus, for differentiated goods, revisits can happen when consumers search sequentially. For the researcher, this means that evaluating the revisit behavior of consumers does not help to discriminate between simultaneous and sequential search in such a setting.

Table 2: Example

Option	Utility (u)	Reservation Utility (z)
A	7	15
B	9	13
C	8	11
D	6	10
E	11	7

De los Santos et al. (2012) point out that a more robust difference between sequential and simultaneous search is that the search behavior depends on observed search outcomes under the former, but not under the latter search method. This insight forms the basis of a second test which uses the following idea: if consumers search sequentially and know the price distribution(s), they should be more likely to stop searching after getting a below-mean price draw as opposed to an above-mean price draw. The reason is as follows: due to the negative price coefficient in the utility function, a below-mean price draw results in an above-mean utility, i.e. $u \geq E[u]$, and an above-mean price draw results in a below-mean utility, i.e.

³⁰Here the assumption of perfect recall made in section 2.2 comes into play.

$u \leq E[u]$. Holding everything else constant, the consumer is more likely to stop the search with an above-mean utility draw than a below-mean utility draw since the stopping rule is more likely to be satisfied, i.e. the maximum utility among the searched options is more likely to be larger than the maximum reservation utility among the non-searched options.

Based on this idea, the search method can be (empirically) identified as follows: if consumers search sequentially, consumers who get a below-mean price draw should be significantly more likely to stop searching after that below-mean price draw. To address store differentiation, this test can be carried out within a store, i.e. if a product has a high price relative to the store’s price distribution, sequentially searching consumers are more likely to continue searching, while the high price (relative to the store’s price distribution) should not affect the behavior of simultaneously searching consumers. In their empirical application, De los Santos et al. (2012) do not find any evidence for search decisions being dependent on observed prices – even within stores. They conclude that a simultaneous search model fits the data better than a sequential search model.

5.2 Honka and Chintagunta (2017)

As stated previously, Honka and Chintagunta (2017) also study search method identification. This paper provides an analytical proof for search method identification. It also shows that it is not necessary to observe the sequences of consumers’ searches (which was a crucial assumption in De los Santos et al. 2012); only data on search sets, purchases, and purchase prices are needed for search method identification.

Suppose prices follow some well-defined (potentially company-specific and/or consumer-specific) distributions and define $\Pr(p < \mu_{ij}^p) = \lambda$, i.e. the probability that a price draw is below the expected price is λ . Further define the event $X = 1$ as a below-price expectation price draw and $X = 0$ as an above-price expectation price draw. Recall that, under simultaneous search, the consumer commits to searching a set S_i consisting of k_i companies. Then Honka and Chintagunta (2017) calculate the expected proportion of below-price expectation prices in a consumer’s consideration set of size k_i as

$$E \left[\frac{1}{k_i} \sum_{m=1}^{k_i} X_m \right] = \frac{1}{k_i} \sum_{m=1}^{k_i} E[X_m] = \frac{\lambda k_i}{k_i} = \lambda. \quad (36)$$

Thus, if consumers search simultaneously, a researcher can expect $\lambda\%$ of the price draws in consumers’ consideration sets to be below and $(1 - \lambda)\%$ to be above the expected price(s). The crucial ingredients for identification are that the researcher observes the means of the price distributions μ_{ij}^p , the actual prices in consumers’ consideration sets p_{ij} and the proba-

bility of a price draw being below its mean λ .

Under sequential search, Honka and Chintagunta (2017) show that, for both homogeneous and differentiated goods allowing for consumer- and company-specific search costs, the expected proportion of below-price expectation prices in consumers' consideration sets of size one, X_1 , is always *larger* than λ , i.e. $X_1 > \lambda$, under the necessary condition that a positive number of consumers is observed to make more than one search.

The characteristic price patterns for simultaneous and sequential search described above hold for all models that satisfy the following set of assumptions: (a) prices are the only source of uncertainty for the consumer which he resolves through search; (b) consumers know the distribution of prices and have rational expectations for these prices; (c) price draws are independent across companies; (d) there is no learning about the price distribution from observing other variables (e.g. advertising); (e) (search costs are sufficiently low so that) all consumers search at least once; and (f) consumers have non-zero search costs. Models that satisfy this set of assumptions include (1) models for homogeneous goods, (2) models for differentiated products, (3) models that include unobserved heterogeneity in preferences and/ or search costs, (4) models with correlations among preferences and search costs, and (5) models with observed heterogeneity in price distribution means μ_{ij} . On the other hand, the researcher would not find the characteristic price patterns when there is *unobserved* heterogeneity in the price distribution means as the researcher would no longer be able to judge whether a price draw is above or below the mean. Note also that the identification arguments in Honka and Chintagunta (2017) are based on the first moments of prices; in principle there could be identification rules based on higher moments as well.

Lastly, Honka and Chintagunta (2017) discuss the modeling assumptions stated in the previous paragraph and to what extent the search method identification results depend on them. Assumptions (a) through (e) are standard in both the theoretical and empirical literature on price search. With regard to assumption (f) that consumers have non-zero search costs, note that search costs have to only be *marginally* larger than zero for search method identification to hold in all model specifications. Alternatively, if the researcher believes that the assumption of non-zero search costs is not appropriate in an empirical setting, search method identification is also given under the assumption that the search cost distribution is continuous, i.e. has support, from 0 to a positive number $A > 0$.³¹

³¹Note that it is not required that the search cost distribution is continuous over its full range. It is only required that it is continuous over the interval 0 to $A > 0$. The search method identification goes through when a search cost distribution has support e.g. from 0 to A and from B to C with $C \geq B > A > 0$.

6 Current Directions

In this section, we review some of the current directions in which the search literature is moving. As discussed in Section 2, this largely involves relaxing some of the rather stringent assumptions made in that section and/or developing new modeling approaches to understand more detailed data on consumer search especially from the online environment.

We start by discussing papers which study search with learning, i.e. research that relaxes the assumption that consumers know the true price or match value distribution (Assumption 2 in Section 2.1). Instead, these papers try to characterize optimal consumer search in the presence of concurrent learning of the price or match value distribution. In the following section, we discuss papers which investigate search for multiple attributes, e.g. price *and* match value. In Section 6.3, we review papers that study the role of advertising in a market which is characterized by consumer search. In the three subsections that follow, we describe research that focuses on how consumers search in an online environment. This includes papers that look at search and rankings in Section 6.4, papers that try to quantify the optimal amount of information shown to consumers in Section 6.5, and papers that work with granular, i.e., extremely detailed, data on consumer search in Section 6.6. We conclude this section by discussing papers that investigate the intensive margin of search, i.e. search duration, allowing for re-visits of sellers in Section 6.7 (relaxing Assumptions 3 and 5 of Sections 2.1 and 2.2) and papers that incorporate dynamic aspects of consumer search in Section 6.8.

6.1 Search and Learning

A standard assumption in the consumer search literature is that consumers know the distribution from which they sample (Assumption 2 in Section 2.1). However, starting with Rothschild (1974), several theoretical papers have studied search behavior in the case that consumers have uncertainty about the distribution from which they sample (Rosenfield and Shapiro 1981; Bikhchandani and Sharma 1996). Although the empirical literature has largely followed Stigler (1961)'s initial assumption that the distribution is known, several recent studies have departed from it and assume that consumers learn about the distribution of prices or utilities while searching.

To quickly recap, Rothschild (1974) studies optimal search rules when individuals are searching from unknown distributions and use Bayesian updating to revise their priors when new information arrives. An important example in his paper is the case of a Dirichlet prior distribution: if prior beliefs follow a Dirichlet distribution, then the reservation value property continues to hold, i.e. the qualitative properties of optimal search rules that apply to

models in which the distribution is known carry over to the case of an unknown distribution. Koulayev (2013) uses this result to derive a model of search for homogenous products with Dirichlet priors that can be estimated using only aggregate data such as market shares and product characteristics. An attractive feature is that Dirichlet priors imply that search decisions can be characterized by the number of searches carried out to date as well as the best offer observed up to that point. This feature simplifies integrating out search histories (which are unobserved in Koulayev 2013’s application) and makes it possible to derive a closed-form expression for ex-ante buying probabilities.

The Dirichlet distribution is a discrete distribution. In settings in which consumers search for a good product match, which is typically modeled as an IID draw from a continuous distribution, a continuous prior may be more appropriate. Bikhchandani and Sharma (1996) extend the Rothschild (1974) model to allow for a continuous distribution of offerings by using a Dirichlet process – a generalization of the Dirichlet distribution. Dirichlet process priors also imply that the only parts of the search history that matter for search decisions are the identity of the best alternative observed so far and the number of searches to date, which simplifies the estimation of such a model. The search model in Häubl et al. (2010) features learning of this type and is empirically tested using data from two experiments. De los Santos et al. (2017) also use this property to develop a method to estimate search costs in differentiated products markets. Specifically, the paper uses observed search behavior to derive bounds on a consumer’s search cost: if a consumer stops searching, this implies that she found a product with a higher realized utility than her reservation utility. If she continues searching, her search costs should have been lower than the gains from search relative to the best utility found so far. Learning enters through the equation describing the gains from search, i.e.,

$$G(\hat{u}_{it}) = \frac{W}{W+t} \int_{\hat{u}_{it}}^{\infty} (u - \hat{u}_{it}) \cdot h(u) \, du, \quad (37)$$

where $h(u)$ is the density of the initial prior distribution, W is the weight put on the initial prior, and t represents the number of searches to date. The term $\frac{W}{W+t}$ differentiates equation (37) from the non-learning case and reflects the updating process of consumers. Intuitively, every time a utility lower than \hat{u}_{it} is drawn, the consumer puts less weight on offers that exceed \hat{u}_{it} . If $t = 0$ and $h(u)$ corresponds to the utility distribution, then equation (37) equals the gains from search equation in the standard sequential search model.

Dzyabura and Hauser (2017) study product recommendations and point out that, in an environment in which consumers are learning about their preference weights while searching, it may not be optimal to recommend the product with the highest probability to be chosen

or the product with the highest option value. Instead, the optimal recommendation system encourages consumers to learn by suggesting products with diverse attribute levels, under-valued products, and products that are most likely to change consumers' priors. Synthetic data experiments show that recommendation systems that have these elements perform well.

Jindal and Aribarg (2018) conduct a lab experiment during which consumers search and learn the price distribution for a household appliance at the same time. The authors elicit each consumer's belief about the price distribution before the first search and after every search the consumer decides to make. Jindal and Aribarg (2018) observe that consumers update their beliefs about the price distribution while searching. Using their experimental data, the authors show that not accounting for belief updating or assuming rational expectations biases search cost estimates. The direction of the bias depends on the position of prior beliefs relative to the true price distribution. Further, Jindal and Aribarg (2018) find that accounting for the means of the belief distribution mitigates the bias in search cost estimates substantially; the standard deviation of the belief distribution has a relative small impact on the distribution of search costs, and hence, the bias.

Most of the previously mentioned papers (e.g. Rothschild 1974, De los Santos et al. 2017, Dzyabura and Hauser 2017) *assume* that consumers are learning while searching and then derive implications for optimal consumer search and/or supply side reactions for such an environment. Crucial, unanswered questions remain: with observational data, is it possible to identify whether consumers know or learn the distribution of interest while searching? What data would be necessary to do so and what would be the identifying data patterns? Furthermore, how quickly do consumers learn? Can companies influence the learning process and how? These and other unanswered questions related to concurrent search and learning represent ample opportunity for future research.

6.2 Search for Multiple Attributes

So far, work discussed in this chapter has modeled consumers' search for the specific value of a *single* attribute, e.g. price or match value. However, in practice, consumers might be searching to resolve uncertainty about more than one attribute.³² Researchers have developed different approaches to address this issue. For example, De los Santos et al. (2012) derive a closed-form solution for the benefit of searching for the case that consumers search simultaneously for both prices and epsilon. However, their solution requires the researcher to assume that both prices and epsilon follow EV Type I distributions and that both distributions are independent.

³²Match value search models sometimes describe the match value as a summary measure for multiple attributes.

Chen and Yao (2017) and Yao et al. (2017) pursue a different approach: while consumers search for multiple attributes in their sequential search models, in both papers, the authors assume that consumers know the *joint* distribution of these attributes. Consumers then search to resolve uncertainty about the (one) joint distribution. Thus Chen and Yao (2017) and Yao et al. (2017) model search for multiple attributes by imposing an additional assumption, i.e. that consumers know the joint distribution of the attributes, which allows them to apply the standard solution for a sequential search model for a single attribute.

While assuming that consumers know the joint distribution of multiple attributes or developing closed-form solutions under specific distributional assumptions are an important steps forward, ample research opportunities remain to develop empirical models of search for multiple attributes with less stringent assumptions. On a different note, Bronnenberg et al. (2016) describe the values of attributes consumers observe while searching. However, an unanswered question is as to how many and which attributes a consumer searches for, i.e. resolves uncertainty about, versus simply observes their values because they are shown to the consumer by default. Field experiments might help shed light on this issue.

6.3 Advertising and Search

Researchers have long been interested in how advertising affects consumers' decision-making in markets that are characterized by consumers' limited information, i.e. markets in which consumers search and form consideration sets. The consideration set literature (Section 3.1) has often modeled consideration as a function of advertising and has often found advertising to have a significant effect on consideration, often larger than its effect on choice. For example, Terui et al. (2011) report advertising to significantly affect consideration but not choice. In the economics literature, by assumption, Goeree (2008) models advertising as affecting consideration but not choice. In a recent paper, using data on individual consumers' awareness, consideration, and choices in the auto insurance industry over a time period of nine year, Tsai and Honka (2018) find advertising to significantly affect consumer awareness, but not conditional consideration or conditional choice.³³ Further, the authors report that the advertising content that leads to consumers' increased awareness is of non-informational nature, i.e. fun/humorous and/or brand name focused, implying that the effect on awareness is coming from non-informational content leading to better brand recall.

³³Tsai and Honka (2018) observe unaided and aided awareness sets to, on average, contain 4.15 and 12.02, respectively. Looking at shoppers and non-shoppers separately, as expected, shoppers have larger unaided and aided awareness sets than non-shoppers. Consideration sets, on average, contain 3.12 brands (which includes the previous insurance provider).

Honka et al. (2017) develop a structural model which describes the three stages of the purchase process: awareness, consideration, and choice. It is one of the first papers that accounts for endogeneity – here: of advertising – within a model of consumer search. Potential endogeneity of advertising may arise in any or all stages of the purchase process and is addressed using the control function approach (Petrin and Train 2010). The model is calibrated with individual-level data from the U.S. retail banking industry in which the authors observe consumers’ (aided) awareness and consideration sets as well as their purchase decisions. Consumers are, on average, aware of 6.8 banks and consider 2.5 banks.

In modeling consumer behavior, Honka et al. (2017) view awareness as a passive occurrence, i.e. the consumer does not exert any costly effort to become aware of a bank. A consumer can become aware of a bank by, for example, seeing an ad or driving by a bank branch. Consideration is an active occurrence, i.e. the consumer exerts effort and incurs costs to learn about the interest rates offered by a bank. The consumer’s consideration set is thus modeled as the outcome of a simultaneous search process given the consumer’s awareness set (à la Honka 2014). And finally, purchase is an active, but effortless occurrence in which the consumer chooses the bank which gives him the highest utility. The consumer’s purchase decision is modeled as a choice model given the consumer’s consideration set. Consideration and choice are modeled in a consistent manner by specifying the same utility function for both stages. This assumption is supported by Bronnenberg et al. (2016) who find that consumers behave similarly during the search and purchase stages.

Honka et al. (2017) find that advertising primarily serves as an awareness shifter. While the authors also report that advertising significantly affects utility, the latter effect is much smaller in terms of magnitude. Advertising makes consumers aware of more options; thus consumers search more and find better alternatives than they would otherwise. In turn, this increases the market share of smaller banks and makes the U.S. banking industry more competitive.

Further study of how advertising interacts with the process through which consumers search/consider products is a potentially very fruitful area for future research. For example, whether advertising in which explicit comparisons with competing products’ attributes and/or prices are made enlarges consumers’ consideration sets is a very interesting question (though how consumers may evaluate firms’ choices of which competitors they compare themselves against is a very interesting question for the literature on strategic information transmission). More broadly, understanding and quantifying the mechanisms through which different types of advertising affect the “purchase funnel” is likely to benefit from the availability of detailed data sets especially on online shopping behavior.

6.4 Search and Rankings

Most of the papers discussed so far assume that the order in which consumers obtain search outcomes is either random or, in the case of differentiated products, the outcome of a consumer's optimal search procedure. However, in certain markets the order in which alternatives are searched may be largely determined by an intermediary or platform. For instance, search engines, travel agents, and comparison sites all present their search results ordered in a certain way and, as such, affect the way in which consumers search. In this section, we discuss several recent papers that study how rankings affect online search behavior.

A particular challenge when estimating how rankings affect search is that rankings are endogenous. More relevant products are typically ranked higher by the intermediary. This endogeneity makes it difficult to estimate the causal effect of rankings on search: being ranked higher makes purchase or clicking more likely, which inflates the effect of relevance or quality. Ursu (2018) deals with this simultaneity problem by using data from a field experiment run by the online travel agent Expedia. Specifically, she compares click and purchase decisions of consumers who were either shown Expedia's standard hotel ranking or a random ranking. Her findings suggest that rankings affect consumers' search decisions in both settings, but conditional purchase decisions are only affected when hotels are ranked according to the Expedia ranking. This finding implies that the position effect of rankings is overestimated when rankings are not randomly generated.

De los Santos and Koulayev (2017) focus on the intermediary's decision of how to rank products. The authors propose a ranking method that optimizes click-through rates: it takes into account that, even though the intermediary typically knows very little about the characteristics of its consumers, the intermediary observes search refinement actions as well as other search actions. De los Santos and Koulayev (2017) find that their proposed ranking method almost doubles click-through rates for a hotel booking website in comparison to the website's default ranking. Using an analytical model, Ursu and Dzyabura (2018) also study how intermediaries should rank products to maximize searches or sales. The authors incorporate the aspect that search costs increase for lower-ranked products. Contrary to common practice, Ursu and Dzyabura (2018) find that intermediaries should not always show the product with the highest expected utility first.

Most online intermediaries give consumers the option to use search refinement tools when going through search results. These tools allow consumers to sort and filter the initial search rankings according to certain product characteristics, and can therefore significantly alter how products are ranked in comparison to the initial search results. Chen and Yao (2017) develop a sequential search model that incorporates consumers' search refinement decisions. Their model is estimated using clickstream data from a hotel booking website. A key finding

is that refinement tools result in 33% more searches leading a 17% higher utility of purchased products.

The intersection of the research areas on rankings and consumer search offers plenty of opportunities for future work. For example, a more detailed investigation of the different types of rankings (e.g. search engine for information, intermediary offering products by different sellers, retailer selling own products) guided by different goals (e.g. maximize click-through, sales, profits) might result in different optimal rankings. Further, do ranking entities want consumers to search more or less? And lastly, some ranking entities offer many filtering and refinement tools, others do not at all or only a small number. The optimality of these decisions and reasons behind them remain open questions.

6.5 Information Provision

The Internet provides a unique environment in which companies can relatively easily and inexpensively change which and how much information to make (more or less) accessible to consumers and, through these website design decisions, to influence consumer search behavior.

Gu (2016) develops a structural model of consumer search describing consumer behavior on the outer and the inner layer of a website. For example, on a hotel booking website, the outer layer refers to the hotel search results page and the inner layer refers to the individual hotel pages. Gu (2016) studies how the amount of information (measured by entropy) displayed on each layer affects consumer search behavior. The amount of information on the outer layer influences the likelihood of searching on the inner layer, i.e. clicking on the hotel page. More information on the outer layer reduces the need to visit the inner layer. At the same time, more information on the outer layer makes it more complex to process, which could decrease the likelihood of consumers searching. Thus there is a cost and a benefit to information on each layer.

Gardete and Antill (2019) propose a dynamic model in which consumers search over alternatives with multiple characteristics. They apply their model to click-stream data from the website of a used car seller. In counterfactual exercises, Gardete and Antill (2019) predict the effects of different website designs and find that the amount of information and the characteristics of the information shown to consumers upfront affect search and conversion rates.

More research is needed to understand how different information provision strategies affect consumer search and equilibrium outcomes. This is a domain where the burgeoning recent theoretical literature on information design (e.g. Kamenica and Gentzkow 2011) may

provide guideposts for empirical model building.

6.6 Granular Search Data

In most studies that use online search data, the search data are either aggregated to the domain level or are restricted to only one retailer. For instance, the comScore data used in De los Santos et al. (2012) and De los Santos (2018) only allow the researcher to observe which domains have been visited, but not the browsing activity within a specific domain. This data limitation means that search is only partially observed, which could lead to biased estimates of search costs. Bronnenberg et al. (2016) use a much more detailed data set in which browsing is available at the URL-level. Their data track consumers' online searches across and within different websites and are used to provide a detailed description of how consumers search when buying digital cameras. Their main finding is that consumers focus on a very small attribute space during the search and purchase process. This pattern can be interpreted as supporting the structural demand model assumption that consumers have the same utility for search and purchase. Moreover, consumers search more extensively than previous studies with more aggregate search data have found: consumers search, on average, 14 times prior to buying a digital camera.

It is typically much easier to obtain search data for online than for brick-and-mortar environments: online browsing data can be used as a proxy for search, whereas data on how consumers move within and across brick-and-mortar stores is typically not available. Seiler and Pinna (2017) use data obtained from radio-frequency identification tags that are attached to shopping carts to infer how consumers search within a grocery store.³⁴ Specifically, the data tell them how much time consumers spent in front of a shelf, which is then used as a proxy for search duration. Using the consumers' walking speed and basket size as instruments for search duration, the authors find that each additional minute spent searching results in a price paid that is lowered by \$2.10. In a related paper, Seiler and Yao (2017) use similar path-track data to study the impact of advertising. They find that, even though advertising leads to more sales, it does not bring more consumers to the category being advertised. This finding suggests that advertising only increases conversion conditional on visiting the category. Moreover, Seiler and Yao (2017) find that search duration, i.e. time spent in front of a shelf, is not affected by advertising.

This emerging literature shows that access to more and more detailed data on consumers'

³⁴An alternative way to capture search behavior is by using eye-tracking equipment. For instance, Stüttgen et al. (2012) use shelf images to analyze search behavior for grocery products and test whether consumers are using satisficing decision rules (see also Shi et al. 2013 for a study that analyzes information acquisition using eye-tracking data in an online environment).

actions on and across websites can enable researchers to get a detailed look into how consumers are actually searching for products. Further attempts to connect these exciting data sets to existing theoretical work on search, and collaboration between theorists and empiricists to build search models that better describe empirical patterns appear to be avenues for fruitful future research.

6.7 Search Duration

In many instances, consumers are observed to re-visit the same seller multiple times before making a purchase – a pattern that cannot be rationalized by the standard simultaneous and sequential search models (see Section 2). To explain this empirical pattern, Ursu et al. (2018) propose that consumers only *partially* resolve their uncertainty through a single search thus necessitating multiple searches of the same product if the consumer desires to know even more precisely about the product before making a purchase decision. Practically, Ursu et al. (2018) combine approaches from two streams of literature: the literature on consumer learning (using Bayesian updating) and the consumer search literature (more specifically, sequential search). Suppose a consumer wants to resolve the uncertainty about his match value with a vacation package. He has some prior belief or expectation of the match value. After searching once, e.g. by reading a review, the consumer receives a signal about the match value and updates his belief about it. The more the consumer searches, i.e. the more signals he receives, the less uncertainty he has about the match value. Thus, at each point in time during the search process, intuitively speaking, the consumer decides whether to search a previously unsearched option or to spend another search on a previously already searched option – allowing the model to capture the empirical observation of re-visits.

A crucial question in this context relates to the characterization of optimal behavior in such a model of sequential search, i.e. how do consumers optimally decide which product to search next (including the same), when to stop, and which one to purchase. For the standard sequential search model, Weitzman (1979) developed the well-known selection, stopping, and choice rules. For the model of sequential search with re-visits, Ursu et al. (2018) point to Chick and Frazier (2012) who developed analogue selection, stopping, and choice rules. The authors take the theoretical results by Chick and Frazier (2012) and apply them within their empirical context. The latter is a restaurant review site. Ursu et al. (2018) observe browsing behavior on this website and assume that a unit of search is represented by spending one minute on a restaurant page. Using this definition, the authors document that both the extensive and the intensive margins of search matter. They find that consumers search very few restaurants, but those that are searched are searched extensively.

Studying search intensity is a new research subfield with plenty of opportunities for future research. Search intensity plays an important role not only in the online, but also in the offline environment. For example, many consumers take multiple test drives with the same car before making a purchase decision or look at a piece of furniture multiple times before buying it. These are big-ticket items and a better understanding of the search process might help both consumers and sellers. From a methodological perspective, collaboration between theorists and empiricists to rationalize search intensity as well as search sequences appears to be an interesting economic modeling challenge that is relatively unexplored.

6.8 Dynamic Search

All papers discussed so far assume static consumer behavior. This assumption is largely driven by data constraints: most papers have access to cross-sectional individual-level data or aggregate data. However, in many instances, consumers are observed to conduct multiple search spells over the course of a month or a year. For example, consumers might search for yoghurt once a week or for laundry detergent once a month.

Seiler (2013) develops a structural model for storable goods in which it is costly for consumers to search, i.e. consumers have limited information. The model is estimated using data on purchases for laundry detergent. Seiler (2013) models the buying process as consisting of two stages: in the first stage, the consumer has to decide whether to search. Search is of the all-or-nothing type, i.e. the consumer can either search and obtain information on all products or not search at all. Only consumers who have searched can purchase. In the second stage, which is similar to a standard dynamic demand model for storable goods, the consumer then decides which product to buy. Dynamic models of demand are typically computationally burdensome and adding an extra layer in which consumers make search decisions will make the model more complex. To obtain closed-form solutions for the value functions in the search and purchase stages, Seiler (2013) adds a separate set of errors to each stage, which are unknown to the consumer before entering that stage. Moreover, he only allows for limited preference heterogeneity and the model does not allow price expectations to be affected by past price realizations, which can be a limitation for some markets. Seiler (2013) finds that search frictions play an important role in the market for laundry detergent: consumers are unaware of prices in 70% of shopping trips. Further, lower search costs by 50% would increase the elasticity of demand from -2.21 to -6.56 .

Pires (2016) builds on Seiler (2013), but instead of all-or-nothing search behavior, Pires (2016) allows consumers to determine which set of products to inspect using a simultaneous search strategy. The author only has access to data on purchases and adds a choice-set

specific error term to the one-period flow utility from searching to deal with the curse of dimensionality of the simultaneous search model. The error term follows a EV Type I distribution and can be integrated out to obtain closed-form expressions. Pires (2016) finds that search costs are substantial. Further, the effects of ignoring search on price elasticity depend on how often a product appears in consumers' consideration sets.

Since many products are purchased multiple times, understanding search behavior across purchase instances is clearly an important avenue for further research. This is an area where models of search come into contact with models of switching costs or customer inertia as well; thus we anticipate further work in this important interface.

7 Conclusions

Although the theoretical literature on search is almost sixty years old, empirical and econometric work in the area continues to develop at a very fast pace thanks to the ever increasing availability of data on actual search behavior. Abundant data on consumers' web searching/browsing/shopping behavior has become available through multiple channels on the Internet, along with the ability to utilize field experiments and randomized controlled trials. The availability of data is not restricted to the online/e-commerce domain; there is increasingly more data on consumer shopping patterns across and within physical retail stores. Data on consumer search and choice patterns is also becoming readily available in finance, insurance, and energy markets. On the public side of the economy, too, search and choice data are becoming increasingly more available (e.g. in educational choices – Kapor 2016, health insurance – Ketcham et al. 2015, and public housing – van Dijk 2019), creating more applications of econometric models that incorporate search and consideration, and also bring methodological challenges of their own.

While our focus was on econometric methods to test between existing theories of search and to estimate the structural parameters of these models, the abundance and complexity of search data also pushes search theory to new frontiers. We also note that, while models of optimizing agents with rational expectations have been used fruitfully to make sense of search data, “behavioral” theories of search that combine insights from both economics and psychology may prove very fruitful in explaining observed patterns in the data.

The mechanisms using which consumers search for and find the products they eventually purchase are important determinants of equilibrium outcomes in product markets. While most of the empirical search models have focused on the demand side, investigating how firms optimize in the presence of consumer search needs further investigation. As we noted in Section 6.5, how firms choose to design their websites or advertising strategies to aid/direct

search is a very important and relatively unexplored area of research, at least from the perspective of economics. Firms spend a lot of time and resources to design and optimize their websites or mobile interfaces. Quantifying how these design decisions affect market outcomes requires careful modeling of the (strategic) interaction between consumer and firm behavior.

Another interesting area of research, highlighted in Section 3.1, is to investigate whether and how one can identify the presence of information/search frictions using conventional data sets (on prices, quantities, and product attributes) that do not contain information on search or credible variation in consumer information. Along with such investigations, efforts to supplement such conventional data sets with data on actual search or credible shifters of search/consumer information will help enrich the analyses that can be done with such data sets.

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