

# Prices and Heterogeneous Search Costs <sup>\*</sup>

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## Abstract

We study price formation in a model of consumer search for differentiated products in which consumers have heterogeneous search costs. We provide conditions under which a pure-strategy symmetric Nash equilibrium exists and is unique. Search costs affect two margins—the intensive search margin (or search intensity) and the extensive search margin (or the decision to search rather than to not search at all). These two margins affect the elasticity of demand in opposite directions and whether lower search costs result in higher or lower prices depends on the properties of the search cost density.

**Keywords:** sequential search, search cost heterogeneity, differentiated products, monotone likelihood ratio

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# 1 Introduction

Throughout history, technological improvements have made it possible for consumers to participate in markets that were previously beyond their reach. A recent example of such a technological development is the Internet. The introduction and sustained growth of e-commerce has significantly lowered the transaction costs consumers experience while shopping. This has not only made it easier for consumers to search for and compare products but it has also made it easier for consumers to get access to new markets and hard-to-find products. This has also led to the Internet's Long Tail phenomenon: the Internet allows consumers to access a much larger product selection, including niche products that were previously difficult to find, thereby creating a long tail in the sales distribution (see, e.g., Brynjolfsson, Hu, and Simester, 2011).<sup>1</sup>

How are technological advances that make it easier for consumers to search expected to affect market competitiveness? The conventional answer is that a reduction in search costs increases the elasticity of demand and as such reduces prices. However, as illustrated by the Internet example above, this is not the complete story. Lower search costs also allow consumers to search for products that previously were not part of their consideration sets. As the new consumers that enter the market are likely to have higher search costs (because otherwise they would have been actively searching before), a reduction in search costs leads to compositional changes in the active consumer population that may *decrease* the elasticity of demand. More consumer participation as a result of a technology-driven reduction in search costs may end up being the dominating factor, which means firms will raise prices, even when search costs go down for all consumers.

Hortaçsu and Syverson (2004) present empirical evidence that is consistent with this particular mechanism in their study of the US mutual fund industry during the late nineties. Not that long ago investors had to go through significant effort to search for adequate investment opportunities while managing their financial investments. However, the rise of Internet banking and online brokerages during the late nineties decreased transaction costs and made it easier for individuals to search for new investment opportunities. Hortaçsu and Syverson (2004) find that during this period investment fund fees went up and that search costs decreased at the lower percentiles of the search cost distribution but increased at the upper percentiles. According to the authors, this surprising finding (i.e., a second-order stochastic dominance decrease in search costs leading to a

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<sup>1</sup>The expansion of the railroad network in the United States in the 19th century also led to significantly lower transportation costs and greater market integration. As with the Internet, lower transportation costs were not only beneficial to consumers because of reduced transaction costs but also because they allowed consumers to get access to new markets (see Donaldson and Hornbeck, 2016, for a recent study on the historical impact of railroads on the U.S. economy that focuses on market access).

price rise) is explained by the large increase in the number of households that participated in the mutual fund market for the first time. These new investors arguably had higher search costs and were less savvy. Hortaçsu and Syverson argue that the entry of these new investors changed the composition of the investor population and made demand more inelastic, explaining the increase in mutual fund fees.

The main objective of our article is to present conditions under which a mechanism like the one described above occurs. In particular, our goal is to derive general conditions under which a first- or second-order stochastic dominance decrease in search costs triggers sufficient entry of consumers with high search costs such that the elasticity of demand decreases and equilibrium prices go up. In order to do so, we develop a model of consumer search for differentiated products in which consumers have heterogeneous search costs and the decision to participate in the market is endogenous. Our model, which is presented in Section 2, builds on the seminal model of consumer search for differentiated products introduced by Wolinsky (1986), and further studied by Anderson and Renault (1999).<sup>2</sup> We extend this model by allowing for arbitrary search cost densities. This extension is crucial for studying the second-order stochastic dominance (SOSD) decreases in search costs observed in the mutual fund industry by Hortaçsu and Syverson (2004).<sup>3</sup>

Section 3 characterizes the pure-strategy symmetric Nash equilibrium of the model. Our first contribution is the study of the existence and uniqueness of a symmetric Nash equilibrium in pure strategies. Because a direct verification of the second order conditions fails to deliver clear-cut results we proceed by studying the conditions under which the profit function of a typical firm is quasi-concave. We show that when the distribution of match values has an increasing hazard rate and the density of search costs is sufficiently log-concave, a symmetric equilibrium exists and is unique. In order to prove our existence result we use an aggregation theorem due to Prékopa (1973) that shows that integration preserves log-concavity (see also Caplin and Nalebuff, 1991).

Our second contribution is the study of the comparative statics effects of lower search costs. In Section 4 we show that a change in search costs affects two margins, namely, the intensity with which consumers search (which we call the *intensive search margin*) and the share of consumers

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<sup>2</sup>Arguably Wolinsky's framework has become the workhorse model of consumer search for differentiated products. Recent work that builds on this article include Bar-Isaac, Caruana, and Cuñat (2012), who extend the model to the case of quality-differentiated firms and study design differentiation, Armstrong, Vickers, and Zhou (2009), who study search and pricing behavior in the presence of a prominent firm, Haan and Moraga-González (2011), who study the emergence and the price effects of prominence, Moraga-González and Petrikaitė (2013), who examine the effect of search costs on mergers, and Zhou (2014), who studies multi-product search.

<sup>3</sup>Our results are not specific to the model we present: we have obtained similar results when firms sell homogeneous products and charge a price randomly drawn from a distribution, while consumers search non-sequentially for good deals. The details can be found in the Supplementary Appendix to this article, which is placed at the journal's website.

who choose to search for a good deal in the first place (which we refer to as the *extensive search margin*). By assuming that all consumers search at least once, the literature has typically focused on the effects of the intensive search margin on price determination and has thereby neglected the role of the extensive search margin.<sup>4</sup>

Recognizing that the magnitude of search costs does not only affect the intensive search margin but may also affect the extensive search margin turns out to be critical for a complete understanding of the functioning of consumer search markets. We show that if the extensive search margin does not play any role whatsoever, any decrease in search costs in the sense of first-order stochastic dominance (FOSD) results in a lower equilibrium price. This would happen when the range of search costs is sufficiently small and all consumers choose to search in market equilibrium. Consumers, facing lower search costs, all choose to search more for products that are more attractive. Firms, anticipating that average demand becomes more elastic, respond by reducing their prices. Under additional conditions on the density of match values (satisfied for example by all increasing and convex densities), a decrease in search costs in the sense of second-order stochastic dominance (SOSD) also results in lower prices.

If the extensive search margin does play a role, we get very different results. Specifically, we show that a shock that reduces search costs leads to more search but also brings new consumers into the market with relatively high search costs. We establish that for search cost densities with the decreasing likelihood ratio property (DLRP), a decrease in search costs in the sense of FOSD or SOSD results in a higher equilibrium price. That is, for densities with the DLRP a decrease in search costs has a bigger effect on the extensive search margin than on the intensive search margin, and as a result prices go up. This is exactly what seems to have occurred in Hortaçsu and Syverson's (2004) study of the mutual fund industry: easier search led to entry of new consumers and this resulted in a more inelastic composition of demand so prices went up in spite of the increasing popularity of Internet banking and online trading. For search cost densities with the increasing likelihood ratio property (ILRP) we find the opposite: a decrease in search costs leads to lower prices. For this type of densities the effect of a fall in search costs on search intensity dominates the effect on demand composition.

We believe that the mechanisms described above may also be important for understanding other existing empirical evidence regarding the effects of the Internet on retail price competition. The commonly held belief is that the Internet improves consumer search technology and this should

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<sup>4</sup>This point is also the central tenet in Anderson and Renault (2006), who, by allowing for arbitrary search costs, are able to reconcile the empirical observation that much of the advertising we observe in arguably search environments does impart only match information and not price information.

naturally lead to lower prices online than offline. Despite this belief, the empirical evidence is mixed: although some studies indeed find lower prices online than offline (Brynjolfsson and Smith, 2000; Scott Morton, Zettelmeyer, Silva-Risso, 2001), others find no effects or even the opposite (Clay, Krishnan, and Wolff, 2001; Clemons, Hann, and Hitt, 2002; Goolsbee, 2001; Lee, 1998). The entry of new buyers as a result of a reduction in transaction costs due to the Internet may very well be part of the explanation for why some studies have found online prices to be higher than offline prices.

A third contribution of our article is more technical in nature and relates to the application of properties of truncated densities. Specifically, as part of our proofs on the comparative statics effects of lower search costs we apply results from the statistics literature that establish stochastic rankings of truncated distributions. For example, if two densities are ranked according to the ILRP, the corresponding distributions truncated from above can be ranked according to the FOSD criterion, irrespective of the location of the truncation point. Conversely, if the densities can be ranked according to the DLRP, then the corresponding distributions truncated from above can be ranked according to the reverse FOSD. The reversed hazard rate ordering has similar implications for the truncated densities.

## Related literature

Most of the previous literature has proceeded under the restrictive assumption that search costs are required to be “low enough,” *de facto* implying that all consumers choose to search at least once in equilibrium rather than to not search at all (e.g., Stahl, 1989; Burdett and Judd, 1983; Wolinsky, 1986). As pointed out by Stiglitz (1979), the alternative assumption that search costs are large may cause the market to collapse.<sup>5</sup> As we have discussed before, this unravelling of the market will not arise when consumer search costs are heterogeneous. Two related articles (Janssen, Moraga-González, and Wildenbeest, 2005; Rauh, 2004) also allow for endogenous consumer participation, although in both articles the type of search cost heterogeneity that is assumed is less general than in our model. Janssen, Moraga-González, and Wildenbeest (2005) employ Stahl’s (1989) setting, with the much used “shoppers and non-shoppers” setup. They show that when the extensive search margin plays a role, prices will always increase if the search cost decreases. Our article shows instead that prices can go both directions and provides conditions under which one or the other

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<sup>5</sup>For example, in the setting of Diamond (1971), the only price that can be part of a market equilibrium is the monopoly price (the well-known “Diamond paradox”). If the search cost is relatively high, the surplus consumers derive at the monopoly price may be insufficient to cover the cost of the first search, in which case consumers rather do not search at all and the market fails to exist.

occurs. This means that by not restricting the extent of consumer heterogeneity in the market, our model may result in qualitatively very different outcomes than a model that uses the standard shoppers and non-shoppers setup. Moreover, the shoppers and non-shoppers setup does not allow for studying the comparative statics effects of SOSD decreases in search costs, such as those that occurred in the US mutual fund industry. In Rauh (2004) consumer participation is endogenous and proves to be important for the welfare analysis of price ceilings; however, he assumes search costs are uniformly distributed and does not examine the comparative statics effects of higher search costs.

A few recent articles have put forward situations in which lower search costs do not necessarily lead to lower prices, although these findings rely on different mechanisms. For instance, Chen and Zhang (2011) enrich Stahl’s (1989) setting by adding loyal consumers, and show that in such an environment a reduction in the search cost sometimes leads to higher equilibrium prices. In a different framework in which search is price directed, Armstrong and Zhou (2011) show that lower search costs lead to higher prices. A similar result is obtained in Haan, Moraga-González and Petrikaitė (2015), who extend the Anderson and Renault’s (1999) model by making prices observable before search. In a model in which consumers search for various products from multi-product firms, Zhou (2014) demonstrates that product externalities can lead firms to raise their prices when search costs go down. In Bar-Isaac, Caruana, and Cuñat’s (2012) study of product-design differentiation, the average price could also increase if search costs fall because a larger share of firms will choose to adopt niche designs. In a related study, Yang (2013) allows for heterogeneity in consumers’ product valuations and shows that when search costs fall, niche consumers enter the market, which drives firms away from catering to mainstream consumers, thereby reducing competition for the latter. Lastly, in a model of vertical relations, Janssen and Shelegia (2015) encounter situations in which retail prices increase as search costs decrease.

## 2 Sequential search for differentiated products

We build on Wolinsky’s (1986) model of consumer search for differentiated products; specifically, we study a version of his model in which there are infinitely many firms and consumers have heterogeneous search costs.<sup>6</sup> Consider a market with infinitely many consumers and firms and, without loss of generality, normalize the number of consumers per firm to 1. Firms produce horizontally

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<sup>6</sup>In the Supplementary Appendix of this article we present numerical results showing that our main insights carry over to the duopoly case. With a finite number of firms, however, proving existence and uniqueness of equilibrium is a challenge because the demand of a typical firm consists of a sum of two functions that is not necessarily quasi-concave functions of its own price (see Anderson and Renault, 1999).

differentiated products using the same constant returns to scale technology of production; let  $r$  be the marginal cost of the firms. Aiming at maximizing their expected profits, firms compete in the market by choosing their prices simultaneously. We focus on pure-strategy symmetric Nash equilibria (SNE); let  $p^*$  denote a SNE price.

A consumer  $\ell$  has tastes for a product  $i$  described by the following indirect utility function:

$$u_{i\ell} = \begin{cases} \varepsilon_{i\ell} - p_i & \text{if she buys product } i \text{ at price } p_i; \\ 0 & \text{otherwise.} \end{cases}$$

The parameter  $\varepsilon_{i\ell}$  is a match value between consumer  $\ell$  and product  $i$ . We assume that the match value  $\varepsilon_{i\ell}$  is the realization of a random variable distributed on the interval  $[0, \bar{\varepsilon}]$  according to a cumulative distribution function (CDF) denoted by  $F$ , with  $\bar{\varepsilon} > r$ . Let  $f$  be the probability density function (PDF) of  $F$ . We assume that  $f$  is differentiable and that  $1 - F$  is log-concave. Match values  $\varepsilon_{i\ell}$  are independently distributed across consumers and products. Moreover, they are private information of consumers so personalized pricing is not possible. Consumers search sequentially in order to maximize their expected utility. While searching, they have correct beliefs about the equilibrium price. The total cost of search of a consumer  $\ell$  who searches  $n$  times is  $nc_\ell$ , where  $c_\ell$  indicates her marginal cost of search.

Consumers differ in their costs of search.<sup>7</sup> Specifically, assume that a buyer  $\ell$ 's search cost  $c_\ell$  is drawn independently from a differentiable cumulative distribution function  $G$  with support  $[\underline{c}, \bar{c}]$ . Let  $g$  be the density of  $G$ . We refer to the difference between the upper and lower bound of the search cost distribution as the *range* of search costs. We require the lower bound of the search cost distribution to be sufficiently low because otherwise no consumer would search and the market would collapse. In what follows we drop the consumer index  $\ell$ .

### 3 Price equilibrium

We now characterize the pure-strategy SNE price. In order to do so, we derive the payoff of a firm, say  $i$ , that deviates from the SNE price  $p^*$  by charging a price  $p_i \neq p^*$ . Next, we compute the first order condition (FOC), apply the symmetry condition  $p_i = p^*$ , and study the existence and

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<sup>7</sup>Most models in the literature abstract from consumer search cost heterogeneity, or only allow for special forms of it, typically with some consumers having a search cost equal to zero (usually referred to as “shoppers”), whereas the rest faces a positive and identical search cost (“non-shoppers”). There are some exceptions to this in the literature (e.g., Bénabou, 1993; Rob, 1985; Rauh, 2009; Stahl, 1996; Tappata, 2009) but in these articles search costs are assumed to be sufficiently low so that all consumers search at least once. We will later show that this assumption is restrictive.

uniqueness of the SNE. For later use, we define the monopoly price as  $p^m = \arg \max_p (p - r)(1 - F(p))$ .

Consider the (expected) payoff to a firm  $i$  that deviates from the equilibrium by charging a price  $p_i$ . In order to compute firm  $i$ 's demand, we first need to characterize consumer search behavior. As consumers do not observe deviations before searching, we can rely on Kohn and Shavell (1974), who study the search problem of a consumer who faces a set of independently and identically distributed options with a known distribution. Kohn and Shavell show that the optimal search rule is static in nature and has the stationary reservation utility property. Accordingly, consider a consumer with search cost  $c$  and denote the solution to

$$h(x) \equiv \int_x^{\bar{\varepsilon}} (\varepsilon - x)f(\varepsilon)d\varepsilon = c \quad (1)$$

in  $x$  by  $\hat{x}(c)$ . The left-hand-side (LHS) of equation (1) is the expected benefit in a symmetric equilibrium from searching one more time for a consumer whose best option so far is  $x$ . Its right-hand-side (RHS) is the consumer's cost of search. Hence  $\hat{x}(c)$  represents the threshold match value above which a consumer with search cost  $c$  will optimally decide not to continue searching for another product. The function  $h$  is monotonically decreasing. Moreover,  $h(0) = E[\varepsilon]$  and  $h(\bar{\varepsilon}) = 0$ . It is readily seen that for any  $c \in [\underline{c}, \min\{\bar{c}, E[\varepsilon]\}]$ , there exists a unique  $\hat{x}(c)$  that solves equation (1). Differentiating equation (1) successively, we obtain

$$\begin{aligned} \hat{x}'(c) &= -\frac{1}{1 - F(\hat{x}(c))} < 0; \\ \hat{x}''(c) &= \frac{f(\hat{x}(c)) [\hat{x}'(c)]^2}{1 - F(\hat{x}(c))} > 0, \end{aligned}$$

which implies that  $\hat{x}(c)$  is a decreasing and convex function of  $c$  on  $[\underline{c}, \min\{\bar{c}, E[\varepsilon]\}]$ , with  $\hat{x}(E[\varepsilon]) = 0$  and  $\hat{x}(\underline{c}) \leq \bar{\varepsilon}$ .<sup>8</sup>

In order to compute firm  $i$ 's demand, consider a consumer with search cost  $c$  who shows up at firm  $i$  to inspect its product after possibly having inspected other products. Let  $\varepsilon_i - p_i$  denote the utility the consumer derives from firm  $i$ 's product. Obviously, if alternative  $i$  is not the best one so far, the consumer will discard it and search again. Therefore, only when the deal offered by firm  $i$  happens to be the best so far, the consumer will contemplate the possibility of stopping searching and buying the product of firm  $i$  right away. For this decision, the consumer compares the gains from an additional search with the costs of such a search. In this comparison, the consumer

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<sup>8</sup>Consumers with search cost  $c > E[\varepsilon]$ , if any, will automatically drop from the market and can therefore be ignored right away. Therefore  $\hat{x}(c)$  is well-defined for every consumer that matters for pricing.



holds correct expectations about the equilibrium price so she expects the other firms to charge  $p^*$ . The expected gains from searching one more firm, say firm  $j$ , are equal to  $\int_{\varepsilon_i - p_i + p^*}^{\bar{\varepsilon}} [\varepsilon_j - (\varepsilon_i - p_i + p^*)] f(\varepsilon_j) d\varepsilon_j$ . Comparing this to equation (1), it follows that, conditional on having arrived at firm  $i$ , the probability that buyer  $c$  stops searching at firm  $i$  is equal to  $\Pr[\varepsilon_i - p_i > \hat{x}(c) - p^*] = 1 - F(\hat{x}(c) + p_i - p^*)$ . With the remaining probability, the consumer finds the product of firm  $i$  not sufficiently satisfactory and prefers to continue searching; with infinitely many firms, such a consumer will surely buy at another firm. Because a consumer with search cost  $c$  may visit firm  $i$  after having visited no, one, two, three, etc. other firms, the unconditional probability she shows up at firm  $i$  and chooses to stop searching and acquire the product of firm  $i$  is

$$\frac{1 - F(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c))}. \quad (2)$$

To obtain the payoff of firm  $i$  we need to integrate expression (2) over the consumers who decide to search for a satisfactory product; in other words, we need to integrate over those consumers who derive expected positive surplus from market participation. To compute the surplus a consumer with search cost  $c$  obtains from participation, we note that she will stop and buy at the first firm she visits whenever the match value she derives at that firm is greater than  $\hat{x}(c)$ ; otherwise she will drop the first option and continue searching. In the latter case she will encounter herself exactly in the same situation as before because, conditional on participating, she will continue searching until she finds a product satisfactory enough to buy it and leave the market. Denoting by  $CS(c)$  her consumer surplus, we have that:<sup>9</sup>

$$CS(c) = \hat{x}(c) - p^*.$$

Setting this surplus equal to zero, and using equation (1), we obtain the critical search cost value above which consumers will refrain from participating in the market:

$$\tilde{c}(p^*) = \int_{p^*}^{\bar{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon. \quad (3)$$

Depending on how large the range of search costs is, more or fewer consumers will choose to search the market for a satisfactory deal. Correspondingly, we define

$$c_0(p^*) \equiv \min\{\bar{c}, \tilde{c}(p^*)\}.$$

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<sup>9</sup>For a formal derivation of this expression, see the Appendix.

The standard assumption in the search cost literature has been that  $c_0(p^*)$  takes on value  $\bar{c}$ , in which case the market is fully covered in the sense that all consumers search at least once. When  $c_0(p^*)$  takes on value  $\tilde{c}(p^*)$ , because  $\tilde{c}(p^*)$  is decreasing in  $p^*$ , fewer consumers will choose to search the market for an acceptable product if they expect a higher equilibrium price.

The payoff to the deviant firm  $i$  is then:

$$\pi(p_i; p^*) = (p_i - r)D(p_i, p^*), \quad (4)$$

where the demand firm  $i$  receives,  $D(p_i, p^*)$ , is

$$D(p_i, p^*) = \begin{cases} \int_{\underline{c}}^{c_0(p^*)} \frac{1 - F(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c))} g(c) dc & \text{if } p_i < p^*, \\ \int_{\max\{\underline{c}, \hat{c}(p_i)\}}^{c_0(p^*)} \frac{1 - F(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c))} g(c) dc & \text{if } p_i \geq p^*, \end{cases} \quad (5)$$

where  $\hat{c}(p_i)$  is the solution of the equation  $\hat{x}(c) + p_i - p^* = \bar{\varepsilon}$  in  $c$ .

The FOC is given by (for  $p_i < p^*$ ):

$$\int_{\underline{c}}^{c_0(p^*)} \frac{1 - F(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c))} g(c) dc - (p_i - r) \int_{\underline{c}}^{c_0(p^*)} \frac{f(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c))} g(c) dc = 0.$$

Applying symmetry, i.e.,  $p_i = p^*$ , we can rewrite the FOC as:<sup>10</sup>

$$p^* = r + \frac{G(c_0(p^*))}{\int_{\underline{c}}^{c_0(p^*)} \frac{f(\hat{x}(c))}{1 - F(\hat{x}(c))} g(c) dc}. \quad (6)$$

We refer to a price  $p^*$  for which the FOC (6) holds as a candidate SNE price.

We now show that a candidate symmetric equilibrium price always exists and is unique. We start by considering the case in which search costs are sufficiently low. Define:

$$c'' \equiv \int_{r + \frac{1}{\int_{\underline{c}}^{\bar{c}} \frac{f(\hat{x}(c))}{1 - F(\hat{x}(c))} g(c) dc}}^{\bar{\varepsilon}} \left( \varepsilon - r - \frac{1}{\int_{\underline{c}}^{\bar{c}} \frac{f(\hat{x}(c))}{1 - F(\hat{x}(c))} g(c) dc} \right) f(\varepsilon) d\varepsilon.$$

**Condition 1.** (*Low search costs condition*)  $\bar{c} \leq c''$ .

Assume Condition 1 holds. In this case, because  $c_0(p^*) = \bar{c}$ , we have  $G(c_0(p^*)) = 1$ . This implies

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<sup>10</sup>Note that the first-order necessary condition for a pure-strategy SNE is the same as equation (6) when we assume that the deviating price  $p_i \geq p^*$ .

that expression (6) gives the candidate equilibrium price explicitly. In this instance, obviously, there exists a unique candidate equilibrium price.

When search costs are not restricted to be sufficiently small and Condition 1 is violated then  $\bar{c} > \tilde{c}(p^*)$  and correspondingly  $G(c_0(p^*)) < 1$ . In this case, the candidate equilibrium price is given implicitly by the solution to equation (6). We now show that equation (6) always has a unique solution. For this we define the function

$$L(p) \equiv G(\tilde{c}(p)) - (p - r) \int_{\underline{c}}^{\tilde{c}(p)} \frac{f(\hat{x}(c))}{1 - F(\hat{x}(c))} g(c) dc$$

for  $p \in [r, p^m]$ , where  $p^m$  denotes the monopoly price. Note that

$$L(r) = G(\tilde{c}(r)) > 0.$$

Also observe that  $L(p^m)$  can be written as

$$L(p^m) = \int_{\underline{c}}^{\tilde{c}(p^m)} \frac{1 - F(\hat{x}(c)) - (p^m - r)f(\hat{x}(c))}{1 - F(\hat{x}(c))} g(c) dc.$$

The sign of this expression depends on the sign of the numerator of the fraction in the integrand. We now argue that  $L(p^m) < 0$  because  $1 - F(\hat{x}(c)) - (p^m - r)f(\hat{x}(c)) \leq 0$  for all  $c \in [\underline{c}, \tilde{c}(p^m)]$  (and strictly negative for some  $c$ 's). In fact, note that by log-concavity of  $f$ , because  $\hat{x}(c)$  decreases in  $c$ , it follows that  $f(\hat{x}(c))/[1 - F(\hat{x}(c))]$  decreases in  $c$ , which implies that  $1 - F(\hat{x}(c)) - (p^m - r)f(\hat{x}(c))$  increases in  $c$ . Because  $\hat{x}(\tilde{c}(p)) = p$ , if we set  $c = \tilde{c}(p^m)$  in the expression  $1 - F(\hat{x}(c)) - (p^m - r)f(\hat{x}(c))$ , we get the monopoly pricing rule  $1 - F(p^m) - (p^m - r)f(p^m) = 0$ . We can now conclude that  $L(p^m) < 0$  because the expression  $1 - F(\hat{x}(c)) - (p^m - r)f(\hat{x}(c))$  is increasing in  $c$  and takes on value zero when we compute it at the upper bound of the integral.

Taken together,  $L(r) > 0$  and  $L(p^m) < 0$  imply that a candidate equilibrium price  $p^* \in [r, p^m]$  exists. We finally note that

$$\begin{aligned} \frac{dL(p)}{dp} &= g(\tilde{c}(p)) \frac{d\tilde{c}(p)}{dp} - (p - r) \frac{f(p)}{1 - F(p)} g(\tilde{c}(p)) \frac{d\tilde{c}(p)}{dp} - \int_{\underline{c}}^{\tilde{c}(p)} \frac{f(\hat{x}(c))}{1 - F(\hat{x}(c))} g(c) dc \\ &= \frac{1 - F(p) - (p - r)f(p)}{1 - F(p)} g(\tilde{c}(p)) \frac{d\tilde{c}(p)}{dp} - \int_{\underline{c}}^{\tilde{c}(p)} \frac{f(\hat{x}(c))}{1 - F(\hat{x}(c))} g(c) dc \end{aligned}$$

is negative for any  $p \in [r, p^m]$ , which implies that the candidate equilibrium price is unique. This follows from the fact that  $1 - F(p) - (p - r)f(p) \geq 0$  (because it is the first order derivative of

the monopoly payoff  $(p - r)(1 - F(p))$ , which is log-concave) and  $d\tilde{c}(p)/dp < 0$  (because, from equation (3),  $\tilde{c}$  is decreasing in  $p$ ). In particular, at the candidate equilibrium price  $p^*$  we must have  $dL(p^*)/dp < 0$ .

Now that we have shown that our pricing game has a unique candidate pure-strategy SNE price, our first result in the article provides a sufficient existence condition.

**Theorem 1** *Let  $\hat{x}^{-1}(t)$  denote the inverse function of  $\hat{x}(c)$ . When the function  $g(\hat{x}^{-1}(t))$  is log-concave in  $t$ , then there exists a unique pure-strategy SNE in the model of sequential search for differentiated products with heterogeneous search costs.*

(A) *When Condition 1 holds so that  $\bar{c} \leq c''$ , all consumers conduct at least a first search and firms charge a price given by expression (6) in which  $c_0(p^*)$  is equal to  $\bar{c}$  and  $\hat{x}(c)$  solves equation (1).*

(B) *When Condition 1 does not hold so that  $\bar{c} > c''$ , only a fraction of consumers*

$$G \left( \int_{p^*}^{\bar{c}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon \right)$$

*conducts at least a first search whereas the rest of the consumers do not search at all and leave the market; firms charge a price given by expression (6) in which  $c_0(p^*)$  is equal to  $\tilde{c}(p^*)$  given by equation (3).*

The proof, whose details are in the Appendix, is as follows. Because a direct verification of the second order conditions does not deliver clear-cut results, we proceed by showing that the demand function of an individual firm is a log-concave function of its own price under the assumption that  $g(\hat{x}^{-1}(t))$  is log-concave in  $t$ . Once this is proven, we know that the firm profit function (4) is quasi-concave in its own price so that the unique candidate equilibrium price given by expression (6) is indeed a symmetric pure-strategy SNE. In order to prove that the demand function of a firm is log-concave in the firm's own price, we first show that the demand from a single consumer is log-concave both in price and in consumer reservation values; after this we make use of Theorem 6 in Prékopa (1973), which shows that integration over consumer reservation values preserves log-concavity.

The assumption that  $g(\hat{x}^{-1}(t))$  is log-concave is needed to apply Prékopa's aggregation theorem. It serves to "log-concavify" the demand of an individual consumer in  $p_i$  and in  $t \equiv \hat{x}(c)$ . This condition is satisfied when search costs follow a uniform distribution, no matter which distribution the match values follow. Moreover, when the match values are uniformly distributed, this condition holds for any log-concave search cost density that is decreasing. Examples are the Pareto and exponential distributions as well as the Weibull and gamma distributions for some parameters (see

also the density we use in Example 1 below). Note however that  $g$  does not necessarily have to be decreasing (see the density we use in Example 2 below). Intuitively, the condition means that the expected number of visitors with search cost  $c$  does not decline very rapidly in  $c$ .

## 4 The comparative statics effects of lower search costs

We now proceed to study the impact of lowering consumer search costs on the unique equilibrium price given in Theorem 1. In order to address this question, we parametrize the distribution of search costs by a positive parameter  $\beta$  that shifts the distribution. Specifically, let  $G(c; \beta)$  denote the parametrized CDF of search costs and let  $g(c; \beta)$  denote the corresponding PDF defined over the support  $[\underline{c}(\beta), \bar{c}(\beta)]$ . We assume that  $G$  and  $g$  are differentiable in  $\beta$ .<sup>11</sup> We study FOSD and SOSD changes in the search costs distribution.

**Definition 1.** *Search costs decrease in the sense of FOSD whenever a decrease in  $\beta$  implies an increase in the search cost distribution for all  $c$ . That is, when for all  $\beta > \beta'$ :*

$$G(c; \beta) \leq G(c; \beta')$$

for all  $c$  (with strict inequality for some  $c$ ). This implies that  $\partial G(c; \beta) / \partial \beta \leq 0$  for all  $c$  (and  $< 0$  for some  $c$ ).

**Definition 2.** *Search costs decrease in the sense of SOSD whenever for all  $\beta > \beta'$ :*

$$\int_{\underline{c}(\beta)}^c G(x; \beta) dx \leq \int_{\underline{c}(\beta')}^c G(x; \beta') dx \text{ for all } c \text{ (and } < \text{ for some } c).$$

This implies that  $\int_{\underline{c}(\beta)}^c \partial G(x; \beta) / \partial \beta dx \leq 0$  for all  $c$  (and  $< 0$  for some  $c$ ).

Note that SOSD is implied by FOSD (but not vice versa). Therefore, in what follows, when we study SOSD changes in the search cost distribution we mean changes that do not satisfy FOSD. A special case of an SOSD change in the distributions is a mean-preserving spread. This occurs when the mean search cost under  $\beta'$  is the same as the mean search cost under  $\beta$ .

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<sup>11</sup>To be as general as possible we allow the support of the density of search costs to depend on  $\beta$ . As will become clear later on, some of our results are completely independent of how the support varies with  $\beta$ ; other results do depend on it.

## Low search costs

Consider first the case in which Condition 1 holds and correspondingly all consumers search in equilibrium (cf. Theorem 1A). Under these circumstances, a change in the search cost distribution only affects the intensive search margin. The equilibrium price is equal to

$$p^*(\beta) = r + \frac{1}{\int_{\underline{c}(\beta)}^{\bar{c}(\beta)} \frac{f(\hat{x}(c))}{1-F(\hat{x}(c))} g(c; \beta) dc}. \quad (7)$$

Inspection of this expression immediately reveals that a decrease in  $\beta$  only affects the equilibrium price via the integral in the denominator of the fraction on the RHS of equation (7). This integral is the expectation of the hazard rate  $f(\hat{x}(c))/(1-F(\hat{x}(c)))$ . By log-concavity of  $1-F$ , this hazard rate increases in  $\hat{x}(c)$ ; hence, it decreases in  $c$ . From Theorem 1 of Hadar and Russell (1971) we know that a FOSD decrease in search costs implies that the expected value of the hazard rate goes up. It follows, then, that the equilibrium price (7) will *unambiguously* decrease.

The intuition is as follows. With a FOSD decrease in search costs, the frequency of low search cost consumers increases and the frequency of high search cost consumers decreases. Because the hazard rate decreases, it puts higher values on low search cost consumers than on high search cost consumers. This makes demand in (5) more elastic because the average consumer searches more and correspondingly the equilibrium price goes down.

To obtain the sign of the effect of SOSD changes in the search cost density we invoke the following condition on the match value density:

**Condition 2.** (*Concavity of the hazard rate in  $c$* )  $f'' [1-F]^2 + 4ff' [1-F] + 3f^3 \geq 0$ .

Under this condition the hazard rate  $f(\hat{x}(c))/(1-F(\hat{x}(c)))$  is decreasing and concave in  $c$  and by virtue of Theorem 2 of Hadar and Russell (1971), a SOSD decrease of the costs of search will lower the equilibrium price. The intuition for why Condition 2 is necessary is as follows. With a SOSD decrease in search costs, the frequency of low search cost consumers increases but the frequency of high search cost consumers rises too. As mentioned above, even though the hazard rate puts higher weight on low search cost consumers than on high search cost consumers, this is not sufficient. Concavity of the hazard rate ensures that a disproportionately higher weight is put on low search cost consumers as compared to high search cost consumers, which ensures that demand in (5) becomes more elastic and correspondingly the equilibrium price goes down. Summarizing:

**Proposition 1** *Let  $G(c; \beta)$  be a parametrized search cost CDF with positive density on  $[\underline{c}(\beta), \bar{c}(\beta)]$ . Assume that Condition 1 holds so that  $\bar{c}(\beta) \leq c''$ . Then:*

(A) *An FOSD decrease (increase) in search costs results in a fall (rise) of the equilibrium price given by Theorem 1A.*

(B) *An SOSD decrease (increase) in search costs results in a fall (rise) of the equilibrium price given by Theorem 1A provided that the density of match values satisfies Condition 2.*

The proof of Proposition 1B can be found in the Appendix. Condition 2 is satisfied by all increasing and convex densities, which includes the uniform distribution, as well as the beta distribution for some parameters. It is not necessary that the density is increasing; for instance, it also holds for the exponential density. Notice that the mentioned densities are also log-concave. When Condition 2 does not hold, we cannot obtain the sign of the effect of SOSD shifts of the search cost distribution.

## High search costs

We now move to the situation in which not all consumers choose to search in equilibrium (cf. Theorem 1B). In this case a change in the distribution of search costs affects both the intensive and the extensive search margins. The equilibrium price is given by the solution to

$$L(p; \beta) \equiv G(\tilde{c}(p); \beta) - (p - r) \int_{\underline{c}(\beta)}^{\tilde{c}(p)} \frac{f(\hat{x}(c))}{1 - F(\hat{x}(c))} g(c, \beta) dc = 0.$$

It is useful to divide this expression by  $G(\tilde{c}(p); \beta)$ , so that we get:

$$\tilde{L}(p; \beta) \equiv 1 - (p - r) \int_{\underline{c}(\beta)}^{\tilde{c}(p)} \frac{f(\hat{x}(c))}{1 - F(\hat{x}(c))} \tilde{g}(c, \beta) dc = 0, \quad (8)$$

where

$$\tilde{g}(c, \beta; \tilde{c}(p)) \equiv \frac{g(c, \beta)}{G(\tilde{c}(p); \beta)}$$

denotes the search cost density truncated from above at  $\tilde{c}(p)$ .

By the implicit function theorem, the impact of a (small) decrease in  $\beta$  on the equilibrium price  $p^*(\beta)$  that solves equation (8) is given by

$$\frac{dp^*(\beta)}{d\beta} = - \frac{\frac{\partial \tilde{L}(p^*; \beta)}{\partial \beta}}{\frac{\partial \tilde{L}(p^*; \beta)}{\partial p}}. \quad (9)$$

The denominator of equation (9) is negative. This follows from the fact that the sign of

$$\frac{\partial \tilde{L}}{\partial p} = \frac{\frac{\partial L}{\partial p} G(\tilde{c}(p)) - L(p) g(\tilde{c}(p)) \frac{\partial \tilde{c}}{\partial p}}{G(\tilde{c}(p))^2}$$

at the equilibrium price is equal to the sign of  $\partial L(p^*)/\partial p$ , which is negative because, as shown above,  $L(p)$  is decreasing in  $p$ . Therefore the sign of  $dp^*(\beta)/d\beta$  depends on the sign of  $\partial \tilde{L}/\partial \beta$  at the equilibrium price, which is equal to

$$\frac{\partial \tilde{L}(p^*; \beta)}{\partial \beta} = -(p^* - r) \frac{\partial}{\partial \beta} \left[ \int_{\underline{c}(\beta)}^{\tilde{c}(p^*)} \frac{f(\hat{x}(c))}{1 - F(\hat{x}(c))} \tilde{g}(c, \beta) dc \right]. \quad (10)$$

Inspection of equation (10) reveals that the change in the equilibrium price as a result of a change in search costs is inversely proportional to the change of the integral in squared brackets. This integral is similar to that in equation (8); the only difference is that we now have the expected value of the hazard rate of the consumers who choose to search. Therefore, answering the question what happens to the equilibrium price when search costs go down, boils down to finding out what happens to the aforementioned conditional expected value.

Denoting the distribution of search costs truncated from above at  $\tilde{c}(p^*)$  by

$$\tilde{G}(c, \beta; \tilde{c}(p^*)) \equiv \int_{\underline{c}(\beta)}^c \tilde{g}(x, \beta; \tilde{c}(p^*)) dx,$$

we can state directly (and without proof) the following extension of Proposition 1.

**Proposition 2** *Let  $G(c; \beta)$  be a parametrized search cost CDF with positive density on  $[\underline{c}(\beta), \bar{c}(\beta)]$ . Assume that Condition 1 does not hold so that some consumers choose not to search at all in equilibrium, i.e.  $\bar{c}(\beta) > c''$ . Then:*

(A) *If a FOSD decrease of the distribution of search costs results in a FOSD decrease (increase) of the distribution truncated from above at  $\tilde{c}(p^*)$ , then the equilibrium price given by Theorem 1B will fall (rise). (Similarly for a FOSD increase of the distribution of search costs.)*

(B) *If a SOSD decrease of the distribution of search costs results in a SOSD decrease (increase) of the distribution truncated from above at  $\tilde{c}(p^*)$ , and if the density of match values satisfies Condition 2 then the equilibrium price given by Theorem 1B will fall (rise). (Similarly for a SOSD increase of the distribution of search costs.)*

The intuition behind this result is as follows. A decrease in consumer search costs lowers the



search costs of consumers who used to search, but also brings new consumers into the market who at the margin did not find it worthwhile to search before. Consider first the consumers who used to search before the fall in search costs. As explained above within the context of Proposition 1, the average participating consumer becomes more picky and searches more, which implies that demand becomes more elastic. Because of this effect, firms have an incentive to lower their prices. Consider now the consumers who at the margin did not find it worthwhile to search before the fall in search costs. These consumers become new participants in the market after search costs decrease. These new consumers do have relatively large search costs because otherwise they would have been searching previously. Because of this effect, firms have an incentive to raise their prices. If the second effect is weak and the distribution of search costs conditional on search has lower search costs after the general search cost decrease, then the price will fall. If the second effect is sufficiently strong, then the price will rise. A SOSD decrease in search costs produces a similar change in the composition of demand and we obtain an analogous result provided that the density of match values satisfies Condition 2.

Proposition 2 clearly shows that what really matters for the relationship between the equilibrium price and  $\beta$  is the way in which the search cost distribution conditional on participation is affected by changes in  $\beta$ . The critical question is then:

Can we find conditions under which a (FOSD or SOSD) decrease in search costs results in a (FOSD or SOSD) decrease or increase in the truncated distribution of search costs?

In order to address this question, we now make use of the likelihood ratio ordering of densities.

**Definition 3.** *The density  $g(c; \beta)$  has the increasing likelihood ratio property (ILRP) if and only if for any  $\beta' < \beta$ ,*

$$g(c; \beta) g(d; \beta') \leq g(c; \beta') g(d; \beta)$$

*for any  $c \leq d$  in the union of the supports of  $g(c; \beta')$  and  $g(c; \beta)$ .*

The increasing likelihood ratio property is used much in the economics literature and is often simply referred to as monotone likelihood ratio property. It is well known that ILRP implies FOSD. What is perhaps less known is that ILRP induces a FOSD ranking of the truncated densities.

**Proposition 3** *Assume that  $g(c; \beta)$  satisfies ILRP. Then the distribution corresponding to  $\beta$  truncated from above at  $\tilde{c}(p^*)$  first-order stochastically dominates the distribution corresponding to  $\beta'$*

truncated from above at  $\tilde{c}(p^*)$ , that is,

$$\tilde{G}(c, \beta'; \tilde{c}(p^*)) \geq \tilde{G}(c, \beta; \tilde{c}(p^*))$$

for all  $\beta' < \beta$  and all  $c$  and  $\tilde{c}(p^*) \leq \min\{\bar{c}(\beta), \bar{c}(\beta')\}$ . As a result, a FOSD decrease in search costs results in a fall of the equilibrium price given in Theorem 1B.

The proof is in the Appendix. In light of Proposition 2, Proposition 3 implies that ILRP suffices to ensure that a fall in search costs results in a decrease of the equilibrium price even if new consumers enter the market. That is, with densities that satisfy ILRP a decrease in search costs will impact the intensive margin more strongly than the extensive margin, hence leading to a fall of the equilibrium price given in Theorem 1B.

The intuition behind the role played by the ILRP of the search cost density is the easiest seen by means of an example. Suppose that search costs are distributed on a fixed interval and that the density is decreasing. Moreover, suppose that the density satisfies the ILRP. Because for  $\beta' \leq \beta$  the relative frequency  $g(c; \beta)/g(c; \beta')$  increases in  $c$ , ILRP implies that  $g(c; \beta')$  must decrease more rapidly than  $g(c; \beta)$ . Because both are densities and their integrals must equal 1, it must be the case that  $g(c; \beta')$  is larger than  $g(c; \beta)$  for low search costs and smaller for high search costs. Hence, for any given truncation point, the average participating consumer under  $g(c; \beta')$  must have a lower search cost than the average participating consumer under  $g(c; \beta)$ . This implies that the elasticity of the demand in (5) under  $g(c; \beta')$  is higher than under  $g(c; \beta)$  and, correspondingly, firms reduce their prices. Consider the following example, which is analyzed in more detail in the Appendix.<sup>12</sup>

**Example 1: ILRP** Assume that match values are distributed on  $[0, 1]$  according to the uniform distribution and that search costs are distributed on  $[0, \beta]$  according to the density function

$$g(c) = \frac{\beta(2 + \delta) - 2c}{\beta^2(1 + \delta)}, \text{ with } \delta \geq 0.$$

The CDF of  $g$  is

$$G(c) = \frac{c(\beta(2 + \delta) - c)}{\beta^2(1 + \delta)}$$

and note that an increase in the parameters  $\beta$  or  $\delta$  signify a FOSD shift of the search cost CDF.

Observe that when  $\beta$  changes, the support changes but when  $\delta$  changes, the support remains the

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<sup>12</sup>A similar argument can be made for increasing densities defined over the same support. In that case,  $g(c; \beta)$  must increase more rapidly than  $g(c; \beta')$  which implies that the frequency of low search cost consumers under  $g(c; \beta)$  is lower than under  $g(c; \beta')$  and the opposite for the frequency of high search cost consumers.

same. Moreover, we observe that  $g$  has the ILRP with respect to parameters  $\beta$  and  $\delta$ . For this example, we can then state that an equilibrium exists and is unique and that an increase in either  $\beta$  or  $\delta$  increases the equilibrium price no matter whether all consumers search (cf. Theorem 1A) or not (cf. Theorem 1B).

The effect of lowering search costs when the extensive margin plays a role is illustrated in Figure 1. The gray curve in graph 1(a) is the search cost density for  $\delta = 1$ ; the black curve is the search cost density for  $\delta = 2$ . In this graph we set  $\beta = 1$ . The move from the black to the gray density represents a FOSD decrease of search costs, as can be seen in graph 1(c). The vertical dashed line indicates the threshold search cost above which consumers choose not to search the market for a satisfactory product. The corresponding truncated densities and distributions are given in graphs 1(b) and 1(d). What we see is that a FOSD decrease of search costs translates into a FOSD decrease of the search costs of the participating consumers.

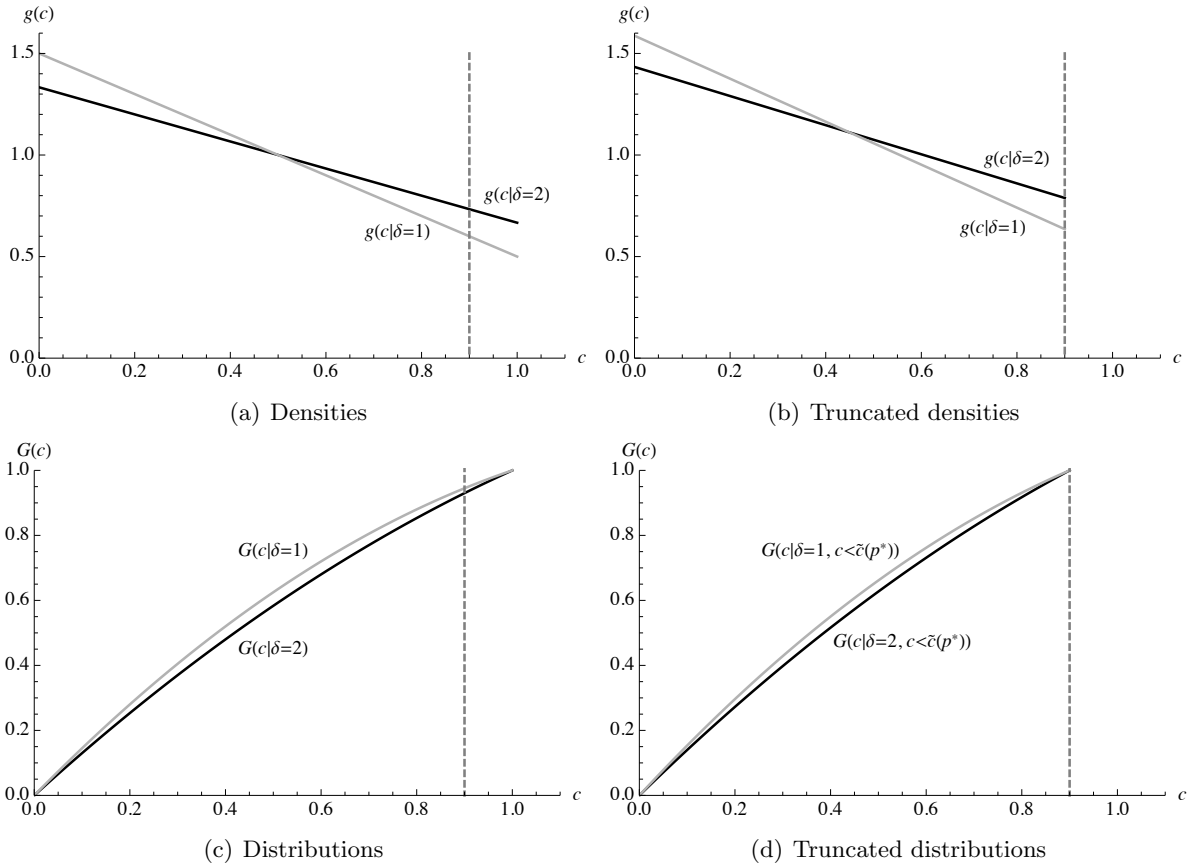


Figure 1: ILRP and the effect of lower search costs on (truncated) densities and distributions

We have seen that the increasing likelihood ratio property induces a FOSD ordering of the truncated densities, which leads to unambiguous results in terms of the direction of the comparative

statics effects of lower search costs. We now show that when the density of search costs satisfies a *decreasing* likelihood ratio property, then we obtain results in the opposite direction.

**Definition 4.** *The density  $g(c; \beta)$  has the decreasing likelihood ratio property (DLRP) if and only if for any  $\beta' < \beta$ ,*

$$g(c; \beta) g(d; \beta') \geq g(c; \beta') g(d; \beta)$$

*for any  $c \leq d$  in  $[\min\{\underline{c}(\beta), \underline{c}(\beta')\}, \min\{\bar{c}(\beta), \bar{c}(\beta')\}]$ .*

Note that compared to ILRP, we do not define DLRP for the union of the supports. This is not accidental: what happens above the truncation point is not relevant for our results and requiring DLRP to hold more generally excludes the interesting search cost shifts discussed below. As a matter of fact, if we define DLRP for the union of the supports, then we exclude FOSD shifts of the search cost distribution altogether, which are central to our article. Defining DLRP as in Definition 4 makes DLRP compatible with FOSD shifts of the search cost distribution provided that the lower bound of the two densities remains the same and the upper bound goes down. Such shifts can be considered central because they include the natural case of a percentage-reduction in consumers' search costs for search costs defined on an interval  $[0, \bar{c}]$ . What is interesting is that a FOSD shift of densities that satisfy DLRP defined according to Definition 4 induces a reverse FOSD ranking of the truncated densities.

**Proposition 4** *Assume that  $g(c; \beta)$  satisfies DLRP. Then the distribution corresponding to  $\beta'$  truncated from above at  $\tilde{c}(p^*)$  first-order stochastically dominates the distribution corresponding to  $\beta$  truncated from above at  $\tilde{c}(p^*)$ , that is,*

$$\tilde{G}(c, \beta; \tilde{c}(p^*)) \geq \tilde{G}(c, \beta'; \tilde{c}(p^*))$$

*for all  $\beta' < \beta$  and all  $c$  and  $\tilde{c}(p^*) \geq \min\{\bar{c}(\beta), \bar{c}(\beta')\}$ . As a result, a FOSD decrease in search costs results in a rise of the equilibrium price given in Theorem 1B. (And similarly for a SOSD decrease in search costs provided that Condition 2 holds.)*

The decreasing likelihood ratio property, as far as we know, has not been used in economic applications.<sup>13</sup> With densities that satisfy the DLRP, the ratio  $g(c; \beta')/g(c; \beta)$  is decreasing in  $c$ . Suppose for example that the densities are increasing. If a fall from  $\beta$  to  $\beta'$  signifies a FOSD

<sup>13</sup>The increasing (decreasing) likelihood ratio property is equivalent to  $g'_\beta/g$  increasing (decreasing) in  $c$  (see Milgrom, 1981). Note that the ILRP (DLRP) is equivalent to log-supermodularity (log-submodularity) (cf. Athey, 2002).

decrease in search costs then DLRP implies that the density under  $\beta$  must increase more rapidly than the density under  $\beta'$ . It then follows straightforwardly that the truncated distributions are ranked according to the reverse FOSD ranking.<sup>14</sup> The following example illustrates the case of densities with the DLRP.

**Example 2: DLRP** *Assume that match values are distributed on  $[0, 1]$  according to the uniform distribution and that the search cost density is*

$$g(c) = \frac{1 + \delta}{\beta(2 + \delta)} \left[ 1 + \left( \frac{c}{\beta} \right)^\delta \right], \text{ with } 0 < \delta \leq 1/2.$$

*The CDF of  $g$  is*

$$G(c) = \frac{c}{\beta(2 + \delta)} \left[ 1 + \delta + \left( \frac{c}{\beta} \right)^\delta \right]$$

*and note that an increase in parameter  $\beta$  implies a FOSD shift of the search cost CDF. Moreover,  $g$  has the DLRP with respect to parameter  $\beta$ . (For DLRP to be consistent with FOSD, as mentioned in Theorem 2, the support has to vary when we move  $\beta$ , which is the case here.) For this example we can then state that an equilibrium exists and is unique and that a decrease in search costs decreases the equilibrium price given by Theorem 1A but increases the equilibrium price given by Proposition 1B.*

Example 2 is illustrated in Figure 2. The black curves represent the densities and distributions, as well as the truncated counterparts when  $\beta = 1.5$ . The gray curves show the case of  $\beta = 1$ . The parameter  $\delta = 1/2$  for both cases. The move from the black curves to the gray ones represents a FOSD fall in search costs, as can be seen in graph 2(c). It can be seen that a decrease in search cost results in a pool of consumers participating that have higher search costs than before (graphs 2(b) and 2(d)). As a result, prices will increase as search costs fall.

## 5 Concluding remarks

Most existing consumer search models are analyzed under the assumption that all consumers search at least once in equilibrium. In this article we have argued that it is hard to reconcile this type of “fully-covered-market” assumption with the general idea that distinct consumers have different

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<sup>14</sup>We note that the likelihood ordering of distributions (ILRP or DLRP) is not necessary for the results we have obtained. The analysis of the homogeneous product model with non-sequential search reveals that we can obtain similar results using the *reversed hazard rate* ordering of distributions (see Shaked and Shanthikumar, 2007). This ordering is implied by the likelihood ratio ordering and is therefore weaker but it is less used and less intuitive from an economics point of view.

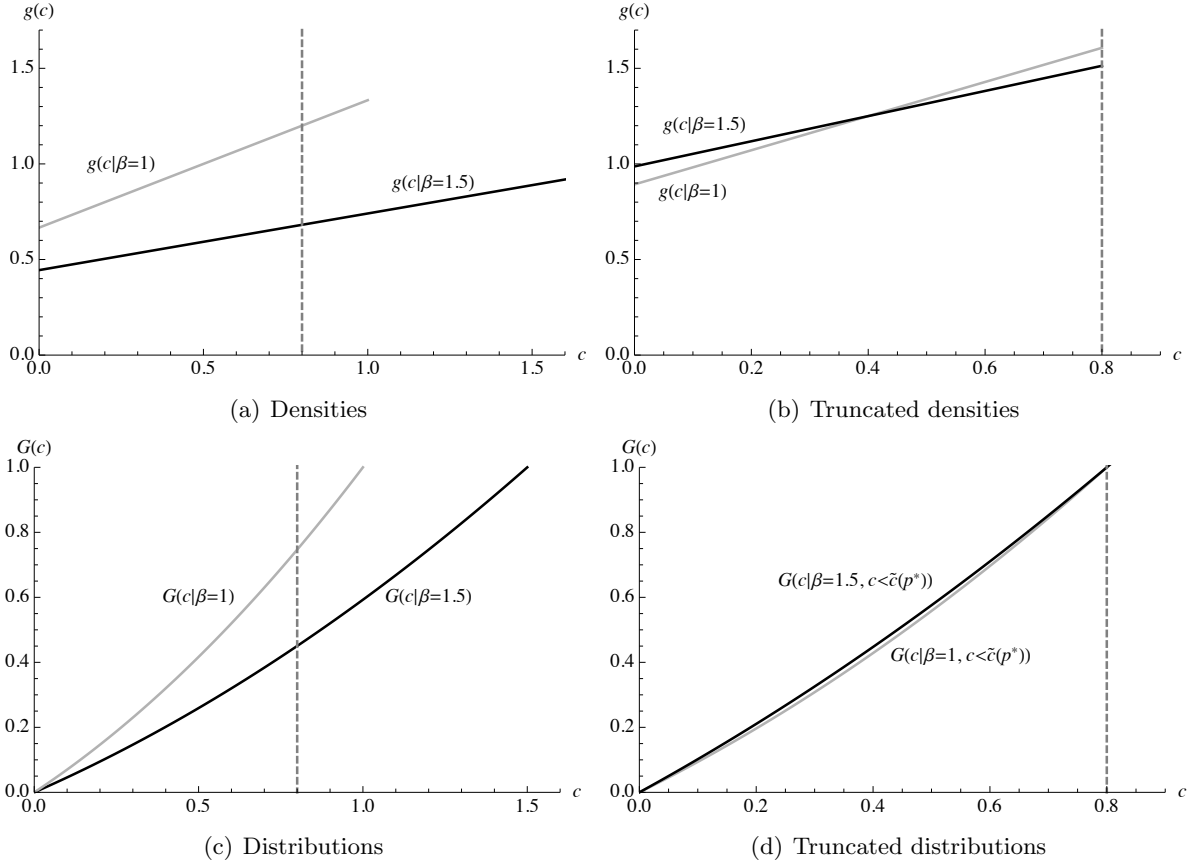


Figure 2: DLRP and the effect of higher search costs on (truncated) densities and distributions

search costs and that in many markets, some consumers may simply find it not worthwhile to start searching for a satisfactory product. As the decision to search depends on the equilibrium price, the existing literature has neglected an important role of the price mechanism, namely, that prices ought to affect the share of consumers who choose to search for a product in the first place.

We have studied how prices are determined in a model of search for differentiated products while allowing for arbitrary search cost heterogeneity. Our first contribution has been to provide results on the existence and uniqueness of pure-strategy symmetric Nash equilibrium. We have shown that when the distribution of match values has increasing hazard rate and the density of search costs is sufficiently log-concave, a pure-strategy symmetric Nash equilibrium exists and is unique. In proving this result, we have exploited a mathematical result due to Prékopa (1973) which shows that log-concavity is preserved by integration.

We have also revisited the question how a decrease in search costs affects the level of prices. A decrease in search costs affects not only *how much* consumers search but also *who* chooses to search. Recognizing that the price mechanism affects the search composition of demand turns out

to be critical for understanding how prices are affected by lower search costs. When search costs decrease there are two effects operating in opposite directions: on the one hand, consumers who used to search increase their search intensity and this puts pressure on firms to cut prices; on the other hand, some consumers who used to not search become active searchers and, because these consumers have relatively high search costs, this gives firms incentives to raise prices. We have referred to the first effect as the effect on the intensive search margin and to the second one as the effect on the extensive search margin. The net effect depends on whether the new average consumer has lower or higher search cost than before.

Our second contribution has been on identifying sufficient and intuitive conditions on search cost densities under which a decrease in search costs in the sense of FOSD or SOSD has an unambiguous effect on the equilibrium price. We have put forward a property of search cost densities that plays a decisive role, namely, log-supermodularity (or monotone increasing likelihood ratio property). When the search cost density is log-supermodular, a decrease in search costs has a relatively stronger impact on the intensive search margin than on the extensive search margin. As a result, the average consumer who chooses to search has a lower search cost and is correspondingly more elastic. This causes the equilibrium price to fall. By contrast, when the search cost density is log-submodular (or has the monotone decreasing likelihood ratio property), a decrease in search frictions raises the relative frequency of consumers with high search costs. Correspondingly, the average consumer who decides to search for an acceptable product becomes less elastic, which results in a price increase.

Our results are in line with Hortaçsu and Syverson's (2004) empirical observation that prices went up in the US mutual fund industry during the late nineties in spite of the decrease in search costs that they estimate. Admittedly, consumers who entered the market for the first time were less savvy and knowledgeable, which could also have been part of the explanation for why prices went up. In future work, we plan to study the role of valuation heterogeneity. Interestingly, higher valuations increase willingness to pay but at the same time also increase the gains from search. We expect that similar interesting results as the ones established in this article can be obtained when dealing with valuation heterogeneity instead of search cost heterogeneity.

In some situations consumers differ in their fixed costs of search, but are relatively similar in their marginal costs of search. Whereas changes in the distribution of fixed search costs do affect consumer participation, these changes do not affect prices. This is because the mechanism by which prices are affected by changes in the extensive search margin works via the changes in the gains from search of the average consumer who chooses to search. Changes in the size of the market that do not affect the gains from search of the average consumer (as happens when fixed search costs

vary) do not have a bearing on the elasticity of demand.

We conclude our article by pointing out an interesting and useful property of models with search cost heterogeneity. As mentioned in the Introduction, in recent years there has been a lively interest in the effects of ordered search on pricing, profits and welfare (for a recent survey, see Armstrong, 2016). One difficulty with the standard search model is that, in addition to the symmetric equilibrium, there may be multiple ordered-search equilibria. This is because the order in which consumers search for satisfactory deals depends on their price beliefs and the prices firms charge depend on the position in which they are visited. When consumers are sufficiently heterogeneous in their search costs, equilibria with ordered search are likely not to exist. The reason is that consumer beliefs about the prices firms charge will no longer completely guide their search. This, in turn, is likely to destabilize the ordered search equilibria. A similar insight emerges if consumers have ex-ante heterogeneous valuations (as in Haan, Moraga-González and Petrikaitė, 2015).



## APPENDIX

**Derivation of consumer surplus.** As mentioned in the main text, a consumer with search cost  $c$  will stop and buy after the first search when  $\varepsilon > \hat{x}(c)$ ; otherwise she will drop the first option and continue searching, in which case she will encounter herself exactly in the same situation as before because, conditional on participating, the consumer will continue searching until she finds a match value for which it is worth to stop searching. Denoting by  $CS(c)$  her consumer surplus, recursively, we must have:

$$CS(c) = -c + (1 - F(\hat{x}(c))) \frac{\int_{\hat{x}(c)}^{\bar{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon}{1 - F(\hat{x}(c))} + F(\hat{x}(c)) CS(c).$$

Solving for  $CS(c)$  gives

$$CS(c) = \frac{\int_{\hat{x}(c)}^{\bar{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon - c}{1 - F(\hat{x}(c))}.$$

Using the value of  $c$  from equation (1) we obtain

$$CS(c) = \frac{\int_{\hat{x}(c)}^{\bar{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon - \int_{\hat{x}(c)}^{\bar{\varepsilon}} (\varepsilon - \hat{x}(c)) f(\varepsilon) d\varepsilon}{1 - F(\hat{x}(c))} = \frac{\int_{\hat{x}(c)}^{\bar{\varepsilon}} (\hat{x}(c) - p^*) f(\varepsilon) d\varepsilon}{1 - F(\hat{x}(c))} = \hat{x}(c) - p^*,$$

which is the expression we give in the text. ■

**Proof of Theorem 1.** It remains to prove that the equilibrium exists when  $g(\hat{x}^{-1}(t))$  is log-concave in  $t$ . For this, we prove that the demand function of a firm  $i$  in equation (5) is log-concave in its own price  $p_i$ . Once this is proven, it follows that the firm profit function (4) is quasi-concave in its own price so that the unique candidate equilibrium price given by equation (6) is indeed an equilibrium.

We start by rewriting the demand function in a more convenient way. Let us use the following change of variables:  $t = \hat{x}(c)$  so that

$$dt = \hat{x}'(c) dc = -\frac{dc}{1 - F(\hat{x}(c))}.$$

Noting that

$$\hat{x}(c_0(p^*)) = \hat{x}(\min\{\bar{c}, \tilde{c}(p^*)\}) = \max\{\hat{x}(\bar{c}), p^*\}$$

and

$$\hat{x}(\max\{\underline{c}, \tilde{c}(p_i)\}) = \min\{\hat{x}(\tilde{c}(p_i)), \hat{x}(\underline{c})\} = \min\{\bar{\varepsilon} - p_i + p^*, \hat{x}(\underline{c})\}$$

we have

$$D(p_i, p^*) = \begin{cases} \int_{\max\{\hat{x}(\bar{c}), p^*\}}^{\hat{x}(\underline{c})} (1 - F(t + p_i - p^*))g(\hat{x}^{-1}(t))dt & \text{if } p_i < p^*, \\ \int_{\max\{\hat{x}(\bar{c}), p^*\}}^{\min\{\bar{\varepsilon} - p_i + p^*, \hat{x}(\underline{c})\}} (1 - F(t + p_i - p^*))g(\hat{x}^{-1}(t))dt & \text{if } p_i \geq p^*. \end{cases} \quad (11)$$

We now show that the integrand function

$$\ell(p_i, t) \equiv (1 - F(t + p_i - p^*))g(\hat{x}^{-1}(t))$$

is jointly log-concave in  $p_i$  and  $t$  under the assumption that  $g(\hat{x}^{-1}(t))$  is log-concave in  $t$ . Because the product of log-concave functions is log-concave,  $\ell(p_i, t)$  is jointly log-concave in  $p_i$  and  $t$  if  $1 - F(t + p_i - p^*)$  is log-concave. Let  $m(p_i, c) \equiv \ln[1 - F(t + p_i - p^*)]$ . Taking derivatives we have:

$$\frac{\partial m}{\partial t} = \frac{\partial m}{\partial p_i} = -\frac{f(t + p_i - p^*)}{1 - F(t + p_i - p^*)}.$$

To construct the Hessian matrix, we now compute the necessary second order derivatives:

$$\frac{\partial^2 m}{\partial t^2} = \frac{\partial^2 m}{\partial p_i^2} = \frac{\partial^2 m}{\partial p_i \partial t} = -\frac{f'(t + p_i - p^*) [1 - F(t + p_i - p^*)] + f(t + p_i - p^*)^2}{[1 - F(t + p_i - p^*)]^2} \leq 0,$$

where the sign follows from the log-concavity of  $f$ . From these second derivatives it can readily be concluded that the Hessian matrix is negative semidefinite; this implies that  $m(p_i, c)$  is concave in  $p_i$  and  $t$ . By implication,  $\ell(p_i, t)$  is log-concave in  $p_i$  and  $t$  on the convex set

$$[r, p^*] \times [\max\{\hat{x}(\bar{c}), p^*\}, \hat{x}(\underline{c})] \cup \{(p_i, t) : p_i \in [p^*, p^m], t \in [\max\{\hat{x}(\bar{c}), p^*\}, \min\{\bar{\varepsilon} - p_i + p^*, \hat{x}(\underline{c})\}]\}$$

so  $\ell(p_i, t)$  is log-concave on  $[r, p^m] \times [p^*, \bar{\varepsilon}]$  because from Prékopa (1971) we know that if a function is log-concave on a convex set and equal to zero elsewhere then the function is log-concave in the entire space.

We now invoke the following result by Prékopa (1973), which shows that integration preserves log-concavity.

**Lemma 1 [Prékopa, 1973]** *Let  $f(x, y)$  be a function of  $n+m$  variables where  $x$  is an  $n$ -component and  $y$  is an  $m$ -component vector. Suppose that  $f$  is log-concave in  $R^{n+m}$  and let  $A$  be a convex*

subset of  $R^m$ . Then the function of the variable  $x$ :

$$\int_A f(x, y) dy$$

is log-concave in the entire space  $R^n$ .

Because  $\ell(p_i, t)$  is log-concave in  $p_i$  and  $t$  and we are integrating for all  $t$  in the convex set  $[p^*, \bar{e}]$ , the theorem implies that the demand function (11) is log-concave in  $p_i$ . We then conclude that an equilibrium exists and is unique. ■

**Proof of Proposition 1.** Consider the function

$$h(c) = -\frac{f(\hat{x}(c))}{1 - F(\hat{x}(c))}.$$

We know that

$$h'(c) = \frac{f'(\hat{x}(c)) [1 - F(\hat{x}(c))] + f^2(\hat{x}(c))}{[1 - F(\hat{x}(c))]^3} \geq 0$$

by the log-concavity of  $1 - F$ . Given this, result (A) follows directly from Theorem 1 in Hadar and Russell (1971).

Taking the derivative again we have:

$$\begin{aligned} h''(c) &= \frac{\{f''(\hat{x}(c)) [1 - F(\hat{x}(c))] + f(\hat{x}(c))f'(\hat{x}(c))\} \hat{x}'(c) [1 - F(\hat{x}(c))]^3}{[1 - F(\hat{x}(c))]^6} \\ &\quad + \frac{\{f'(\hat{x}(c)) [1 - F(\hat{x}(c))] + f^2(\hat{x}(c))\} 3 [1 - F(\hat{x}(c))]^2 f(\hat{x}(c)) \hat{x}'(c)}{[1 - F(\hat{x}(c))]^6} \\ &= -\frac{f''(\hat{x}(c)) [1 - F(\hat{x}(c))]^2 + 4f(\hat{x}(c))f'(\hat{x}(c)) [1 - F(\hat{x}(c))] + 3f^3(\hat{x}(c))}{[1 - F(\hat{x}(c))]^5}. \end{aligned}$$

Under the assumption  $f'' [1 - F]^2 + 4ff' [1 - F] + 3f^3 \geq 0$  we know that  $h(c)$  is an increasing and concave function. Given this, result (B) follows directly from Theorem 2 in Hadar and Russell (1971). ■

**Proof of Proposition 3.** For simplicity of notation, let us drop the truncation point  $\tilde{c}(p^*)$  from the arguments of the truncated distribution  $\tilde{G}$  and density  $\tilde{g}$ . We use the following Lemma stating that likelihood ratio orderings are preserved under general truncations of the distributions of the involved random variables.

**Lemma 2** *Let  $h$  and  $h'$  be two densities with the increasing likelihood ratio property, i.e.  $h(c)h'(d) \leq$*

$h(d)h'(c)$  for all  $c \leq d$  in the union of the supports of  $h$  and  $h'$ . Then, the densities truncated from above at any point  $t$  also satisfy the increasing likelihood ratio property.

For a proof of this Lemma see Theorem 1.C.6 in Shaked and Shanthikumar (2007).

When  $g(c; \beta)$  has the ILRP, by Lemma 2 we know that the truncated densities  $\tilde{g}(c; \beta')$  and  $\tilde{g}(c; \beta)$  also satisfy the ILRP. As ILRP implies FOSD, it follows that  $\tilde{G}(c; \beta)$  dominates  $\tilde{G}(c; \beta')$  in the sense of FOSD. ■

**Proof of Proposition 4.** Take an arbitrary number  $x$  such that  $c \leq x \leq d \leq \min\{\bar{c}(\beta), \bar{c}(\beta')\}$  and integrate the inequality

$$g(c; \beta)g(d; \beta') \geq g(c; \beta')g(d; \beta)$$

on the interval  $[\underline{c}, x]$ , where for simplicity we put  $\underline{c} \equiv \min\{\underline{c}(\beta), \underline{c}(\beta')\}$ . We obtain that for any  $x \leq d$

$$\int_{\underline{c}}^x g(c; \beta)g(d; \beta')dc \geq \int_{\underline{c}}^x g(c; \beta')g(d; \beta)dc.$$

This is equivalent to

$$G(x; \beta)g(d; \beta') \geq G(x; \beta')g(d; \beta) \text{ for any } x \leq d.$$

Integrating this for  $d \in [x, \tilde{c}(p^*)]$ , we get

$$G(x; \beta) [G(\tilde{c}(p^*); \beta') - G(x; \beta')] \geq G(x; \beta') [G(\tilde{c}(p^*); \beta) - G(x; \beta)],$$

which is equivalent to

$$G(x; \beta)G(\tilde{c}(p^*); \beta') \geq G(x; \beta')G(\tilde{c}(p^*); \beta),$$

which implies that  $\tilde{G}(c; \beta')$  dominates  $\tilde{G}(c; \beta)$  in the sense of FOSD, as required. ■

**Example 1.** We first need to prove that an equilibrium exists and is unique for the search cost density

$$g(c) = \frac{\beta(2 + \delta) - 2c}{\beta^2(1 + \delta)}, \text{ with } \delta \geq 0.$$

Because match values are uniformly distributed on  $[0, 1]$ , for existence and uniqueness of symmetric equilibrium we only need to check that the function  $g(\hat{x}^{-1}(t))$  is log-concave in  $t$ .

Solving the consumers equilibrium condition (1) gives  $\hat{x}(c) = 1 - \sqrt{2c}$ ; therefore  $\hat{x}^{-1}(t) = (1 - t)^2/2$ . Plugging this into the density function gives

$$g(\hat{x}^{-1}(t)) = \frac{(1 - t)^2 - \beta(2 + \delta)}{\beta^2(1 + \delta)}.$$

Inspection of this function immediately reveals that it is strictly concave in  $t$ ; therefore it is log-concave in  $t$ . We conclude that an equilibrium exists and is unique.

We now show that  $g$  satisfies the ILRP (and by implication FOSD) with respect to parameters  $\beta$  and  $\delta$ . From Milgrom (1981),  $g$  has the ILRP with respect to parameter  $\beta$  if and only if the ratio  $g'_\beta/g$  is increasing in  $c$ , where  $g'_\beta$  denotes the derivative of  $g$  with respect to  $\beta$ . We have

$$\frac{g'_\beta}{g} = \frac{4c - \beta(2 + \delta)}{\beta(\beta(2 + \delta) - 2c)},$$

and

$$\frac{d(g'_\beta/g)}{dc} = \frac{2(2 + \delta)}{(\beta(2 + \delta) - 2c)^2} > 0.$$

For the case of parameter  $\delta$  we have

$$\frac{g'_\delta}{g} = \frac{\beta}{\beta(2 + \delta) - 2c} - \frac{1}{1 + \delta},$$

and

$$\frac{d(g'_\delta/g)}{dc} = \frac{2\beta}{(\beta(2 + \delta) - 2c)^2} > 0.$$

The details of the example are now complete. ■

**Example 2.** Consider now the case when search costs are distributed on  $[0, \beta]$  according to the density

$$g(c; \beta) = \frac{1 + \delta}{\beta(2 + \delta)} \left[ 1 + \left( \frac{c}{\beta} \right)^\delta \right], \text{ with } 0 < \delta \leq 1/2,$$

and again assume match values are distributed uniformly on  $[0, 1]$ .

For this density function we have

$$g(\hat{x}^{-1}(t)) = \frac{(1 + \delta) \left( 2^\delta + \left( \frac{(1-t)^2}{\beta} \right)^\delta \right)}{2^\delta \beta (2 + \delta)}.$$

Taking the second derivative with respect to  $t$  gives

$$\frac{d^2 g(\hat{x}^{-1}(t))}{dt^2} = \frac{2^{1-\delta} \delta (1 - \delta - 2\delta^2) (1 - t)^{2\delta-2}}{\beta^{1+\delta} (2 + \delta)}$$

which is clearly negative for  $0 < \delta \leq 1/2$ . Therefore  $g(\hat{x}^{-1}(t))$  is concave in  $t$  and by implication it

is log-concave in  $t$ . We conclude that an equilibrium exists and is unique.

We now observe that  $g$  has the DLRP with respect to parameter  $\beta$ . To see this, we note that

$$\frac{g'_\beta}{g} = \frac{-\frac{1+\delta+(1+\delta)^2\left(\frac{c}{\beta}\right)^\delta}{\beta^2(2+\delta)}}{\frac{1+\delta}{\beta(2+\delta)}\left[1+\left(\frac{c}{\beta}\right)^\delta\right]} = -\frac{1+(1+\delta)\left(\frac{c}{\beta}\right)^\delta}{\beta\left[1+\left(\frac{c}{\beta}\right)^\delta\right]}$$

Taking the derivative with respect to  $c$  gives

$$\frac{\partial(g'_\beta/g)}{\partial c} = -\frac{\delta^2\left(\frac{c}{\beta}\right)^{\delta-1}}{\beta^2\left[1+\left(\frac{c}{\beta}\right)^\delta\right]^2} < 0.$$

Finally, we note an increase in  $\beta$  shifts the search cost distribution to the right, so higher  $\beta$  implies a FOSD shift of the search cost distribution. This is seen directly from the following derivative:

$$\frac{dG}{d\beta} = -\frac{c(1+\delta)\left(1+\left(\frac{c}{\beta}\right)^\delta\right)}{\beta^2(2+\delta)} < 0.$$

The details of the example are now complete. ■

## References

- Anderson, S. and Renault, R. “Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model.” *RAND Journal of Economics*, Vol. 30 (1999), pp. 719–735.
- Anderson, S. and Renault, R. “Advertising Content.” *American Economic Review*, Vol. 96 (2006), pp. 93–113.
- Armstrong, M., Vickers, J., and Zhou, J. “Prominence and Consumer Search.” *RAND Journal of Economics*, Vol. 40 (2009), pp. 209–233.
- Armstrong, M. and Zhou, J. “Paying for Prominence.” *Economic Journal*, Vol. 121 (2011), pp. F368–F395.
- Armstrong, M. “Ordered Consumer Search.” Unpublished Manuscript, Department of Economics, University of Oxford, 2016.
- Athey, S. “Monotone Comparative Statics under Uncertainty.” *Quarterly Journal of Economics*, Vol. 117 (2002), pp. 187–223.
- Bar-Isaac, H., Caruana, G., and Cuñat, V. “Search, Design, and Market Structure.” *American Economic Review*, Vol. 102 (2012), pp. 1140–1160.
- Bénabou, R. “Search Market Equilibrium, Bilateral Heterogeneity, and Repeat Purchases.” *Journal of Economic Theory*, Vol. 60 (1993), pp. 140–158.
- Brynjolfsson, E., Hu, Y., and Simester, D. “Goodbye Pareto Principle, Hello Long Tail: The Effect of Search Costs on the Concentration of Product Sales.” *Management Science*, Vol. 57 (2011), pp. 1373–1386.
- Brynjolfsson, E. and Smith, M. “Frictionless Commerce? A Comparison of Internet and Conventional Retailers.” *Management Science*, Vol. 46 (2000), pp. 563–585.
- Burdett, K. and Judd, K. “Equilibrium Price Dispersion.” *Econometrica*, Vol. 51 (1983), pp. 955–969.
- Caplin, A. and Nalebuff, B. “Aggregation and Imperfect Competition: On the Existence of Equilibrium.” *Econometrica*, Vol. 59 (1991), pp. 25–59.
- Chen, Y. and Zhang, T. “Equilibrium Price Dispersion with Heterogeneous Searchers.” *International Journal of Industrial Organization*, Vol. 29 (2011), pp. 645–654.

- Clay, K., Krishnan, R., and Wolff, E. “Prices and Price Dispersion on the Web: Evidence from the Online Book Industry.” *Journal of Industrial Economics*, Vol. 49 (2001), pp. 521–540.
- Clemons, E., Hann, I., and Hitt, L. “Price Dispersion and Differentiation in Online Travel: An Empirical Investigation.” *Management Science*, Vol. 48 (2002), pp. 534–549.
- Diamond, P. “A Model of Price Adjustment.” *Journal of Economic Theory*, Vol. 3 (1971), pp. 156–168.
- Donaldson, D. and Hornbeck, R. “Railroads and American Economic Growth: A ‘Market Access’ Approach.” *Quarterly Journal of Economics*, Vol. 131 (2016), pp. 799–858.
- Goolsbee, A. “Competition in the Computer Industry: Online Versus Retail.” *Journal of Industrial Economics*, Vol. 49 (2001), pp. 487–499.
- Haan, M. and Moraga-González, J. “Advertising for Attention in a Consumer Search Model.” *Economic Journal*, Vol. 121 (2011), pp. 552–579.
- Haan, M., Moraga-González, J., and Petrikaitė, V. “Price and Match-Value Advertising with Directed Consumer Search.” SOM Research Report no. 15012-EEF, Faculty of Economics and Business, University of Groningen, 2015.
- Hadar, J. and Russell, W. “Stochastic Dominance and Diversification.” *Journal of Economic Theory*, Vol. 3 (1971), pp. 288–305.
- Hortaçsu, A. and Syverson, C. “Product Differentiation, Search Costs, and Competition in the Mutual Fund Industry: A Case Study of S&P 500 Index Funds.” *Quarterly Journal of Economics*, Vol. 119 (2004), pp. 403–456.
- Janssen, M., Moraga-González, J., and Wildenbeest, M. “Truly Costly Sequential Search and Oligopolistic Pricing.” *International Journal of Industrial Organization*, Vol. 23 (2005), pp. 451–466.
- Janssen, M. and Shelegia, S. “Consumer Search and Double Marginalization.” *American Economic Review*, Vol. 105 (2015), pp. 1–29.
- Kohn, M. and Shavell, S. “The Theory of Search.” *Journal of Economic Theory*, Vol. 9 (1974), pp. 93–123.



- Lee, H. “Do Electronic Marketplaces Lower the Price of the Goods?” *Communications of the ACM*, Vol. 41 (1998), pp. 73–80.
- Moraga-González, J. and Petrikaitė, V. “Search Costs, Demand-Side Economies and the Incentives to Merge under Bertrand Competition.” *RAND Journal of Economics*, Vol. 44 (2013), pp. 391–424.
- Milgrom, P. “Good News and Bad News: Representation Theorems and Applications.” *Bell Journal of Economics*, Vol. 12 (1981), pp. 380–391.
- Prékopa, A. “Logarithmic Concave Measures with Application to Stochastic Programming.” *Acta Scientiarum Mathematicarum*, Vol. 32 (1971), pp. 301–316.
- Prékopa, A. “On Logarithmic Concave Measures and Functions.” *Acta Scientiarum Mathematicarum*, Vol. 34 (1973), pp. 335–343.
- Rauh, M. “Wage and Price Controls in the Equilibrium Sequential Search Model.” *European Economic Review*, Vol. 48 (2004), pp. 1287–1300.
- Rauh, M. “Strategic Complementarities and Search Market Equilibrium.” *Games and Economic Behavior*, Vol. 66 (2009), pp. 959–978.
- Rob, R. “Equilibrium Price Distributions.” *Review of Economic Studies*, Vol. 52 (1985), pp. 452–504.
- Scott Morton, F., Zettelmeyer, F., and Silva-Risso, J. “Internet Car Retailing.” *Journal of Industrial Economics*, Vol. 49 (2001), pp. 509–519.
- Shaked, M. and Shanthikumar, J. *Stochastic Orders*. New York: Springer-Verlag, 2007.
- Stahl, D. “Oligopolistic Pricing with Sequential Consumer Search.” *American Economic Review*, Vol. 79 (1989), pp. 700–712.
- Stahl, D. “Oligopolistic Pricing with Heterogeneous Consumer Search.” *International Journal of Industrial Organization*, Vol. 14 (1996), pp. 243–268.
- Stiglitz, J. “Equilibrium in Product Markets with Imperfect Information.” *American Economic Review*, Vol. 69 (1979), pp. 339–345.
- Tappata, M. “Rockets and Feathers: Understanding Asymmetric Pricing.” *RAND Journal of Economics*, Vol. 40 (2009), pp. 673–687.

Wolinsky, A. “True Monopolistic Competition as a Result of Imperfect Competition.” *Quarterly Journal of Economics*, Vol. 101 (1986), pp. 493–511.

Yang, H. “Targeted Search and the Long Tail Effect.” *RAND Journal of Economics*, Vol. 44 (2013), pp. 733–756.

Zhou, J. “Multiproduct Search and the Joint Search Effect.” *American Economic Review*, Vol. 104 (2014), pp. 2918–2939.