Properties of Profit Premium in an Equilibrium Framework*

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Abstract

This paper proposes a profit premium concept that, in addition to brand equities, also accounts for brand-specific features in the marginal costs of products. This is justified in markets where brand characteristics are pronounced since in such markets marginal costs are likely to show higher variation across brands. The paper shows that brands with a sufficiently large ratio of marginal cost and utility brand-specific intercepts may face a negative profit premium. We argue that this is an important property of our method that allows profit premium to signal when it is not profitable to invest in brand development. In order to establish the results we analyze the monotonicity of profit premium with respect to brand equity in a structural demand and supply model. We obtain monotonicity results analytically for the simple (non-random coefficient) logit demand model through comparative statics with respect to brand equity and by Monte Carlo simulations for the random coefficient logit model. An empirical study of the new car market from the Netherlands confirms our theoretical findings. Six out of the brands with the highest ratios of marginal cost and utility brand-specific intercepts have negative profit premiums. This result questions the generally believed positive relationship between brand equity and brand value.

Keywords: brand equity, brand value, Nash-Bertrand, random coefficient logit, empirical IO methods, car market

JEL codes: M31, D43

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1 Introduction

An important problem in marketing is to quantify the efficiency of marketing activities devoted to improve the image of brands. Researchers have focused on measuring, on the one hand, the effect of brands on consumers' preferences, and on the other hand, the value of a brand to its producer. Following Goldfarb et al. (2009), we use the terms *brand value* and *brand equity* distinctively to mean the performance of a brand from the perspective of its producer and the contribution of a brand to consumers' utilities, respectively. Because brands are capable of incorporating the positive effects of marketing activities, by measuring brand value one is able to quantify the effect of marketing activities.

Brand value is in general measured by the difference between a factual measure (like price or revenue) and a corresponding counterfactual. We follow the recent proposal by Goldfarb et al. (2009) and Ferjani et al. (2009) and define the counterfactual as the unbranded equilibrium measure, that is, as the measure computed in a new equilibrium when the product is deprived of its brand equity. We operationalize brand equity as a brand-specific intercept in the utility that is common to all consumers (Kamakura and Russell 1993, Sriram et al. 2007, Goldfarb et al. 2009, Borkovsky et al. 2017). This way the counterfactual depends on the search attributes of a specific product, which are just the product attributes available to consumers from the description of the product. Consequently, brand value is measured as the extra value to the producer that can be attributed to brand equity.

The literature has conceptualized brand value measurement through different measures. For example, Aaker (1991) proposed the price premium; Kamakura and Russell (1993) used the sales premium, which is based on market share as a quantity for computing the brand value; Ailawadi et al. (2003) proposed the revenue premium. An important discovery was that, employing the methodology from Berry et al. (1995), one can estimate marginal costs, which makes it possible to compute profit premium as the difference between the profit from the products belonging to a brand and the profit from the unbranded versions of the same products (Kartono and Rao 2006, Goldfarb et al. 2009, Borkovsky et al. 2017). This is arguably a potentially superior brand value measure since it contains relevant information regarding the financial performance of the brand.

Goldfarb et al. (2009) propose to compute profit premiums in the ready-to-eat cereal market by assuming that marginal cost does not depend on brand-specific parameters, that is, they compute unbranded marginal cost by assuming that the production technology is preserved. There are markets, however, where this assumption does not appear to be plausible. For example, in markets where brand characteristics are pronounced, marginal costs are likely to show higher variation across brands and, therefore, brand-specific intercepts in the marginal cost are expected to capture an important part of this variation.

There are several types of expenditure on brand equity that may affect marginal cost. One example is product development expenditure for improving experience attributes. In the special case of the car market studied in this paper, examples of product development expenditure can be buying a higher quality gearbox from suppliers or more comfortable seats. Variable costs in an average car production process exceed 60% of total costs (Rogozhin et al. 2010). This high proportion of the marginal cost in the price of a car influences the financial performance of the brand significantly, so one cannot ignore its role in studying the relationship between brand equity and profit premium.

In this paper we propose a modification of the profit premium concept of Goldfarb et al. (2009) that takes brand-specific features in marginal cost into account. In order to do so we consider a model of demand and supply along the lines of Berry et al. (1995). This model defines demand as a random coefficient logit model that is derived from utilities that depend on search attributes and brand equity. Products are also characterized by experience attributes, which can signal features beyond search attributes (Nelson 1970). We assume that the experience attributes of a given brand are captured by brand-specific intercepts, which correspond to brand equities in our methodology. The supply side of the model specifies prices as the outcome of a Nash equilibrium for profit maximizing firms. Similar to the demand side, we capture brand-specific features in marginal cost by including brand-specific intercepts (Kartono and Rao 2006). Our proposed profit premium defines the counterfactual marginal cost of a product by depriving it of its brand-specific intercept.

The main contribution of this paper is the finding that the profit premium we propose is *qualitatively* different from the one proposed by Goldfarb et al. (2009). We show that, when marginal costs do not contain brand-specific features (this is the assumption adopted by Goldfarb et al.), the profit premium of any brand is positively related to its equity. This implies that an increase in the equity of the brand necessarily increases its profit, and hence this fact supports the decision to invest in brand development. On the other hand, we also show that, when marginal costs contain brand-specific features, then whether a brand's profit premium is positively or negatively related to brand equity depends on the ratio of marginal cost and utility brand-specific intercepts. Specifically, we show that if for a brand the corresponding ratio is sufficiently large then the profit premium of the brand will be negatively related to its equity. In this case it is not profitable to invest in the brand since it disproportionally raises marginal cost. We stress that, due to this property, the profit premium concept we propose is qualitatively superior to previously proposed concepts, as it signals the possible risk of unprofitable investment.

We derive the above-mentioned results on the relationship between profit premium and brand equity analytically for the simple (non-random coefficient) logit demand model through comparative statics with respect to brand equity. Monte Carlo simulations suggest that the analytical results are also valid for the random coefficient logit model. We verify the validity of these analytical and Monte Carlo results both in the case when brands are firm-specific and product-specific.

In order to verify these findings empirically and demonstrate their practical importance, we conduct an empirical study of the new car market in the Netherlands. Using yearly sales and car characteristics data in the period 2003-2008, we estimate demand and brand equities as well as marginal cost specifications both without and with brand-specific intercepts. Our findings indicate that in the former case all profit premiums are positive while in the latter case some profit premiums are negative. These results support our theoretical findings.

The remainder of the paper is structured as follows. Section 2 describes the model, with Section 2.3 providing the definition of profit premium. Section 3 presents analytical results on how profit premium is related to brand equity for the simple logit model. Section 4 presents the corresponding Monte Carlo simulations for the random coefficient logit model. Section 5 presents the empirical results for the Dutch car market and includes a brief description of the data and the estimation method used. Section 6 concludes and provides some recommendations for managers. Some of the more technical results are included in an Appendix.

2 The model

We use a model that allows for measuring brand effects on both the demand and supply sides. It is based on the well-known Berry et al. (1995) model, which features a random coefficient logit demand model combined with a Nash-Bertrand supply side model.

2.1 Demand

Let F denote the number of firms active in the market. The utility of consumer i from buying product $j \in \mathcal{G}_f$, where $\mathcal{G}_f$ denotes the set of products produced by firm $f \in \{1, \ldots, F\}$, is given by

$$u_{ij} = \beta_f - \alpha_i p_j + \mathbf{x}_j \boldsymbol{\beta}_i + \delta_i M_j + \xi_j + \varepsilon_{ij}.$$

In this indirect utility function, β_f is a parameter common to all products of firm f, $\mathbf{x}_j$ is a K-dimensional row vector of search attributes of product j whose first component is 1 for the intercept, p_j is the unit price of product j, M_j is a measure of marketing expenditures, ξ_j is a product characteristic unknown to the econometrician but observed by consumers, and ε_{ij} is an iid type I extreme value distributed error term. Further, the random coefficients have distributions $\alpha_i \sim N(\alpha, \sigma_\alpha^2)$, $\boldsymbol{\beta}_i \sim N(\boldsymbol{\beta}, \Sigma)$, and $\delta_i \sim N(\delta, \sigma_\delta^2)$, where Σ is a diagonal matrix with diagonal elements $\left(\sigma_1^2, \ldots, \sigma_K^2\right)$. Consumers can choose from J products or can opt for an outside alternative, which represents the option of not purchasing any of the J products. We normalize the utility of the outside good to $u_{i0} = \varepsilon_{i0}$.

The utility specification yields that the probability that product j is purchased is

$$s_j = \int \frac{\exp\left(\beta_f - \alpha_i p_j + \mathbf{x}_j \boldsymbol{\beta}_i + \delta_i M_j + \xi_j\right)}{1 + \sum_{g=1}^{F} \sum_{r \in \mathcal{G}_g} \exp\left(\beta_g - \alpha_i p_r + \mathbf{x}_r \boldsymbol{\beta}_i + \delta_i M_r + \xi_r\right)} f\left(\alpha_i, \boldsymbol{\beta}_i, \delta_i\right) d\alpha_i d\boldsymbol{\beta}_i d\delta_i, \tag{1}$$

where $f\left(\alpha_i, \boldsymbol{\beta}_i, \delta_i\right)$ is the joint density function of the random coefficients α_i, $\boldsymbol{\beta}_i$, and δ_i. If the number of purchases is large, this choice probability is equal to the market share of product j. Therefore, in what follows we use the term 'market share' to refer to both quantities.

We define brand equity as the demand side effect of the brand, and, since we assume that all products of firm f have the same brand name, we measure brand equity by the firm-specific parameter β_f.[1] This approach is rather common in the literature (e.g., Jedidi et al. 1999; Chintagunta 1994; Chintagunta et al. 2005, Sriram et al. 2007; Aribarg and Arora 2008; Goldfarb et al. 2009). Since search attributes are included in the utility, we expect β_f to measure the brand-specific effect of experience attributes on utility.

2.2 Supply

We assume that prices are determined as a Nash equilibrium, where each firm maximizes its own profit with respect to own prices. The profit of firm f is

$$\pi_f = \sum_{h \in \mathcal{G}_f} \left(p_h - c_h\right) s_h,$$

where c_h denotes the marginal cost of producing product $h \in \mathcal{G}_f$. The fixed costs of production and the number of consumers in the market are omitted because they do not depend on prices. We specify the marginal cost of product $j \in \mathcal{G}_f$ as

$$c_j = \gamma_f + \mathbf{w}_j \gamma + \omega_j, \tag{2}$$

where γ_f is a parameter that measures the brand-specific effect on the marginal cost, $\mathbf{w}_j$ is a vector of attributes that affect marginal cost and ω_j is a marginal cost characteristic unobserved by the econometrician. Intuitively, γ_f

[1] Below in Section 3.2 we also analyze the case when the products of a firm have different brand names. Bronnenberg and Dubé (2017, footnote 3) raise the concern that the brand equity measured by a brand-specific parameter captures all unobserved product-level features, including some features that should not be part of brand equity. The unobserved product characteristic ξ_j in the utility attempts to alleviate this concern.

is expected to be positively correlated with β_f across firms $f = 1, \ldots, F$ because higher experience attributes for a product are likely to increase the marginal cost of the product. Therefore, in the paper we refer to the brand-specific intercept γ_f as the experience attribute effect on marginal cost. In order to model the dependence of γ_f on β_f, we assume that $\gamma_f = \phi_f \beta_f$. By specifying the coefficient ϕ_f of β_f as firm f-dependent, we allow marginal costs to be heterogenous with respect to experience attributes. This allows for imperfect correlation between the brand-specific parameters in the demand and supply side. For example, if the ϕ_f coefficients are close to each other for different firms f then the demand and supply brand-specific parameters will be highly correlated. If, however, the ϕ_f's differ from each other significantly, then the correlation will be low. Throughout the paper we maintain that $\phi_f \geq 0$, which reflects our expectation that β_f and γ_f are positively correlated. As mentioned in the Introduction, this positive correlation is realistic because several types of brand equity cost determinants affect marginal cost. Examples include product development expenditure that is meant to improve experience attributes.

As is common in the literature, we assume that prices can be determined from the first order conditions for profit maximization. These are equivalent to the equations (Berry et al. 1995)

$$\mathbf{p}_f - \mathbf{c}_f = \Delta_f(\mathbf{p})^{-1} \mathbf{s}_f, \ \ f = 1, \ldots, F, \tag{3}$$

where $\mathbf{p}_f$, $\mathbf{c}_f$ and $\mathbf{s}_f$ are the vectors of prices, marginal costs, and market shares for the products of firm f, respectively, and $\Delta_f(\mathbf{p})$ is a conformable square matrix with the element in row j and column r equal to $-\partial s_r / \partial p_j$.

2.3 Profit premium

According to the widely accepted definition of Keller (1993), brand value measurement involves a comparison between a certain factual measure and a corresponding counterfactual. The literature offers various solutions for choosing the brand used for the counterfactual, including a private label brand (Ailawadi et al. 2003), a hypothetical unbranded product (Ferjani et al. 2009), or the brand with the lowest market share. Following the proposal of Goldfarb et al. (2009), we define the counterfactual for a brand to be an unbranded quantity, that is, the quantity computed by setting the brand-specific parameters equal to zero. Along these lines, we define the brand value of a specific brand as the incremental gain realized over the unbranded state of the same brand. In the unbranded state the brand enters the computations without brand equity but it retains its search attributes. Specifically, in order to compute the counterfactual prices and market shares for the products of firm f, we take the unbranded version of these products by putting $\beta_f = \gamma_f = 0$, while keeping the parameters and variables corresponding to the other firms unchanged.

Within this framework we define *profit premium* as the difference between the profit from the products belonging to a brand and the profit from the same unbranded products. Specifically, the profit premium for firm f is $prp_f = \sum_{j \in \mathcal{G}_f} \left[(p_j - c_j)\, s_j - \left(p_j^c - c_j^c\right) s_j^c \right]$, where p_j^c, s_j^c, and c_j^c are product j's counterfactual equilibrium price, market share, and marginal cost, respectively, computed by putting $\beta_f = \gamma_f = 0$ in (2), (3), and (1).

3 Properties of profit premium in the simple logit

According to the above definition, profit premium can be regarded as an explicit function of brand equity. Here we present results on how profit premium behaves as a function of brand equity in a version of the model that does not allow for consumer heterogeneity. Specifically, we provide conditions under which profit premium is either increasing or decreasing in brand equity. We observe that if profit is increasing in brand equity, then profit premium is also increasing in brand equity. Therefore, we study whether profit is increasing or decreasing in brand equity by computing its derivatives with respect to its own brand equity.

An exogenous increase in the brand equity of a product implies changes in virtually all endogenous variables of the model, that is, prices and market shares of all products in the market.[2] Intuitively, when the brand equity of a product increases the market share of the same product will increase, if prices stay unchanged, but the price of the product is also expected to increase. Now, even if the prices of the other products change only insignificantly, the effect on the market share of the product will be ambiguous because the price increase lowers the market share. The literature has not clarified whether price will increase or not, if the corresponding brand equity increases, and the effect on profit is even more complicated. Therefore, in this section we derive comparative statics for profit by computing the derivative with respect to brand equity.

The simpler version of the model considered in this section is the simple logit, which maintains all the variables but eliminates the random coefficients.[3] This yields a model that is analytically tractable in certain dimensions. Exploiting this analytical tractability, below we present comparative statics results on the signs of the derivative of profit. Section 3.1 treats the case when brands are firm-specific, while Section 3.2 studies the case when brands are product-specific. The simulation results for the random coefficient logit in Section 4 suggest that the results derived for the simple logit are likely to hold for the random coefficient logit as well.

3.1 The case of firm-specific brands

As mentioned, we consider the special case when consumer preferences are not heterogenous, that is, we assume that all random coefficients are deterministic (i.e., $\alpha_i = \alpha$, $\beta_i = \beta$, and $\delta_i = \delta$). Then, by denoting $d_j = x_j\beta + \delta M_j + \xi_j$, we obtain that the market share of product j is the logit expression

$$s_j = \frac{\exp\left(\beta_f - \alpha p_j + d_j\right)}{1 + \sum_{g=1}^{F}\sum_{r\in\mathcal{G}_g}\exp\left(\beta_g - \alpha p_r + d_r\right)}. \tag{4}$$

In the simple logit model, the first order condition for profit maximization (3) can be written in closed form as

$$p_j - c_j = \frac{1}{\alpha}\frac{1}{1 - \sum_{r\in\mathcal{G}_j} s_r} \quad \text{for } \ j \in \mathcal{G}_f, \ \ f = 1, \dots, F.$$

We know that a unique Nash equilibrium in prices exists under the assumption $\alpha > 0$ (Konovalov and Sándor 2010), and therefore, we maintain this assumption throughout the paper. Note that for products j belonging to the same firm, the equilibrium markups $p_j - c_j$ are the same. Denote the common equilibrium markup of the products belonging to firm f by m_f. Then the first order condition for profit maximization can be rewritten as

$$m_f = \frac{1}{\alpha}\frac{1}{1 - \overline{s}_f}, \tag{5}$$

where $\overline{s}_f = \sum_{r\in\mathcal{G}_j} s_r$ is the market share of firm f. In this case the profit of firm f is $\pi_f = m_f\overline{s}_f$.

[2] In reality brand equity is endogenous, as modeled for example by Borkovsky et al. (2017) in a dynamic context. However, in the literature it is not uncommon that for analytical purposes, marketing activities devoted for creating brand equity or brand equity itself are treated as exogenous. For example, Sriram et al. (2007) treat advertising as exogenous, whereas Stahl et al. (2012) study the effect of exogenous changes in brand equity.

[3] This means that the simple logit cannot model consumer heterogeneity regarding the observed product attributes (which includes price), and as such it generates restrictive substitution patterns. Due to this drawback the simple logit is not used very often in empirical work, despite its analytical tractability (e.g., Berry 1994).

Note that the market shares can be rewritten to depend on the markup $m_f = p_j - c_j,\ f = 1, \ldots, F$ instead of $p_j,\ j = 1, \ldots, J$, that is,

$$s_j = \frac{\exp\left(\beta_f - \alpha m_f - \alpha c_j + d_j\right)}{1 + \sum_{g=1}^{F} \sum_{r \in \mathcal{G}_g} \exp\left(\beta_g - \alpha m_g - \alpha c_r + d_r\right)}.$$

Consequently, the market share of firm f is

$$\overline{s}_f = \frac{\exp\left(\rho_f \beta_f - \alpha m_f + \ell_f\right)}{1 + \sum_{g=1}^{F} \exp\left(\rho_g \beta_g - \alpha m_g + \ell_g\right)}, \tag{6}$$

where $\rho_f = 1 - \alpha \phi_f$ and $\ell_f = \ln\left[\sum_{j \in \mathcal{G}_j} \exp\left(-\alpha\left[w_j \gamma + \omega_j\right] + d_j\right)\right]$ for $f = 1, \ldots, F$.[4]

In Appendix A.1, equation (29), we derive that

$$\frac{d\pi_f}{d\beta_f} = \frac{\overline{s}_f}{\alpha D}\left(\rho_f\left(E - \overline{s}_f\right) + \overline{s}_f \sum_{g \neq f} \frac{\left(\rho_f - \rho_g\right)\overline{s}_g^2}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g}\right) \tag{7}$$

for any firm f and any product j of firm f, where

$$E = 1 - \sum_{g \neq f} \frac{\overline{s}_g^2}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g},\quad D = \left[\left(1 - \overline{s}_f\right)^2 + \overline{s}_f\right]\left(1 - \sum_{g=1}^{F} \frac{\overline{s}_g^2}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g}\right). \tag{8}$$

Here and throughout the paper, $\sum_{g \neq f}$ denotes summation with respect to $g = 1, \ldots, F$ with $g \neq f$. Note that

$$E > 1 - \sum_{g \neq f} \overline{s}_g = s_0 + \overline{s}_f > \overline{s}_f \text{ and} \tag{9}$$

$$D > \left[\left(1 - \overline{s}_f\right)^2 + \overline{s}_f\right]\left(1 - \sum_{g=1}^{F} \overline{s}_g\right) > 0. \tag{10}$$

We obtain the following result on the derivative of profit.

Proposition 1 *The following statements hold for any firm f.*

1. *If $\phi_f = \min\{\phi_g : g = 1, \ldots, F\}$ and $\rho_f = 1 - \alpha\phi_f > 0$, the derivative $d\pi_f / d\beta_f$ is positive.*

2. *If $\phi_f = \max\{\phi_g : g = 1, \ldots, F\}$ and $\rho_f = 1 - \alpha\phi_f < 0$, the derivative $d\pi_f / d\beta_f$ is negative.*

Proof. 1. We have $\rho_f - \rho_g \geq 0$ for all g and $\rho_f > 0$, so equation (7) implies that $d\pi_f / d\beta_f$ is positive.
2. In this case $\rho_f - \rho_g \leq 0$ for all g and $\rho_f < 0$, so equation (7) implies that $d\pi_f / d\beta_f$ is negative. ■

Part 1 of this result states that if firm f has the lowest ratio of marginal cost and utility brand-specific intercepts (i.e., the lowest $\phi_f = \gamma_f / \beta_f$) and $\rho_f = 1 - \alpha\phi_f > 0$, then the profit of firm f is increasing in the common brand

[4] The details of this derivation:

$$\begin{aligned}
\overline{s}_f &= \frac{\sum_{j \in \mathcal{G}_j} \exp\left(\left[1 - \alpha\varphi_f\right]\beta_f - \alpha m_f - \alpha\left[w_j\gamma + \omega_j\right] + d_j\right)}{1 + \sum_{g=1}^{F}\sum_{r \in \mathcal{G}_g} \exp\left(\left[1 - \alpha\varphi_g\right]\beta_g - \alpha m_g - \alpha\left[w_r\gamma + \omega_r\right] + d_r\right)} \\
&= \frac{\exp\left(\left[1 - \alpha\varphi_f\right]\beta_f - \alpha m_f\right)\sum_{j \in \mathcal{G}_j} \exp\left(-\alpha\left[w_j\gamma + \omega_j\right] + d_j\right)}{1 + \sum_{g=1}^{F} \exp\left(\left[1 - \alpha\varphi_g\right]\beta_g - \alpha m_g\right)\sum_{r \in \mathcal{G}_g} \exp\left(-\alpha\left[w_r\gamma + \omega_r\right] + d_r\right)} \\
&= \frac{\exp\left(\left[1 - \alpha\varphi_f\right]\beta_f - \alpha m_f + \ell_f\right)}{1 + \sum_{g=1}^{F} \exp\left(\left[1 - \alpha\varphi_g\right]\beta_g - \alpha m_g + \ell_g\right)}.
\end{aligned}$$

equity of the firm's products. Part 2 of the result states that if firm f has the highest ratio of marginal cost and utility brand-specific intercepts and $\rho_f = 1 - \alpha\phi_f < 0$, then the profit of firm f is decreasing in the brand equity of the firm's products. The latter result is remarkable because it means that the profit of a firm decreases if the equity of its brand increases. This is not unrealistic because firms are willing to give up profitability over a short period at the expense of offering consumers brands with higher utility. Part 2 of Proposition 1 shows that the profit premium is not positively related to brand equity during such periods.

Next we consider the case when marginal costs do not contain brand-specific intercepts.

Corollary 2 *When for all $f = 1, \ldots, F$ the brand-specific marginal cost intercepts ϕ_f are zero, the derivative $d\pi_f/d\beta_f$ is positive.*

Proof. This follows from part 1 of Proposition 1 for $\phi_f = 0$ for all $f = 1, \ldots, F$. ■

This corollary implies that profit premium is positive when marginal costs do not contain brand-specific intercepts. This case is important because it is the case considered by Goldfarb et al. (2009). An important consequence of the corollary is that, if the analyst omits brand-specific effects from the marginal cost when in reality they are present, then profit premium will not be able to capture brand-specific signals from the supply side. This may imply that profit premium is measured to be positive even when in reality it is negative. This may erroneously induce managers to invest more in brand equity, which in this case leads to lower profit. In the empirical study discussed in Section 5 we present such an example.

3.2 The case of product-specific brands

Here we discuss the case in which each product of a firm has a different brand name. In this case the utility of consumer i for product $j \in \mathcal{G}_f$ is

$$u_{ij} = \beta_j - \alpha_i p_j + x_j \boldsymbol{\beta}_i + \delta_i M_j + \xi_j + \varepsilon_{ij}.$$

As before, we consider the special case when consumer preferences are not heterogenous, that is, we assume that all random coefficients are deterministic (i.e., $\alpha_i = \alpha$, $\boldsymbol{\beta}_i = \boldsymbol{\beta}$, $\delta_i = \delta$). Then, by denoting $d_j = x_j\boldsymbol{\beta} + \delta M_j + \xi_j$, we obtain that the market share of product j is the simple logit expression

$$s_j = \frac{\exp(\beta_j - \alpha p_j + d_j)}{1 + \sum_{r=1}^{J} \exp(\beta_r - \alpha p_r + d_r)}.$$

Define the marginal cost of product j as

$$c_j = \phi\beta_j + w_j\gamma + \omega_j,$$

which implies that the demand and supply brand-specific parameters are perfectly correlated; here we maintain the assumption $\phi \geq 0$.[5] The market shares can be rewritten to depend on $m_f = p_j - c_j,\ f = 1, \ldots, F$ instead of $p_j,\ j = 1, \ldots, J$, that is,

$$s_j = \frac{\exp(\rho\beta_j - \alpha m_f + \ell_j)}{1 + \sum_{g=1}^{F} \sum_{r \in \mathcal{G}_g} \exp(\rho\beta_r - \alpha m_g + \ell_r)}, \tag{11}$$

where $\rho = 1 - \alpha\phi$ and $\ell_j = d_j - \alpha(w_j\gamma + \omega_j)$ for all j. The market share of firm f is

$$\overline{s}_f = \frac{\sum_{j \in \mathcal{G}_f} \exp(\rho\beta_j - \alpha m_f + \ell_j)}{1 + \sum_{g=1}^{F} \sum_{r \in \mathcal{G}_g} \exp(\rho\beta_r - \alpha m_g + \ell_r)}. \tag{12}$$

[5]The derivation of the comparative statics in the case of brand-specific ϕ_f is not tractable.

In equations (38) and (37) of Appendix A.2 we derive that

$$\frac{d\pi_j}{d\beta_j} = \frac{\rho s_j}{\alpha D}\left(\frac{1-s_j}{1-\overline{s}_f}\overline{s}_f\left(E-\overline{s}_f\right)+\left(1-\overline{s}_f\right)\left(E-s_j\right)\right) \text{ and}$$
$$\frac{d\pi_f}{d\beta_j} = \frac{\rho s_j}{\alpha D}\left(E-\overline{s}_f\right),$$

where $\pi_j = (p_j - c_j)\, s_j \equiv m_f s_j$ is the profit obtained from product j of firm f. Since by (9) the inequalities $E - s_j \geq E - \overline{s}_f > 0$ and $1 - s_j \geq 1 - \overline{s}_f > 0$ hold, the derivatives $d\pi_j/d\beta_j$ and $d\pi_f/d\beta_j$ have signs identical to the sign of $\rho = 1 - \alpha\phi$. So we can state the following result.

Proposition 3 *For any firm f and any product j of firm f, if $\alpha\phi < 1$ (> 1) the derivatives $d\pi_j/d\beta_j$, $d\pi_f/d\beta_j$ are positive (negative).*

Proposition 3 shows that when the ratio of marginal cost and utility brand-specific intercepts is relatively small (i.e., $\alpha\phi < 1$ or $\rho = 1 - \alpha\phi < 0$), then the firm's profit and the product's profit are increasing in the corresponding brand equity. It is important to note that this also covers the case when there is no brand-specific intercept included in the marginal cost. However, when the ratio of marginal cost and utility brand-specific intercepts is relatively large (i.e., $\alpha\phi > 1$, or $\rho = 1 - \alpha\phi < 0$), then both firm- and product-level profit are decreasing in brand equity. In this case profit premium is not positively related to brand equity.

4 Simulations in the random coefficient logit

This section presents Monte Carlo simulation results for the random coefficient logit model. These simulations are meant to study the statistical relationship between brand equity and profit premium. The simulations are useful because they show if the results derived in Section 3 for the simple logit model are valid for the random coefficient logit model as well. We study the statistical relationship via two indicators that are estimated based on simulated data. The first indicator is the slope coefficient of a linear regression of profit premium on a constant and brand equity. This can be regarded as an estimate of the derivative of profit premium with respect to the corresponding brand equity, which we derive in Section 3.[6] The second indicator is the Kendall correlation coefficient between profit premium and brand equity, which captures statistical monotonicity between these two variables.[7] Correlation has often been used in the literature to validate brand equity or brand value measures, and it can be regarded as a complement to the other indicator.

First we generate the exogenous variables of the model. Next, we generate 100 random replications of the brand equities and their coefficients in the marginal cost equation from the distributions specified below. Then we determine the market shares and prices of the products as well as the profit premium for each replication. Finally, for each brand we determine the OLS estimates when regressing profit premium on brand equity across the replications and also calculate the correlations between these two variables.

First we consider the case in which brands are firm-specific. In the simulations we consider cases in which the number of products in the market ranges from small to large while we also let the number of products per firm vary. Specifically, the number of products is either $J = 10$, 50, or 100. For $J = 10$ we consider cases of $F = 10$ single

[6] Since we define the profit premium as the difference between profit and counterfactual profit (which does not depend on own brand equity β_f), the derivative of profit premium with respect to brand equity is equal to the derivative of profit with respect to brand equity.

[7] We have also computed Pearson correlations, which are supposed to capture linearity, although we do not report them in the paper. The Pearson correlations tend to be numerically higher for the positive correlations, but their relative magnitudes are qualitatively similar.

product firms, $F = 4$ firms with 3 – 3 – 3 – 1 products, and $F = 2$ firms with 5 products each. For $J = 50$ we consider cases with $F = 10$ firms with 5 products each and $F = 5$ firms with 10 products each. For $J = 100$ we consider the case of $F = 10$ firms with 10 products each.

The three attributes x_{jk}, ($k = 2, 3, 4$) and the marketing activities variable M_j are generated as uniform on $[1, 2]$; the marginal cost variables are computed as $w_{jk} = \ln(x_{jk})$. The utility parameters used are $\alpha = 4$, $\sigma_\alpha = 1$, $\beta = (2, 1.5, 2, 2.5)'$, $(\sigma_1, \sigma_2, \sigma_3, \sigma_4) = (2.5, 2, 2, 2)$, $\delta = 2$, $\sigma_\delta = 2$.[8] The marginal cost parameters are $\gamma = (0.5, 1, 1-, 1.5)$. We generate the unobserved demand and marginal cost characteristics ξ_j, ω_j as standard normal with $corr\,(\xi_j, \omega_j) = 0.6$, the firm-specific constants β_f as uniform on $[0, 3]$. Regarding the coefficients ϕ_f in the marginal cost we consider two situations. In the first we assume that ϕ_f is uniform on $[0, 0.2]$; therefore, $\alpha\phi_f < 1$ for any brand f. We refer to this as the case when the ratios of marginal cost and utility brand-specific intercepts are small. In the second situation we assume that ϕ_f is uniform on $[0, 0.8]$, where it can happen that $\alpha\phi_f > 1$ for some f. We refer to this as the case when the ratios of marginal cost and utility brand-specific intercepts are large.

We also present simulation results for the case when brands are product-specific, which is studied above in Section 3.2. In order to simplify the exposition, here we only consider markets with 10 products and two different market structures, namely, when there are 5 firms with 2 products each and when there are 2 firms with 5 products each. We present slope coefficients and correlations corresponding to both product- and firm-level profit.

We present the results corresponding to different cases in Tables 1 and 2. The left hand side panels in the tables contain the OLS estimates of the slope coefficients while the right hand side panels contain the (Kendall) correlations. For example, the first entry in Table 1, 0.02, is the estimate of the slope coefficient in the linear regression of the profit premium on the equity of brand 1 across the 100 replications. In this case there are 10 firms in the market each having 10 products; this appears in the table as case A. Each column corresponds to a different case. Specifically, case B: 50 products/10; C: 10 products/10 firms; D: 50 products/5 firms; E: 10 products/4 firms; and F: 10 products/2 firms. We do not report the standard errors of the estimates in order to save space, but we selectively mention below those estimates that are statistically significant at 5% significance level.

The upper parts of Tables 1 and 2 present results when the ratios of marginal cost and utility brand-specific intercepts are small. Notice that in this case slope coefficients are positive and the vast majority of them are statistically different from zero. All correlations in the upper parts of Tables 1 and 2 are positive and rather high.[9] These results are in line with part 1 of Proposition 1 as well as Proposition 3. In conclusion, in the random coefficient logit model considered in the Monte Carlo simulations, when the ratios of marginal cost and utility brand-specific intercepts are small, profit premium is positively related to brand equity, which suggests that the results established in Section 3 also hold for the random coefficient logit model.

The outcome of the simulations is quite different when the ratios of marginal cost and utility brand-specific intercepts are large; see the lower parts of Tables 1 and 2. We notice that some slope coefficients are negative in both tables. Specifically, in case B of Table 1, which corresponds to 50 products/10 firms, the slope coefficients for brands 2 and 7 corresponding to profit premium are negative and significantly different from zero. Moreover, there are brands for which the slope coefficients corresponding to profit premium are negative and significantly different from zero in case C (10 products/10 firms), D (50 products/5 firms), E (10 products/4 firms), and F (10 products/2 firms) as well. Similarly, in the case of product-specific brands reported in Table 2, in both the 5-firm and 2-firm

[8] We have also experimented with different values for the price coefficient, for example $\alpha = -1$, $\sigma_\alpha = 0.2$, while keeping the rest of the data generating process unchanged. The results were qualitatively similar.

[9] The corresponding Pearson correlations (not reported) imply R^2's for the estimated linear regressions in the range $0.4 - 0.9$, which suggests that there is an approximate linear relationships between the profit premiums and brand equity.

Table 1. OLS estimates of slope coefficients and corresponding correlations when brands are firm-specific

Small ratios of marginal cost and utility brand-specific intercepts												
Brands	A	B	C	D	E	F	A	B	C	D	E	F
	Slope coefficients						Correlations					
1	0.02	0.01	0.04	0.03	0.05	0.06	0.63	0.68	0.63	0.66	0.64	0.60
2	0.00	0.02	0.00	0.01	0.05	0.03	0.72	0.67	0.75	0.69	0.64	0.65
3	0.01	0.00	0.01	0.01	0.03	–	0.70	0.76	0.71	0.73	0.66	–
4	0.02	0.01	0.02	0.09	0.00	–	0.70	0.68	0.63	0.64	0.64	–
5	0.01	0.01	0.03	0.01	–	–	0.70	0.70	0.62	0.69	–	–
6	0.03	0.00	0.01	–	–	–	0.65	0.74	0.67	–	–	–
7	0.01	0.09	0.01	–	–	–	0.66	0.64	0.70	–	–	–
8	0.04	0.00	0.02	–	–	–	0.63	0.76	0.64	–	–	–
9	0.02	0.00	0.00	–	–	–	0.63	0.72	0.73	–	–	–
10	0.00	0.01	0.00	–	–	–	0.72	0.64	0.64	–	–	–
Large ratios of marginal cost and utility brand-specific intercepts												
	Slope coefficients						Correlations					
1	-0.01	-0.00	-0.02	-0.02	-0.04	-0.06	-0.19	-0.14	-0.23	-0.18	-0.23	-0.21
2	0.00	-0.02	0.00	-0.00	-0.04	-0.04	-0.11	-0.20	-0.08	-0.16	-0.23	-0.22
3	0.01	0.00	0.01	0.01	-0.00	–	-0.06	-0.03	-0.05	-0.04	-0.12	–
4	-0.00	-0.00	-0.01	-0.07	-0.00	–	-0.10	-0.12	-0.17	-0.19	-0.10	–
5	0.00	0.00	0.00	0.00	–	–	-0.13	-0.09	-0.16	-0.09	–	–
6	-0.00	0.00	0.00	–	–	–	-0.13	-0.03	-0.12	–	–	–
7	0.00	-0.08	0.00	–	–	–	-0.15	-0.24	-0.13	–	–	–
8	-0.02	0.00	-0.00	–	–	–	-0.16	-0.01	-0.12	–	–	–
9	-0.00	0.00	0.00	–	–	–	-0.17	-0.04	0.01	–	–	–
10	0.00	-0.00	-0.00	–	–	–	-0.07	-0.11	-0.14	–	–	–

Notes. The columns correspond to A:100 products/10 firms, B: 50 products/10, C: 10 products/10 firms, D: 50 products/5 firms, E: 10 products/4 firms, F: 10 products/2 firms. Products are distributed evenly across firms, except for the case of 10 products/4 firms, where the distribution is 3-3-3-1. Small ratios of marginal cost and utility brand-specific intercepts correspond to ϕ_f uniform on [0,0.2]. Large ratios of marginal cost and utility brand-specific intercepts correspond to ϕ_f uniform on [0,0.8].

cases the slope coefficients corresponding to profit premiums are negative and significantly different from zero. Further, the correlations reported in the lower parts of Tables 1 and 2 are mostly negative. These results are in line with part 2 of Proposition 1 and Proposition 3 obtained for the simple logit model.

To conclude this section we note that the results obtained in both the firm-specific and product-specific brand cases are similar to those obtained for the simple logit in Section 3. Consequently, the specific feature of the random coefficient logit model that captures substitution of products with similar characteristics does not alter how profit premium behaves with respect to brand equity.

5 Profit premium in the Dutch car market

In this section we present profit premium estimates for new cars sold in the Netherlands in 2008. We first provide a brief description of the data used, and then describe estimation of the model, and present the results.

5.1 Data

The data sets we use contain prices, sales, car characteristics, and advertising expenditure of cars sold in the Netherlands between 2003 and 2008. We exclude car makes that did not have positive sales for each year of the sample. We define a market to consist of the car models that appear in a given year. A car model in a given year is included if its sales exceed fifty. This leads to a total of 309 different car models that were sold during this period; this corresponds to about 226 different models on average per year. We regard each model-year combination as one observation, which results in 1,355 observations in total.

Table 2. OLS estimates of slope coefficients and corresponding correlations when brands are product-specific

	A		B		A		B	
Brands	$prpp$	prp	$prpp$	prp	$prpp$	prp	$prpp$	prp
Small ratios of marginal cost and utility brand-specific intercepts								
	Slope coefficients				Correlations			
1	0.04	0.04	0.05	0.03	0.63	0.63	0.62	0.61
2	0.00	0.00	0.00	0.00	0.74	0.75	0.74	0.38
3	0.01	0.01	0.02	0.01	0.71	0.71	0.71	0.70
4	0.03	0.02	0.04	0.01	0.63	0.62	0.63	0.62
5	0.03	0.03	0.05	0.02	0.62	0.61	0.61	0.59
6	0.02	0.01	0.03	0.01	0.67	0.65	0.67	0.67
7	0.01	0.01	0.01	0.00	0.69	0.69	0.66	0.70
8	0.02	0.02	0.03	0.02	0.64	0.65	0.64	0.63
9	0.00	0.00	0.00	0.00	0.73	0.74	0.72	0.67
10	0.00	0.00	0.00	0.00	0.63	0.63	0.61	0.63
Large ratios of marginal cost and utility brand-specific intercepts								
	Slope coefficients				Correlations			
1	-0.02	-0.02	-0.03	-0.02	-0.23	-0.23	-0.23	-0.21
2	0.00	0.00	0.00	0.00	-0.08	-0.07	-0.06	0.04
3	0.01	0.01	0.01	0.01	-0.05	-0.04	-0.05	-0.03
4	-0.01	-0.01	-0.01	-0.00	-0.16	-0.16	-0.17	-0.08
5	-0.00	0.00	0.00	0.01	-0.15	-0.17	-0.16	-0.11
6	0.00	0.00	-0.00	0.00	-0.12	-0.11	-0.13	-0.10
7	0.00	0.00	-0.00	0.00	-0.13	-0.12	-0.14	-0.07
8	-0.00	-0.00	-0.01	0.00	-0.12	-0.12	-0.14	-0.08
9	0.00	0.00	0.00	0.00	0.01	0.01	-0.02	0.22
10	-0.00	-0.00	-0.00	0.00	-0.14	-0.14	-0.12	-0.09

Notes. A: 5 firms, 2 products per firm, B: 2 firms, 5 products per firm prp and $prpp$ refer to profit premium measured at the firm and product level, respectively. Products are distributed evenly across firms. Small ratios of marginal cost and utility brand-specific intercepts correspond to to ϕ_f uniform on [0,0.2]. Large ratios of marginal cost and utility brand-specific intercepts correspond to to ϕ_f uniform on [0,0.8].

We collected prices, sales, and car characteristics data from Autoweek Carbase, which is an open online database that contains data on all car models sold in the Netherlands.[10] Car characteristics include engine power, fuel consumption (as kilometers per liter), weight, size, dummy variables for whether the car's standard equipment includes cruise control, the car class the car belongs to, as well as other technical characteristics. All prices available are listed (post-tax) prices; although transaction prices would be more desirable, they are not available. We have normalized all prices to 2006 euros by using the Consumer Price Index in the corresponding years.

In order to create all necessary variables for estimation of the model, we have supplemented the data set with several variables from the Dutch statistical office (i.e., Statistics Netherlands).[11] For example, we use the total number of households to construct market shares and we use average yearly gasoline prices to construct a fuel consumption variable, where the latter is defined as kilometers per liter divided by the average price of gasoline per liter. In addition, we use data on the distribution of disposable household income.

We use information from 2007 on brand ownership structure to specify which car brands belong to the same parent car producer. There are 36 different brands in our sample over the 2003-2008 period that are owned by 16 different companies. For instance, in 2007 the Volkswagen Group owned Volkswagen, Audi, Seat, and Škoda. We use data on brand-level advertising expenditure obtained from Nielsen.

Table 3 presents the means weighted by sales for the main variables used in the estimation of demand. The number of different car models sold increased from 208 in 2003 to 236 in 2008. The lowest sales are observed in 2005 while the highest in 2007. Prices had an increasing tendency in real terms with a remarkable drop in 2008,

[10] See http://www.autoweek.nl/carbase.php.
[11] See http://www.cbs.nl.

which is probably the result of the upcoming economic crisis. Regarding the share of European cars sold, the period 2003-2008 witnessed a downward trend, which was accompanied by an upward trend of brands that originate from Eastern Asia. Acceleration of cars measured by the ratio of horsepower to weight (denoted HP/weight) showed a steady increase. The average size of cars (measured as length times width times height) increased slightly during the sampling period. The share of cars sold with cruise control as standard equipment went up significantly in 2004, but afterwards showed a slight decrease. Average fuel efficiency improved during the sampling period as shown by the kilometers per liter (denoted KPL) variable. This improvement, however, was not matched by kilometers per euro (denoted KP€), which is a fuel efficiency variable more relevant to consumers. This means that the improvement in fuel efficiency was not sufficient to offset the increase in gasoline prices over the sampling period. The proportion of family cars sold is rather high in each year and shows a slight decrease. The proportion of luxury cars has a similar trend, although the overall share of luxury cars is rather low. The share of sport cars is extremely low and does not change much during the sampling period. The proportion of MPV's shows a decreasing trend while the proportion of SUV's shows an increasing trend. Finally, we observe that advertising expenditures peaked in 2004, after which they dropped considerably.

Table 3. Summary statistics

Year	No. of Models	Sales	Price	HP/ Weight	Size	Cruise Control	KPL	KP€	Family Car	Luxury	Sport	MPV	SUV	Adver-tising
2003	208	480,309	19,553	0.786	7.152	0.230	14.482	12.498	0.426	0.031	0.006	0.184	0.033	1.686
2004	222	475,032	19,945	0.788	7.184	0.309	14.699	11.739	0.408	0.029	0.008	0.198	0.038	2.152
2005	227	457,094	20,540	0.794	7.270	0.301	14.863	10.989	0.403	0.030	0.007	0.189	0.054	1.802
2006	229	474,452	20,388	0.804	7.271	0.309	15.122	10.709	0.360	0.030	0.006	0.172	0.063	0.843
2007	233	491,723	20,552	0.809	7.328	0.282	15.121	10.363	0.359	0.027	0.007	0.171	0.071	0.867
2008	236	483,807	18,699	0.812	7.268	0.297	15.834	10.305	0.377	0.021	0.006	0.127	0.061	0.855
All	1,355	477,070	19,941	0.799	7.246	0.288	15.023	11.097	0.389	0.028	0.007	0.173	0.053	1.361

Notes: Prices are in 2006 euros. Advertising is in million euros. All variables are sales weighted means, except for the number of models and sales.

Table 4 presents the means weighted by sales for additional variables used in the estimation of marginal cost. Cylinder volume (CC) shows an upward trend between 2003-2007, while it drops to its lowest average in 2008. Acceleration (seconds elapsed for reaching 100km/h) and maximum speed both improve during the sampling period, which means that acceleration decreases and maximum speed increases. The proportions of cars having air-conditioning, power steering, sport chairs, and xenon lights show increasing trends and tend to reach their peaks around 2007 after which they decline. The proportion of cars with a board computer is rather variable. The proportion of cars with an anti-roll bar increases steadily over the sampling period.

Table 4. Summary statistics additional marginal cost variables

Year	CC	Accel-eration	Maximum speed	Aircon-ditioning	Board computer	Power steering	Sports chairs	Anti-roll bar	Xenon lights
2003	1,478	13.555	173.655	0.248	0.409	0.882	0.037	0.141	0.001
2004	1,476	13.523	173.805	0.283	0.495	0.917	0.038	0.163	0.003
2005	1,500	13.402	174.936	0.350	0.527	0.908	0.042	0.229	0.006
2006	1,498	13.334	174.578	0.380	0.465	0.915	0.058	0.224	0.006
2007	1,512	13.287	174.756	0.396	0.442	0.920	0.060	0.239	0.006
2008	1,464	13.163	175.112	0.353	0.464	0.904	0.047	0.298	0.004
All	1,488	13.376	174.473	0.335	0.466	0.908	0.047	0.216	0.004

Notes: All variables are sales weighted means.

5.2 Estimation

We estimate a version of the model presented in Section 2. In the demand model the price coefficient is defined as $\alpha_i = \alpha/y_i$, where y_i is the income of household i, and only the constant is specified as random. The supply side is specified slightly differently from the model in Section 2 since, following Berry et al. (1995), we use the logarithm of marginal costs as the dependent variable:

$$\ln c_j = \gamma_f + \mathbf{w}_j\gamma + \omega_j, \tag{13}$$

where $\mathbf{w}_j$ is a vector of supply side characteristics of product j, where the included characteristics are listed in the lower part of Table 5.

Table 5. Estimation results

	(A)		(B)		(C)	
Variable	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.
Base coefficients						
constant	-19.486	$(7.408)^{***}$	-19.486	$(7.408)^{***}$	-19.486	$(7.408)^{***}$
HP/weight	2.444	$(0.548)^{***}$	2.444	$(0.548)^{***}$	2.444	$(0.548)^{***}$
cruise control	0.478	$(0.110)^{***}$	0.478	$(0.110)^{***}$	0.478	$(0.110)^{***}$
KM per euro	0.764	$(0.298)^{***}$	0.764	$(0.298)^{***}$	0.764	$(0.298)^{***}$
size	10.910	$(1.494)^{***}$	10.910	$(1.494)^{***}$	10.910	$(1.494)^{***}$
advertising	0.557	$(0.040)^{***}$	0.557	$(0.040)^{***}$	0.557	$(0.040)^{***}$
family car	-0.773	$(0.165)^{***}$	-0.773	$(0.165)^{***}$	-0.773	$(0.165)^{***}$
luxury	-0.193	(0.216)	-0.193	(0.216)	-0.193	(0.216)
sport	-0.775	$(0.238)^{***}$	-0.775	$(0.238)^{***}$	-0.775	$(0.238)^{***}$
MPV	-0.240	(0.153)	-0.240	(0.153)	-0.240	(0.153)
SUV	0.544	$(0.217)^{**}$	0.544	$(0.217)^{**}$	0.544	$(0.217)^{**}$
Random coefficients						
price/income	-6.755	$(2.323)^{***}$	-6.755	$(2.323)^{***}$	-6.755	$(2.323)^{***}$
constant	3.352	(5.418)	3.352	(5.418)	3.352	(5.418)
Marginal cost parameters						
constant	-4.553	$(0.942)^{***}$	-0.594	(0.883)	-0.462	(0.815)
log(HP/weight)	0.160	$(0.072)^{**}$	0.334	$(0.066)^{***}$	0.173	$(0.064)^{***}$
cruise control	0.019	(0.015)	0.023	$(0.014)^{*}$	0.046	$(0.013)^{***}$
log(KM per liter)	-0.665	$(0.057)^{***}$	-0.687	$(0.052)^{***}$	-0.599	$(0.048)^{***}$
log(size)	0.511	$(0.097)^{***}$	0.780	$(0.089)^{***}$	1.075	$(0.084)^{***}$
log(CC)	0.409	$(0.051)^{***}$	0.263	$(0.047)^{***}$	0.333	$(0.044)^{***}$
log(acceleration)	-0.091	(0.084)	-0.119	(0.076)	-0.194	$(0.070)^{***}$
log(maximum speed)	0.822	$(0.126)^{***}$	0.274	$(0.119)^{***}$	0.170	(0.113)
airconditioning	-0.000	(0.016)	0.052	$(0.014)^{***}$	0.059	$(0.013)^{***}$
board computer	0.046	$(0.013)^{***}$	0.039	$(0.012)^{***}$	0.032	$(0.012)^{***}$
power steering	0.114	$(0.026)^{***}$	0.103	$(0.023)^{***}$	0.094	$(0.021)^{***}$
sports chairs	0.075	$(0.024)^{***}$	0.047	$(0.021)^{***}$	0.068	$(0.020)^{***}$
anti-roll bar	0.249	$(0.015)^{***}$	0.127	$(0.015)^{***}$	0.065	$(0.014)^{***}$
xenon lights	0.033	(0.027)	0.121	$(0.025)^{***}$	0.122	$(0.023)^{***}$
brand equity	—		0.111	$(0.006)^{***}$	—	
Brand fixed effects supply side	no		no		yes	
R^2 supply side	0.916		0.933		0.952	

Notes: * significant at 10%; ** significant at 5%; *** significant at 1%. The number of observations is 1,355. The number of simulated consumers used for the aggregate moments is 2,209. Standard errors are in parenthesis. The same demand side estimates are used for the marginal cost specifications (A), (B), (C). Kia is the base brand.

The assumption used for identification is that the demand and supply side unobserved characteristics corresponding to a given product are mean independent of the observed characteristics of all products, which is the assumption used by Berry et al. (1995). In a first stage we estimate the demand parameters by GMM with moments based on the unobserved demand characteristics and instruments based on predicted prices and differentiation instruments (Gandhi and Houde 2016) constructed from the observed characteristics. The estimates are shown in the upper part of Table 5. In all specifications we include brand-specific intercepts, which are reported as brand equity estimates in the fist column of Table 6. In the second stage we use the demand estimates as well as the first order conditions for profit maximization in equation (3) to estimate the supply side parameters. We do so by substituting the c_j's from equation (3) into equation (13), followed by OLS estimation of equation (13).

Table 6. Brand-specific intercepts and profit premiums

	Brand-specific intercepts									
	Brand equity (A-C)		Marginal cost (C)		Profit premiums					
Variable	Coeff.	Std. Err.	Coeff.	Std. Err.	Specification (A)		Specification (B)		Specification (C)	
bmw	3.482	(0.412)***	0.556	(0.034)***	61.650		53.383		44.083	
mini	2.728	(0.397)***	0.498	(0.067)***	10.184		9.400		8.673	
chrysler	0.378	(0.322)	0.190	(0.037)***	1.673		0.986		-2.441	
jeep	1.039	(0.440)**	0.267	(0.041)***	2.127		1.389		-0.208	
mercedes-benz	3.577	(0.481)***	0.591	(0.032)***	44.795		39.230		31.586	
smart	1.437	(0.582)**	0.924	(0.049)***	1.810		1.656		0.782	
alfa romeo	1.292	(0.319)***	0.304	(0.036)***	7.847		5.974		2.821	
fiat	0.660	(0.277)**	0.173	(0.032)***	27.590		23.883		18.424	
lancia	0.261	(0.326)	0.323	(0.042)***	0.542		0.400		-1.532	
ford	1.828	(0.263)***	0.162	(0.030)***	118.571		101.760		105.723	
jaguar	3.012	(0.544)***	0.537	(0.041)***	5.216		3.871		1.022	
land rover	3.020	(0.641)***	0.493	(0.039)***	11.237		8.726		5.071	
mazda	0.930	(0.292)***	0.303	(0.031)***	16.009		12.365		2.521	
volvo	3.008	(0.369)***	0.379	(0.031)***	67.220		57.552		55.235	
subaru	0.367	(0.295)	0.308	(0.035)***	1.418		1.000		-3.142	
cadillac	0.044	(0.364)	0.216	(0.053)***	0.039		0.019		-1.342	
chevrolet	0.276	(0.262)	0.051	(0.030)	5.427		4.186		3.293	
opel	1.850	(0.313)***	0.356	(0.030)***	107.956		94.581		81.353	
saab	1.867	(0.326)***	0.314	(0.046)***	6.460		5.030		3.857	
honda	0.914	(0.283)***	0.377	(0.034)***	16.348		12.641		-2.543	
hyundai	0.478	(0.260)*	0.115	(0.029)***	18.940		15.287		10.569	
kia	0.000		0.000		0.000		0.000		0.000	
mitsubishi	0.943	(0.285)***	0.230	(0.031)***	14.383		11.735		7.753	
porsche	4.206	(0.643)***	0.682	(0.048)***	4.439		3.723		2.165	
citroen	1.243	(0.275)***	0.251	(0.030)***	55.991		45.523		34.297	
peugeot	1.240	(0.281)***	0.227	(0.031)***	88.810		75.014		64.401	
nissan	1.152	(0.309)***	0.235	(0.031)***	23.224		18.666		13.797	
renault	1.699	(0.291)***	0.260	(0.031)***	94.842		82.449		76.395	
suzuki	0.934	(0.264)***	0.174	(0.033)***	24.460		21.166		18.709	
daihatsu	0.762	(0.324)**	0.371	(0.037)***	13.413		12.026		6.824	
lexus	1.954	(0.412)***	0.344	(0.043)***	4.735		3.279		1.487	
toyota	1.865	(0.298)***	0.378	(0.030)***	103.000		88.910		71.226	
audi	3.220	(0.392)***	0.513	(0.034)***	70.038		60.965		52.312	
seat	1.111	(0.309)***	0.209	(0.033)***	28.330		23.700		19.909	
skoda	1.342	(0.367)***	0.257	(0.040)***	22.504		19.295		16.480	
volkswagen	2.283	(0.347)***	0.334	(0.029)***	139.605		123.458		116.309	
Correlations (p-values) with brand equity:										
Pearson			0.671	(0.000)	0.295	(0.085)	0.300	(0.080)	0.307	(0.073)
Kendall			0.550	(0.000)	0.284	(0.016)	0.281	(0.018)	0.308	(0.009)

Notes: * significant at 10%; ** significant at 5%; *** significant at 1%. The number of observations is 1,355. The number of simulated consumers used for the aggregate moments is 2,209. Standard errors of estimates other than correlations are in parenthesis. Numbers are based on the estimates in Table 5. Profits are measured in € mln. Profit premium is calculated as profits minus counterfactual profits (setting brand to the minimum brand found).

Regarding the marginal cost brand effect γ_f, we consider three specifications. In specification (A) there is no brand intercept included in the marginal cost, which corresponds to the approach followed by Goldfarb et al. (2009). Specification (B) assumes no heterogeneity of the experience attribute effect on marginal cost, which implies perfect correlation between the brand equity β_f and γ_f, that is, $\gamma_f = \phi\beta_f$. The estimate of ϕ is shown in the lower part of Table 5. Specification (C) allows for full heterogeneity of the brand effect on marginal cost, which means that γ_f is allowed to be different for each brand f. The estimates of the constant and slope coefficients are presented in the lower part of Table 5 while those of the brand effects on marginal cost are presented in Table 6.

5.3 Results

The upper part of Table 5 contains the demand estimates obtained in the first stage. *Base coefficients* refer to estimates of the coefficients of observed demand characteristics and *Random coefficients* refer to the estimates of price over income as well as the random constant. We include a relatively large number of characteristics in order to exploit as much product-level variation as possible and to obtain more precise brand equity estimates. This also explains why we include a large number of characteristics and only a few random coefficients. Note that according to our simulations from Section 4, including more random coefficients does not have any qualitative implications for profit premiums estimates.

The base coefficient estimates have the expected signs. The constant is large and negative reflecting that only a small proportion of households buy a new car in a given year. The characteristics HP/weight, cruise control, kilometers per euro, and size affect utility positively and are statistically significant. Advertising expenditure has a positive significant effect on utility. The special car class dummy variables have effects with mixed signs on utility, which can be explained by the popularity of the respective class in comparison to a car with similar characteristics in the tiny class (the base group). Specifically, both family cars and sport cars generate lower marginal utility, SUV's are relatively popular, while the luxury car and MPV dummies do not affect utility significantly. The coefficient of price/income is estimated to be highly negative and statistically significant, which reflects the negative marginal utility of price. The random constant (i.e., the standard deviation parameter corresponding to the random constant parameter) estimate is not significantly different from zero.

The brand-specific intercept estimates presented in Table 6 are obtained by using Kia as the base brand for both the demand and supply sides.[12] Recall that we define brand equity as the brand-specific intercepts in the demand model. We note that most estimates of brand equity are significant at a 5% level. These estimates show several patterns. First, European luxury brands tend to have higher equities: German luxury brands (Porsche, Mercedes, BMW, Audi) tend to have the highest brand equities (above 3), followed by luxury brands from the United Kingdom (Jaguar, Land Rover). Second, American brands, including American luxury brands (e.g., Cadillac) tend to have low equities. Third, highly popular brands (Ford, Opel, Volkswagen, Toyota, Renault) have equities between 1.7 and 2.3. Overall, we believe these brand equity estimates are not unrealistic.

The lower part of Table 5 contains the marginal cost estimates obtained in the second stage. In order to capture brand-specific effects as precisely as possible, we include a large number of covariates in the marginal cost specification. The estimates across the three specifications differ to some extent. However, it is remarkable that apart from the sign of log(KM per liter), all estimates have the expected signs; in all three specifications the estimates suggest that characteristics that increase utility shift marginal costs upwards.[13] In specification (B) we find the effect of the brand equities (estimated in the first stage) on marginal cost to be positive and statistically significant. We can conclude from this that in the Dutch new car market, experience attributes increase marginal cost significantly. Specification (C) includes brand-specific intercepts. These estimates are statistically significant and positive, apart from that of Chevrolet. We also report the R^2's for the three specifications. These are rather high for all specifica-

[12]Borkovsky et al. (2017) use the outside alternative as the base brand. Since we do not specify the marginal cost of the outside alternative, we cannot use it as the base brand in the supply side.

[13]Berry et al. (1995) obtained a similar result with respect to fuel efficiency for their main specification. Their explanation is that fuel efficiency is positively correlated with sales, so if sales is negatively correlated with marginal costs (which happens if there are increasing returns to scale), the fuel efficiency parameter may be biased downward. By adding a log(sales) variable to one of their specifications in order to proxy for log(production), the sign on fuel efficiency got reversed. However, with our data this is not feasible because almost none of the cars is produced in the Netherlands, so domestic sales are not a good predictor of total production.

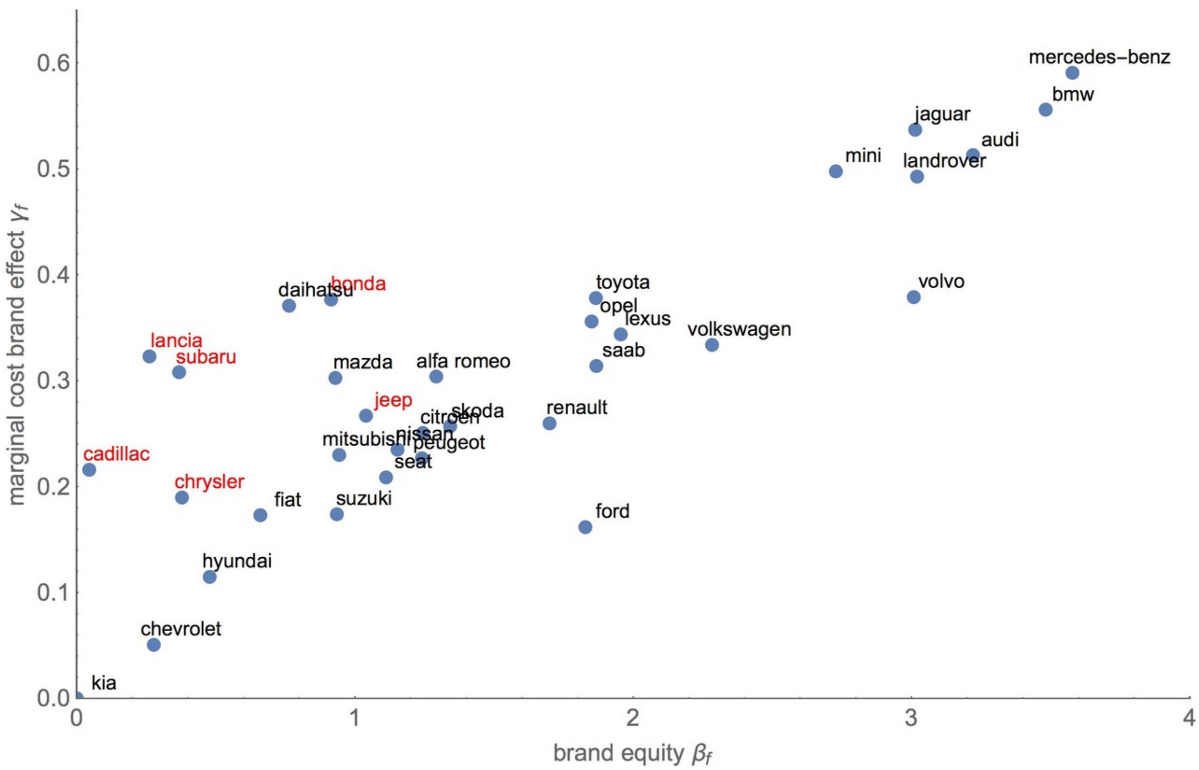

Figure 1. Brand equity and marginal cost brand effect

tions $(0.91 - 0.95)$, but specification (C) has a somewhat higher R^2 than the other two. This shows among others that brand-specific intercepts capture variation in marginal costs beyond that of brand equities.

The estimates of the brand-specific intercepts are related to brand equities. For example, luxury brands tend to have higher brand-specific marginal costs. The correlation coefficients between the brand equities and the marginal cost intercepts are reported at the bottom of Table 6 and confirm the strong positive relationship between these two variables. Both the Pearson and Kendall correlations are rather large and statistically significant (0.671 and 0.55, respectively, with both p-values equal to 0.000). This can be also seen in Figure 1, which shows how the marginal cost brand effect estimates relate to the brand equity estimates. It is important to mention that some brands with low equities have disproportionately high marginal cost brand-specific intercepts; an example of this is Cadillac. This is intuitively plausible: even though Cadillac is a luxury brand and therefore has a relatively high marginal cost brand-specific intercept due to the superior technology used, it is not very popular in the Netherlands, which manifests itself by a relatively low brand equity estimate.

The profit premium estimates based on data from 2008 are presented in Table 6. Those corresponding to specifications (A) and (B) are all positive, while some of them corresponding to specification (C) are negative. The profit premiums in different specifications are highly correlated with each other (Kendall correlations range between $0.86 - 0.98$; not shown in the tables), so they show strong similarities. According to all three specifications, highly popular brands like Volkswagen, Toyota, Ford, Opel, and Renault have the highest profit premiums, although their equities are clearly lower than those of the European luxury brands (Porsche, Mercedes, BMW, Audi, Jaguar, Land Rover). On the opposite side we have obtained the lowest profit premiums for brands with low equities like Chrysler, Lancia, Subaru, Cadillac, and Chevrolet, but also for some brands with relatively high equities like Smart and Jeep. Below we argue that these results can be explained by using the negative profit premiums obtained for specification (C) and our results from Section 3.1.

Although profit premiums corresponding to specification (C) highly correlate with those corresponding to the other two specifications, there are some notable differences. First, the former are systematically lower than the latter, which suggests that they are less optimistic than the other two. Second, as mentioned above, some profit premiums corresponding to specification (C) are negative (brands in red in Figure 1). Four out of these, namely, Chrysler, Lancia, Subaru, and Cadillac are among those brands mentioned above as having the lowest equities, while Jeep and Honda have relatively high brand equities. This suggests that brand equity alone cannot explain negative (or low) profit premium. According to our conclusion in Proposition 1, the magnitudes of the ratios γ_f/β_f of brand-specific intercepts corresponding to marginal cost and utility are expected to carry information on the sign of the profit premium. Despite their high brand equity, this ratio is rather high for Jeep and Honda, so the findings in Proposition 1 provide an explanation why their profit premiums are negative.

In order to investigate this issue statistically, we compare the Kendall correlations of profit premiums with brand equities to those with the ratios γ_f/β_f for specification (C). The lower part of Table 6 presents the former along with the Pearson correlations. Although none of the Pearson correlations is significant at the 5% level, the Kendall correlations are more relevant here, and they are statistically significant. The Kendall correlations between profit premiums and brand equity range between $0.281 - 0.308$, where the correlation corresponding to specification (C) is the highest. The correlation between profit premiums and the ratios γ_f/β_f for specification (C) is 0.486 (not presented in the table), which is clearly higher than the correlation 0.308 of the same profit premiums with brand equities. This suggests that the ratios γ_f/β_f of brand-specific intercepts are indeed more informative than brand equities regarding variation in profit premiums.

Among the brands with a negative profit premium, Honda deserves special attention because this case illustrates the advantages of including brand-specific intercepts in marginal cost when computing profit premiums, as done in specification (C). Indeed, we can notice that in terms of brand equity Honda is rather similar to Mazda and Mitsubishi while it has a brand intercept in marginal cost higher than the other two brands. The profit premiums computed for specification (A) are rather similar for these three brands, so they do not capture the differences in the marginal cost brand effects. The profit premiums corresponding to specification (C), however, are quite different. For example, the profit premium of Mitsubishi drops by less than 50% compared to that in specification (A) from 14.383 to 7.753, while that of Honda goes down by more than 100% from 16.348 to -2.543.

Consequently, the specific profit premiums computed for Honda and the other two brands illustrate the theoretical findings from Section 3. Specifically, the profit premium computed based on brand-specific effects in the marginal cost (specification (C)) allows for both positive and negative profit premiums, which may lead to the conclusion that it is not worth to invest in brand development. On the other hand, the profit premiums computed without taking brand-specific effects in marginal cost into account (specification (A)) are positive for all brands, and therefore, they only allow for the conclusion that it is profitable to invest in the brands.

6 Conclusions

This paper proposes a modification of the profit premium concept introduced by Goldfarb et al. (2009) that accounts for brand-specific features in the marginal costs of products. According to this modification the counterfactual profit of a brand is computed by depriving the brand of its specific features both in utility and marginal cost. This profit premium will generally have values lower than the one originally proposed by Goldfarb et al., and it could be negative for some brands, even when the brand-specific effects in the utility and marginal cost specifications are positive. We argue in the paper that this is neither an anomaly nor just a quantitative artifact, but it is a rather important feature that allows profit premium to signal situations in which it is not profitable to invest in brand development.

In order to elucidate this issue, we study the relationship between brand equity and profit premium in the framework of a static equilibrium model. Specifically, by employing the explicit demand and supply structure of the model, we study the monotonicity of profit premium in different situations. We derive comparative statics results for the simple logit model and verify them by Monte Carlo simulations for the random coefficient logit model. We have obtained that profit premium has a similar behavior irrespective of whether each product of a firm represents a different brand or all products of a firm belong to the same brand. Our results imply that the ratio of marginal cost and utility brand-specific intercepts is a factor that drives the behavior of profit premium. When this ratio is sufficiently high for a brand and the price coefficient is sufficiently high, the profit premium of the brand may be negative. The case when marginal costs do not contain brand-specific features represents the opposite extreme; in this case all profit premiums are positive. In an empirical study we compute profit premiums in the Dutch new car market. We have obtained negative profit premiums for 6 out of the 36 brands. The relationship between profit premiums and brand equities as well as the relationship between profit premiums and ratios of marginal cost and utility brand-specific intercepts confirm our theoretical findings.

A practical implication of our profit premium concept is that it can provide useful information about return on investment in brand equity. Specifically, a negative profit premium signals that brand equity development is a suboptimal strategy since in this case investment in branding—contrary to what the literature suggests—lowers the profitability of the brand. According to the equilibrium model employed in this paper, an increase in the equity of a brand implies changes in prices and market shares so that the profit of the brand decreases. This does not exclude the possibility of a short term increase in brand equity on its own due to past investment in the brand, or the possibility of enhancing brand equity through advertising, which does not affect marginal cost. In this way, a higher brand equity can reduce the ratio of brand-specific intercepts in marginal cost and utility, which is expected to improve the profitability of the brand. In the case of a brand with a negative profit premium such as Cadillac, which because of a lack of a core consumer base is underperforming in Europe, it would be optimal to execute advertising campaigns instead of developing experiential attributes until the brand becomes stronger.

Both our conceptual and empirical procedures are based on assumptions that may be restrictive in certain situations. First, in order for the comparative statics to be well defined we assume that brand equity is exogenous. This assumption is not realistic since brand equity is regularly monitored by firms and its value is influenced by various instruments like advertising. This implies a relationship between brand equity and advertising that is not taken into account by our model. Advertising in a market with so many brands as the car market is associated with limited consumer information that may have nontrivial implications on the brand equity estimates (Sovinsky Goeree 2008, Draganska and Klapper 2011). Second, we assume that brand equities and brand effects in marginal cost are constant over the sampling period. From a conceptual point of view this is not necessary, but we needed to adopt this assumption in the estimation due to the limitations of our data set. One could potentially use additional data (e.g., consumer-level data) or a more restrictive method of estimation (e.g., maximum likelihood) to estimate brand equities. Inter-temporal variation of brand equities can be modeled by taking depreciation and rebuilding through advertising into account in a dynamic framework (Borkovsky et al. 2017). Adapting our profit premium concept with the less restrictive model features mentioned above are tasks that we will take on in the future.

A Appendix

This Appendix contains technical results for deriving comparative statics of profit with respect to brand equity. **Notation**. Vectors appear as boldface characters; $\mathbf{v}'$ denotes the transpose of the vector $\mathbf{v}$. For $\mathbf{m} = (m_1, \ldots, m_F)'$ and $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_F)'$ we use the notation for the derivative

$$\frac{d\mathbf{m}}{d\boldsymbol{\beta}'} = \begin{pmatrix} \frac{dm_1}{d\beta_1} & \cdots & \frac{dm_1}{d\beta_F} \\ & \ddots & \\ \frac{dm_F}{d\beta_1} & \cdots & \frac{dm_F}{d\beta_F} \end{pmatrix}.$$

A.1 The case of firm-specific brands

The derivative of the profit $\pi_f = m_f \overline{s}_f$ of firm f with respect to brand equity β_f is

$$\frac{d\pi_f}{d\beta_f} = \frac{dm_f}{d\beta_f}\overline{s}_f + m_f \frac{d\overline{s}_f}{d\beta_f}. \tag{14}$$

In order to calculate this first we calculate $dm_f/d\beta_f$. Let $\mathbf{m} = (m_1, \ldots, m_F)'$ and $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_F)'$ It will be easier to compute first $d\mathbf{m}/d\boldsymbol{\beta}'$. We have

$$\frac{d\mathbf{m}}{d\boldsymbol{\beta}'} = \frac{d\mu(\mathbf{m}, \boldsymbol{\beta})}{d\boldsymbol{\beta}'} = \frac{\partial\mu(\mathbf{m}, \boldsymbol{\beta})}{\partial\mathbf{m}'}\frac{d\mathbf{m}}{d\boldsymbol{\beta}'} + \frac{\partial\mu(\mathbf{m}, \boldsymbol{\beta})}{\partial\boldsymbol{\beta}'},$$

where $\mu(\mathbf{m}, \boldsymbol{\beta}) = (\mu_1(\mathbf{m}, \boldsymbol{\beta}), \ldots, \mu_F(\mathbf{m}, \boldsymbol{\beta}))'$ with $\mu_f(\mathbf{m}, \boldsymbol{\beta}) = 1/[\alpha(1 - \overline{s}_f)]$. Here

$$\frac{\partial\mu(\mathbf{m}, \boldsymbol{\beta})}{\partial\mathbf{m}'} = \mathbf{a}\overline{\mathbf{s}}' - A \quad \text{and} \quad \frac{\partial\mu(\mathbf{m}, \boldsymbol{\beta})}{\partial\boldsymbol{\beta}'} = \frac{1}{\alpha}(B - \mathbf{b}\overline{\mathbf{s}}'),$$

where

$$\mathbf{a} = \left(\frac{\overline{s}_1}{(1-\overline{s}_1)^2}, \ldots, \frac{\overline{s}_F}{(1-\overline{s}_F)^2}\right)', \quad \overline{\mathbf{s}} = (\overline{s}_1, \ldots, \overline{s}_F)',$$

$$\mathbf{b} = \left(\frac{\rho_1\overline{s}_1}{(1-\overline{s}_1)^2}, \ldots, \frac{\rho_F\overline{s}_F}{(1-\overline{s}_F)^2}\right)', \quad \rho_f = 1 - \alpha\phi_f,$$

and A and B are the diagonal matrix with main diagonal equal to $\mathbf{a}$ and $\mathbf{b}$, respectively.

Therefore,

$$\frac{\partial\mathbf{m}}{\partial\boldsymbol{\beta}'} = \frac{1}{\alpha}(I_F + A - \mathbf{a}\overline{\mathbf{s}}')^{-1}(B - \mathbf{b}\overline{\mathbf{s}}'), \tag{15}$$

where I_F is the identity matrix of dimension F. We know that (Dhrymes 1984, p.40)

$$(I_F + A - \mathbf{a}\overline{\mathbf{s}}')^{-1} = (I_F + A)^{-1} + \frac{1}{1 - \overline{\mathbf{s}}'(I_F + A)^{-1}\mathbf{a}}(I_F + A)^{-1}\mathbf{a}\overline{\mathbf{s}}'(I_F + A)^{-1}, \tag{16}$$

where

$$\left(I_F+A\right)^{-1}=\begin{pmatrix} \frac{(1-\overline{s}_1)^2}{(1-\overline{s}_1)^2+\overline{s}_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{(1-\overline{s}_F)^2}{(1-\overline{s}_F)^2+\overline{s}_F} \end{pmatrix},$$

$$1-\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\mathbf{a}=1-\sum_{f=1}^{F}\frac{\overline{s}_f^2}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}>0,$$

$$\mathbf{a}\overline{\mathbf{s}}'=\begin{pmatrix} \frac{\overline{s}_1^2}{(1-\overline{s}_1)^2} & \cdots & \frac{\overline{s}_1\overline{s}_F}{(1-\overline{s}_1)^2} \\ \vdots & \ddots & \vdots \\ \frac{\overline{s}_1\overline{s}_F}{(1-\overline{s}_F)^2} & \cdots & \frac{\overline{s}_F^2}{(1-\overline{s}_F)^2} \end{pmatrix},$$

$$\left(I_F+A\right)^{-1}\mathbf{a}\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}=\begin{pmatrix} \frac{\overline{s}_1^2(1-\overline{s}_1)^2}{\left[(1-\overline{s}_1)^2+\overline{s}_1\right]^2} & \cdots & \frac{\overline{s}_1\overline{s}_F(1-\overline{s}_F)^2}{\left[(1-\overline{s}_1)^2+\overline{s}_1\right]\left[(1-\overline{s}_F)^2+\overline{s}_F\right]} \\ \vdots & \ddots & \vdots \\ \frac{\overline{s}_1\overline{s}_F(1-\overline{s}_1)^2}{\left[(1-\overline{s}_1)^2+\overline{s}_1\right]\left[(1-\overline{s}_F)^2+\overline{s}_F\right]} & \cdots & \frac{\overline{s}_F^2(1-\overline{s}_F)^2}{\left[(1-\overline{s}_F)^2+\overline{s}_F\right]^2} \end{pmatrix}.$$

Therefore,

$$\frac{dm_f}{d\beta_f}=\mathbf{e}_f'\frac{d\mathbf{m}}{d\boldsymbol{\beta}'}\mathbf{e}_f,$$

where $\mathbf{e}_f$ is an $F\times 1$ vector having components equal to 0 except for component f, which is 1. Using (15) and (16) we get

$$\begin{aligned}\frac{dm_f}{d\beta_f}&=\frac{1}{\alpha}\mathbf{e}_f'\left(\left(I_F+A\right)^{-1}+\frac{1}{1-\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\mathbf{a}}\left(I_F+A\right)^{-1}\mathbf{a}\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\right)\left(B-\mathbf{b}\overline{\mathbf{s}}'\right)\mathbf{e}_f\\&=\frac{1}{\alpha}\mathbf{e}_f'\left(I_F+A\right)^{-1}\left(B-\mathbf{b}\overline{\mathbf{s}}'\right)\mathbf{e}_f\\&+\frac{1}{\alpha}\frac{1}{1-\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\mathbf{a}}\mathbf{e}_f'\left(I_F+A\right)^{-1}\mathbf{a}\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\left(B-\mathbf{b}\overline{\mathbf{s}}'\right)\mathbf{e}_f.\end{aligned}\tag{17}$$

Note that

$$\mathbf{e}_f'\left(I_F+A\right)^{-1}=\begin{pmatrix} 0 & \ldots & \frac{(1-\overline{s}_f)^2}{(1-\overline{s}_f)^2+\overline{s}_f} & \ldots & 0\end{pmatrix},\tag{18}$$

$$\mathbf{e}_f'\left(I_F+A\right)^{-1}\mathbf{a}=\frac{\overline{s}_f}{\left(1-\overline{s}_f\right)^2+\overline{s}_f},\tag{19}$$

$$\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}=\begin{pmatrix} \frac{\overline{s}_1(1-\overline{s}_1)^2}{(1-\overline{s}_1)^2+\overline{s}_1} & \ldots & \frac{\overline{s}_F(1-\overline{s}_F)^2}{(1-\overline{s}_F)^2+\overline{s}_F}\end{pmatrix},\tag{20}$$

$$\left(B-\mathbf{b}\overline{\mathbf{s}}'\right)\mathbf{e}_f=\overline{s}_f\left(-\frac{\rho_1\overline{s}_1}{\left(1-\overline{s}_1\right)^2},\ldots,\frac{\rho_f}{1-\overline{s}_f},\ldots,-\frac{\rho_F\overline{s}_F}{\left(1-\overline{s}_F\right)^2}\right)',\tag{21}$$

$$\begin{aligned}\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\left(B-\mathbf{b}\overline{\mathbf{s}}'\right)\mathbf{e}_f&=\begin{pmatrix} \frac{\overline{s}_1(1-\overline{s}_1)^2}{(1-\overline{s}_1)^2+\overline{s}_1} & \ldots & \frac{\overline{s}_F(1-\overline{s}_F)^2}{(1-\overline{s}_F)^2+\overline{s}_F}\end{pmatrix}\\&\cdot\left(-\frac{\rho_1\overline{s}_1}{\left(1-\overline{s}_1\right)^2},\ldots,\frac{\rho_f}{1-\overline{s}_f},\ldots,-\frac{\rho_F\overline{s}_F}{\left(1-\overline{s}_F\right)^2}\right)'\\&=\overline{s}_f\left(\frac{\rho_f\overline{s}_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}-\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right).\end{aligned}\tag{22}$$

Collecting these expressions we obtain

$$\frac{dm_f}{d\beta_f} = \frac{\overline{s}_f}{\alpha D}\left(\rho_f\left(E-\overline{s}_f\right)+\overline{s}_f\sum_{g\neq f}\frac{\left(\rho_f-\rho_g\right)\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right). \tag{23}$$

This is because by (17) and straightforward calculations we obtain

$$\begin{aligned}
\frac{dm_f}{d\beta_f} &= \frac{1}{\alpha}\frac{\rho_f\overline{s}_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}+\frac{1}{\alpha}\frac{1}{1-\overline{s}'\left(I_F+A\right)^{-1}a}\frac{\overline{s}_f^2}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}\left(\frac{\rho_f\overline{s}_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}-\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)\\
&= \frac{\overline{s}_f}{\alpha}\left[\frac{\rho_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}+\frac{1}{1-\sum_{g=1}^F\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}}\frac{\overline{s}_f}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}\left(\frac{\rho_f\overline{s}_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}-\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)\right]\\
&= \frac{\overline{s}_f}{\alpha D}\left[\rho_f\left(1-\overline{s}_f\right)\left(1-\sum_{g=1}^F\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)+\overline{s}_f\left(\frac{\rho_f\overline{s}_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}-\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)\right]\\
&= \frac{\overline{s}_f}{\alpha D}\left[\rho_f\left(1-\overline{s}_f\right)\left(\frac{1-\overline{s}_f}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}-\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)\right.\\
&\quad\left.+\overline{s}_f\left(\frac{\rho_f\overline{s}_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}-\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)\right]
\end{aligned}$$

$$\begin{aligned}
&= \frac{\overline{s}_f}{\alpha D}\left[\frac{\rho_f\left(1-\overline{s}_f\right)^2}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}-\rho_f\left(1-\overline{s}_f\right)\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}+\frac{\rho_f\overline{s}_f^2\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}-\overline{s}_f\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right]\\
&= \frac{\overline{s}_f}{\alpha D}\left(\rho_f\left(1-\overline{s}_f\right)-\rho_f\left(1-\overline{s}_f\right)\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}-\overline{s}_f\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)\\
&= \frac{\overline{s}_f}{\alpha D}\left(\rho_f\left(E-\overline{s}_f\right)+\rho_f\overline{s}_f\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}-\overline{s}_f\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)\\
&= \frac{\overline{s}_f}{\alpha D}\left(\rho_f\left(E-\overline{s}_f\right)+\overline{s}_f\sum_{g\neq f}\frac{\left(\rho_f-\rho_g\right)\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right).
\end{aligned}$$

We turn now to the derivation of $d\overline{s}_f/d\beta_f$. We start with the derivative of (6), that is,

$$\frac{d\overline{s}_f}{d\beta_f} = \overline{s}_f\left[\rho_f\left(1-\overline{s}_f\right)-\alpha\frac{dm_f}{d\beta_f}\left(1-\overline{s}_f\right)+\alpha\sum_{g\neq f}\frac{dm_g}{d\beta_f}\overline{s}_g\right]. \tag{24}$$

The (g,f) off-diagonal element of the matrix $d\mathbf{m}/d\boldsymbol{\beta}'$ is

$$\begin{aligned}
\frac{dm_g}{d\beta_f} &= \mathbf{e}_g'\frac{d\mathbf{m}}{d\boldsymbol{\beta}'}\mathbf{e}_f\\
&= \frac{1}{\alpha}\mathbf{e}_g'\left(I_F+A\right)^{-1}\left(B-\mathbf{b}\overline{\mathbf{s}}'\right)\mathbf{e}_f+\frac{1}{\alpha}\frac{1}{1-\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\mathbf{a}}\mathbf{e}_g'\left(I_F+A\right)^{-1}\mathbf{a}\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\left(B-\mathbf{b}\overline{\mathbf{s}}'\right)\mathbf{e}_f.
\end{aligned}$$

Using the formulas (18)-(22) we obtain

$$\begin{aligned}
\frac{dm_g}{d\beta_f} &= -\frac{1}{\alpha}\frac{\rho_g \overline{s}_f \overline{s}_g}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} + \frac{1}{\alpha}\frac{1}{1-\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\mathbf{a}}\frac{\overline{s}_f\overline{s}_g}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\left(\frac{\rho_f\overline{s}_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2+\overline{s}_f} - \sum_{h\neq f}\frac{\rho_h\overline{s}_h^2}{\left(1-\overline{s}_h\right)^2+\overline{s}_h}\right) \\
&= \frac{\overline{s}_f\overline{s}_g}{\alpha\left[\left(1-\overline{s}_g\right)^2+\overline{s}_g\right]}\left(-\rho_g + \frac{1}{1-\sum_{h=1}^{F}\frac{\overline{s}_h^2}{\left(1-\overline{s}_h\right)^2+\overline{s}_h}}\left(\frac{\rho_f\overline{s}_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2+\overline{s}_f} - \sum_{h\neq f}\frac{\rho_h\overline{s}_h^2}{\left(1-\overline{s}_h\right)^2+\overline{s}_h}\right)\right) \\
&= \frac{\overline{s}_f\overline{s}_g}{\alpha\left[\left(1-\overline{s}_g\right)^2+\overline{s}_g\right]D}\left(\rho_f\overline{s}_f\left(1-\overline{s}_f\right) - \rho_g D - \left[\left(1-\overline{s}_f\right)^2+\overline{s}_f\right]\sum_{h\neq f}\frac{\rho_h\overline{s}_h^2}{\left(1-\overline{s}_h\right)^2+\overline{s}_h}\right). \qquad (25)
\end{aligned}$$

Further, from (25) we get

$$\begin{aligned}
\sum_{g\neq f}\frac{dm_g}{d\beta_f}\overline{s}_g &= \sum_{g\neq f}\frac{\overline{s}_f\overline{s}_g^2}{\alpha\left[\left(1-\overline{s}_g\right)^2+\overline{s}_g\right]D}\left(\rho_f\overline{s}_f\left(1-\overline{s}_f\right) - \rho_g D - \left[\left(1-\overline{s}_f\right)^2+\overline{s}_f\right]\sum_{h\neq f}\frac{\rho_h\overline{s}_h^2}{\left(1-\overline{s}_h\right)^2+\overline{s}_h}\right) \\
&= \frac{\overline{s}_f}{\alpha D}\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\left(\rho_f\overline{s}_f\left(1-\overline{s}_f\right) - \rho_g D - \left[\left(1-\overline{s}_f\right)^2+\overline{s}_f\right]\sum_{h\neq f}\frac{\rho_h\overline{s}_h^2}{\left(1-\overline{s}_h\right)^2+\overline{s}_h}\right) \\
&= \frac{\overline{s}_f}{\alpha D}\left(\rho_f\overline{s}_f\left(1-\overline{s}_f\right) - \left[\left(1-\overline{s}_f\right)^2+\overline{s}_f\right]\sum_{h\neq f}\frac{\rho_h\overline{s}_h^2}{\left(1-\overline{s}_h\right)^2+\overline{s}_h}\right)\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} \\
&\quad - \frac{\overline{s}_f}{\alpha}\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} \\
&= \frac{\overline{s}_f}{\alpha D}\left(\rho_f\overline{s}_f\left(1-\overline{s}_f\right)\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} - \left[\left(1-\overline{s}_f\right)^2+\overline{s}_f\right]\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\sum_{h\neq f}\frac{\rho_h\overline{s}_h^2}{\left(1-\overline{s}_h\right)^2+\overline{s}_h}\right) \\
&\quad - \frac{\overline{s}_f}{\alpha}\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}. \qquad (26)
\end{aligned}$$

Note that

$$\left[\left(1-\overline{s}_f\right)^2+\overline{s}_f\right]\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} = 1-\overline{s}_f-D,$$

so by further straightforward calculations we obtain that

$$\begin{aligned}
\sum_{g\neq f}\frac{dm_g}{d\beta_f}\overline{s}_g &= \frac{\overline{s}_f}{\alpha D}\left(\rho_f\overline{s}_f\left(1-\overline{s}_f\right)\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} - \left(1-\overline{s}_f-D\right)\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right) \\
&\quad - \frac{\overline{s}_f}{\alpha}\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}
\end{aligned}$$

$$= \frac{\rho_f \overline{s}_f^2 (1-\overline{s}_f)}{\alpha D} \sum_{g \neq f} \frac{\overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} - \frac{\overline{s}_f}{\alpha D} (1 - \overline{s}_f - D) \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g}$$
$$- \frac{\overline{s}_f}{\alpha} \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g}$$
$$= \frac{\rho_f \overline{s}_f^2 (1-\overline{s}_f)}{\alpha D} \sum_{g \neq f} \frac{\overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} - \frac{\overline{s}_f (1-\overline{s}_f)}{\alpha D} \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g}$$
$$= \frac{\overline{s}_f (1-\overline{s}_f)}{\alpha D} \left(\rho_f \overline{s}_f \sum_{g \neq f} \frac{\overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} - \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} \right).$$

Substituting this and (23) into (24), we obtain

$$\frac{d\overline{s}_f}{d\beta_f} = \overline{s}_f \left[\rho_f (1-\overline{s}_f) - \frac{\overline{s}_f (1-\overline{s}_f)}{D} \left(\rho_f (E - \overline{s}_f) + \overline{s}_f \sum_{g \neq f} \frac{(\rho_f - \rho_g) \overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} \right) \right.$$
$$\left. + \frac{\overline{s}_f (1-\overline{s}_f)}{D} \left(\rho_f \overline{s}_f \sum_{g \neq f} \frac{\overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} - \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} \right) \right]$$
$$= \frac{\overline{s}_f (1-\overline{s}_f)}{D}$$
$$\times \left[\rho_f D - \overline{s}_f \left(\rho_f (E - \overline{s}_f) + \overline{s}_f \sum_{g \neq f} \frac{(\rho_f - \rho_g) \overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} - \rho_f \overline{s}_f \sum_{g \neq f} \frac{\overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} + \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} \right) \right]$$
$$= \frac{\overline{s}_f (1-\overline{s}_f)}{D} \left[\rho_f D - \overline{s}_f \left(\rho_f (E - \overline{s}_f) - \overline{s}_f \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} + \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} \right) \right]$$
$$= \frac{\overline{s}_f (1-\overline{s}_f)}{D} \left[\rho_f D - \overline{s}_f \left(\rho_f (E - \overline{s}_f) + (1-\overline{s}_f) \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} \right) \right]$$
$$= \frac{\overline{s}_f (1-\overline{s}_f)}{D} \left[\rho_f D - \rho_f \overline{s}_f (E - \overline{s}_f) - \overline{s}_f (1-\overline{s}_f) \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} \right]$$
$$= \frac{\overline{s}_f (1-\overline{s}_f)}{D} \left[\rho_f [D - \overline{s}_f (E - \overline{s}_f)] - \overline{s}_f (1-\overline{s}_f) \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} \right].$$

Now we use

$$D - \overline{s}_f (E - \overline{s}_f) = \left[(1-\overline{s}_f)^2 + \overline{s}_f \right] E - \overline{s}_f^2 - \overline{s}_f E + \overline{s}_f^2 = (1-\overline{s}_f)^2 E, \tag{27}$$

so

$$\frac{d\overline{s}_f}{d\beta_f} = \frac{\overline{s}_f (1-\overline{s}_f)}{D} \left(\rho_f (1-\overline{s}_f)^2 E - (1-\overline{s}_f) \overline{s}_f \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} \right)$$
$$= \frac{\overline{s}_f (1-\overline{s}_f)^2}{D} \left(\rho_f (1-\overline{s}_f) E - \overline{s}_f \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} \right)$$
$$= \frac{\overline{s}_f (1-\overline{s}_f)^2}{D} \left(\rho_f (1-\overline{s}_f) E - \overline{s}_f \sum_{g \neq f} \frac{\rho_f \overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} + \overline{s}_f \sum_{g \neq f} \frac{\rho_f \overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} - \overline{s}_f \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} \right)$$

$$= \frac{\overline{s}_f\left(1-\overline{s}_f\right)^2}{D}\left(\rho_f\left(1-\overline{s}_f\right)E-\rho_f\overline{s}_f\left(1-E\right)+\overline{s}_f\sum_{g\neq f}\frac{\rho_f\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}-\overline{s}_f\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)$$

$$= \frac{\overline{s}_f\left(1-\overline{s}_f\right)^2}{D}\left(\rho_f\left(E-\overline{s}_f\right)+\overline{s}_f\sum_{g\neq f}\frac{\left(\rho_f-\rho_g\right)\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right). \tag{28}$$

Substituting the derivatives from (23) and (28) and the equilibrium markup expression (5) we get

$$\frac{d\pi_f}{d\beta_f}=\frac{\overline{s}_f}{\alpha D}\left(\rho_f\left(E-\overline{s}_f\right)+\overline{s}_f\sum_{g\neq f}\frac{\left(\rho_f-\rho_g\right)\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right). \tag{29}$$

A.2 The case of product-specific brands

The derivative of the profit of firm f with respect to the brand equity β_j of product j that belongs to firm f is

$$\frac{d\pi_f}{d\beta_j}=\frac{dm_f}{d\beta_j}\overline{s}_f+m_f\frac{d\overline{s}_f}{d\beta_j}.$$

so we need to calculate $\partial m_f/\partial\beta_j$ and $\partial\overline{s}_f/\partial\beta_j$. The former will follow from $\partial\mathbf{m}/\partial\boldsymbol{\beta}'$, which is equal to

$$\frac{d\mathbf{m}}{d\boldsymbol{\beta}'}=\frac{d\mu\left(\mathbf{m},\boldsymbol{\beta}\right)}{d\boldsymbol{\beta}'}=\frac{\partial\mu\left(\mathbf{m},\boldsymbol{\beta}\right)}{\partial\mathbf{m}'}\frac{d\mathbf{m}}{d\boldsymbol{\beta}'}+\frac{\partial\mu\left(\mathbf{m},\boldsymbol{\beta}\right)}{\partial\boldsymbol{\beta}'},$$

where $\mu\left(\mathbf{m},\boldsymbol{\beta}\right)=\left(\mu_1\left(\mathbf{m},\boldsymbol{\beta}\right),\ldots,\mu_F\left(\mathbf{m},\boldsymbol{\beta}\right)\right)'$ with $\mu_f\left(\mathbf{m},\boldsymbol{\beta}\right)=1/\left[\alpha\left(1-\overline{s}_f\right)\right]$. Here

$$\frac{\partial\mu\left(\mathbf{m},\boldsymbol{\beta}\right)}{\partial\mathbf{m}'}=\mathbf{a}\overline{\mathbf{s}}'-A\quad\text{and}\quad\frac{\partial\mu\left(\mathbf{m},\boldsymbol{\beta}\right)}{\partial\boldsymbol{\beta}'}=\frac{\rho}{\alpha}\left(B-\mathbf{a}\mathbf{s}'\right),$$

where

$$\mathbf{a}=\left(\frac{\overline{s}_1}{\left(1-\overline{s}_1\right)^2},\ldots,\frac{\overline{s}_F}{\left(1-\overline{s}_F\right)^2}\right)',\quad\mathbf{s}=\left(s_1,\ldots,s_J\right)',$$

$$B=\begin{pmatrix}\frac{\mathbf{s}_1'}{\left(1-\overline{s}_1\right)^2} & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \frac{\mathbf{s}_F'}{\left(1-\overline{s}_F\right)^2}\end{pmatrix}.$$

Therefore,

$$\frac{d\mathbf{m}}{d\boldsymbol{\beta}'}=\frac{\rho}{\alpha}\left(I_F+A-\mathbf{a}\overline{\mathbf{s}}'\right)^{-1}\left(B-\mathbf{a}\mathbf{s}'\right),$$

where I_F is the identity matrix of dimension F. We know from Dhrymes (1984) that

$$\left(I_F+A-\mathbf{a}\overline{\mathbf{s}}'\right)^{-1}=\left(I_F+A\right)^{-1}+\frac{1}{1-\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\mathbf{a}}\left(I_F+A\right)^{-1}\mathbf{a}\overline{\mathbf{s}}'\left(I_F+A\right)^{-1},$$

so

$$\frac{d\mathbf{m}}{d\boldsymbol{\beta}'}=\frac{\rho}{\alpha}\left(\left(I_F+A\right)^{-1}\left(B-\mathbf{a}\mathbf{s}'\right)+\frac{1}{1-\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\mathbf{a}}\left(I_F+A\right)^{-1}\mathbf{a}\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\left(B-\mathbf{a}\mathbf{s}'\right)\right). \tag{30}$$

Element (f,j) of the matrix $d\mathbf{m}/d\boldsymbol{\beta}'$ for $j\in\mathcal{G}_f$ is

$$\frac{dm_f}{d\beta_j}=\frac{\rho}{\alpha}\left(\mathbf{e}_f'\left(I_F+A\right)^{-1}\left(B-\mathbf{a}\mathbf{s}'\right)\mathbf{e}_j+\frac{1}{1-\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\mathbf{a}}\mathbf{e}_f'\left(I_F+A\right)^{-1}\mathbf{a}\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\left(B-\mathbf{a}\mathbf{s}'\right)\mathbf{e}_j\right),$$

where $\mathbf{e}_f$ and $\mathbf{e}_j$ are column vectors of conformable size having 1 on position f and j, respectively, and 0 elsewhere.

It holds that

$$\begin{aligned}
\mathbf{e}_f' \left(I_F + A\right)^{-1} &= \begin{pmatrix} 0 & \dots & \frac{(1-\overline{s}_f)^2}{(1-\overline{s}_f)^2+\overline{s}_f} & \dots & 0 \end{pmatrix}, \\
\mathbf{e}_f' \left(I_F + A\right)^{-1} \mathbf{a} &= \frac{\overline{s}_f}{\left(1-\overline{s}_f\right)^2 + \overline{s}_f}, \\
\overline{\mathbf{s}}' \left(I_F + A\right)^{-1} &= \begin{pmatrix} \frac{\overline{s}_1(1-\overline{s}_1)^2}{(1-\overline{s}_1)^2+\overline{s}_1} & \dots & \frac{\overline{s}_F(1-\overline{s}_F)^2}{(1-\overline{s}_F)^2+\overline{s}_F} \end{pmatrix}, \\
\left(B - \mathbf{a}\mathbf{s}'\right) \mathbf{e}_j &= s_j \begin{pmatrix} -\frac{\overline{s}_1}{(1-\overline{s}_1)^2} & \dots & \frac{1}{1-\overline{s}_f} & \dots & -\frac{\overline{s}_F}{(1-\overline{s}_F)^2} \end{pmatrix}', \\
\overline{\mathbf{s}}' \left(I_F + A\right)^{-1} \left(B - \mathbf{a}\mathbf{s}'\right) \mathbf{e}_j &= s_j \left(\frac{\overline{s}_f \left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2 + \overline{s}_f} - \sum_{g \neq f} \frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2 + \overline{s}_g} \right).
\end{aligned}$$

So

$$\frac{dm_f}{d\beta_j} = \frac{\rho s_j}{\alpha D} \left(E - \overline{s}_f\right). \tag{31}$$

This is because

$$\begin{aligned}
\frac{dm_f}{d\beta_j} &= \frac{\rho}{\alpha} \left(\frac{s_j \left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2 + \overline{s}_f} + \frac{\frac{s_j \overline{s}_f}{(1-\overline{s}_f)^2+\overline{s}_f} \left(\frac{\overline{s}_f(1-\overline{s}_f)}{(1-\overline{s}_f)^2+\overline{s}_f} - \sum_{g\neq f} \frac{\overline{s}_g^2}{(1-\overline{s}_g)^2+\overline{s}_g} \right)}{1 - \sum_{g=1}^F \frac{\overline{s}_g^2}{(1-\overline{s}_g)^2+\overline{s}_g}} \right) \\
&= \frac{\rho}{\alpha} \frac{s_j}{\left(1-\overline{s}_f\right)^2 + \overline{s}_f} \left(\left(1-\overline{s}_f\right) + \frac{\overline{s}_f \left(\frac{\overline{s}_f(1-\overline{s}_f)}{(1-\overline{s}_f)^2+\overline{s}_f} - \sum_{g\neq f} \frac{\overline{s}_g^2}{(1-\overline{s}_g)^2+\overline{s}_g} \right)}{1 - \sum_{g=1}^F \frac{\overline{s}_g^2}{(1-\overline{s}_g)^2+\overline{s}_g}} \right) \\
&= \frac{\rho s_j}{\alpha D} \left[\left(1-\overline{s}_f\right) \left(E - \frac{\overline{s}_f^2}{\left(1-\overline{s}_f\right)^2 + \overline{s}_f} \right) + \overline{s}_f \left(\frac{\overline{s}_f \left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2 + \overline{s}_f} - \sum_{g\neq f} \frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2 + \overline{s}_g} \right) \right] \\
&= \frac{\rho s_j}{\alpha D} \left[\left(1-\overline{s}_f\right) \left(E - \frac{\overline{s}_f^2}{\left(1-\overline{s}_f\right)^2 + \overline{s}_f} \right) + \overline{s}_f \left(\frac{\overline{s}_f \left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2 + \overline{s}_f} - 1 + E \right) \right] \\
&= \frac{\rho s_j}{\alpha D} \left(E - \overline{s}_f\right),
\end{aligned}$$

where for the third equality we use (8).

In order to compute $\partial \overline{s}_f / \partial \beta_j$ we compute $\partial s_r / \partial \beta_j$ for $r = j$ and $r \neq j$. The derivative with respect to β_j of the market share s_r of product r different from j is

$$\begin{aligned}
\frac{ds_r}{d\beta_j} &= -s_r \left(\rho s_j + \alpha \frac{\partial m_f}{\partial \beta_j} \left(1-\overline{s}_f\right) - \alpha \sum_{g\neq f} \frac{\partial m_g}{\partial \beta_j} \overline{s}_g \right) \\
&= -s_r \left(\rho s_j + \frac{\rho}{D} s_j \left(E - \overline{s}_f\right) \left(1-\overline{s}_f\right) + \frac{\rho s_j \left(1-\overline{s}_f\right)^2}{D} \sum_{g\neq f} \frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2 + \overline{s}_g} \right) \\
&= -\frac{\rho s_j s_r}{D} \left(D + \left(E - \overline{s}_f\right)\left(1-\overline{s}_f\right) + \left(1-\overline{s}_f\right)^2 \left(1-E\right) \right) \\
&= -\frac{\rho s_j s_r}{D} \left(\overline{s}_f \left(E - \overline{s}_f\right) + \left(1-\overline{s}_f\right)^2 E + \left(E - \overline{s}_f\right)\left(1-\overline{s}_f\right) + \left(1-\overline{s}_f\right)^2 \left(1-E\right) \right) \\
&= -\frac{\rho s_j s_r}{D} \left(E - \overline{s}_f + \left(1-\overline{s}_f\right)^2 \right).
\end{aligned} \tag{32}$$

The derivative of the market share s_j from (11) with respect to its own brand equity β_j is

$$\frac{ds_j}{d\beta_j} = s_j \left(\rho \left(1 - s_j\right) - \alpha \frac{dm_f}{d\beta_j} \left(1 - \overline{s}_f\right) + \alpha \sum_{g \neq f} \frac{dm_g}{d\beta_j} \overline{s}_g \right). \tag{33}$$

The derivative $dm_g/d\beta_j$ is element (g, j) of the matrix $\partial m/\partial \boldsymbol{\beta}'$ for $j \in \mathcal{G}_f$ and $g \neq f$, which by (30) is

$$\frac{dm_g}{d\beta_j} = \frac{\rho}{\alpha} \left(\mathbf{e}_g' \left(I_F + A\right)^{-1} \left(B - \mathbf{a}\mathbf{s}'\right) \mathbf{e}_j + \frac{1}{1 - \overline{\mathbf{s}}' \left(I_F + A\right)^{-1} \mathbf{a}} \mathbf{e}_g' \left(I_F + A\right)^{-1} \mathbf{a}\overline{\mathbf{s}}' \left(I_F + A\right)^{-1} \left(B - \mathbf{a}\mathbf{s}'\right) \mathbf{e}_j \right),$$

where

$$\begin{aligned}
\mathbf{e}_g' \left(I_F + A\right)^{-1} &= \begin{pmatrix} 0 & \dots & \frac{(1-\overline{s}_g)^2}{(1-\overline{s}_g)^2+\overline{s}_g} & \dots & 0 \end{pmatrix}, \\
\mathbf{e}_g' \left(I_F + A\right)^{-1} \mathbf{a} &= \frac{\overline{s}_g}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g}, \\
\overline{\mathbf{s}}' \left(I_F + A\right)^{-1} &= \begin{pmatrix} \frac{\overline{s}_1(1-\overline{s}_1)^2}{(1-\overline{s}_1)^2+\overline{s}_1} & \dots & \frac{\overline{s}_F(1-\overline{s}_F)^2}{(1-\overline{s}_F)^2+\overline{s}_F} \end{pmatrix}, \\
\left(B - \mathbf{a}\mathbf{s}'\right) \mathbf{e}_j &= s_j \begin{pmatrix} -\frac{\overline{s}_1}{(1-\overline{s}_1)^2} & \dots & \frac{1}{1-\overline{s}_f} & \dots & -\frac{\overline{s}_F}{(1-\overline{s}_F)^2} \end{pmatrix}', \\
\overline{\mathbf{s}}' \left(I_F + A\right)^{-1} \left(B - \mathbf{a}\mathbf{s}'\right) \mathbf{e}_j &= s_j \left(\frac{\overline{s}_f}{\left(1 - \overline{s}_f\right)^2 + \overline{s}_f} - \sum_{h=1}^{F} \frac{\overline{s}_h^2}{\left(1 - \overline{s}_h\right)^2 + \overline{s}_h} \right).
\end{aligned}$$

Substituting these, we get

$$\frac{dm_g}{d\beta_j} = -\frac{\rho s_j}{\alpha} \frac{\overline{s}_g}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g} \frac{\left(1 - \overline{s}_f\right)^2}{D}. \tag{34}$$

This is because

$$\begin{aligned}
\frac{dm_g}{d\beta_j} &= \frac{\rho}{\alpha} \left(-s_j \frac{\overline{s}_g}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g} + \frac{\frac{\overline{s}_g}{(1-\overline{s}_g)^2+\overline{s}_g} s_j \left(\frac{\overline{s}_f}{(1-\overline{s}_f)^2+\overline{s}_f} - \sum_{h=1}^F \frac{\overline{s}_h^2}{(1-\overline{s}_h)^2+\overline{s}_h} \right)}{1 - \sum_{h=1}^F \frac{\overline{s}_h^2}{(1-\overline{s}_h)^2+\overline{s}_h}} \right) \\
&= \frac{\rho s_j}{\alpha} \frac{\overline{s}_g}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g} \left(-1 + \frac{\frac{\overline{s}_f}{(1-\overline{s}_f)^2+\overline{s}_f} - \sum_{h=1}^F \frac{\overline{s}_h^2}{(1-\overline{s}_h)^2+\overline{s}_h}}{1 - \sum_{h=1}^F \frac{\overline{s}_h^2}{(1-\overline{s}_h)^2+\overline{s}_h}} \right) \\
&= \frac{\rho s_j}{\alpha} \frac{\overline{s}_g}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g} \left(\frac{\frac{\overline{s}_f}{(1-\overline{s}_f)^2+\overline{s}_f} - \sum_{h=1}^F \frac{\overline{s}_h^2}{(1-\overline{s}_h)^2+\overline{s}_h} - 1 + \sum_{h=1}^F \frac{\overline{s}_h^2}{(1-\overline{s}_h)^2+\overline{s}_h}}{1 - \sum_{h=1}^F \frac{\overline{s}_h^2}{(1-\overline{s}_h)^2+\overline{s}_h}} \right) \\
&= \frac{\rho s_j}{\alpha} \frac{\overline{s}_g}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g} \left(\frac{\frac{\overline{s}_f}{(1-\overline{s}_f)^2+\overline{s}_f} - 1}{1 - \sum_{h=1}^F \frac{\overline{s}_h^2}{(1-\overline{s}_h)^2+\overline{s}_h}} \right) \\
&= -\frac{\rho s_j}{\alpha} \frac{\overline{s}_g}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g} \frac{\left(1 - \overline{s}_f\right)^2}{D}.
\end{aligned}$$

Substituting the derivatives (34) and (31) into (33), we get

$$\begin{aligned}
\frac{ds_j}{d\beta_j} &= s_j \left(\rho \left(1 - s_j\right) - \frac{\rho}{D} s_j \left(E - \overline{s}_f\right)\left(1 - \overline{s}_f\right) - \frac{\rho s_j \left(1 - \overline{s}_f\right)^2}{D} \sum_{g \neq f} \frac{\overline{s}_g^2}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g} \right) \\
&= \frac{\rho s_j}{D} \Big(\left(1 - s_j\right) D - s_j \left(E - \overline{s}_f\right)\left(1 - \overline{s}_f\right) - s_j \left(1 - \overline{s}_f\right)^2 \left(1 - E\right) \Big) \\
&= \frac{\rho s_j}{D} \Big(D - s_j \left[\left(E - \overline{s}_f\right)\left(1 - \overline{s}_f\right) + D \right] - s_j \left(1 - \overline{s}_f\right)^2 \left(1 - E\right) \Big) \\
&= \frac{\rho s_j}{D} \Big(D - s_j \Big[\left(E - \overline{s}_f\right)\left(1 - \overline{s}_f\right) + \overline{s}_f \left(E - \overline{s}_f\right) + \left(1 - \overline{s}_f\right)^2 E + \left(1 - \overline{s}_f\right)^2 \left(1 - E\right) \Big] \Big) \\
&= \frac{\rho s_j}{D} \Big(\overline{s}_f \left(E - \overline{s}_f\right) + \left(1 - \overline{s}_f\right)^2 E - s_j \Big[E - \overline{s}_f + \left(1 - \overline{s}_f\right)^2 \Big] \Big) \\
&= \frac{\rho s_j}{D} \Big(\left(\overline{s}_f - s_j\right)\left(E - \overline{s}_f\right) + \left(1 - \overline{s}_f\right)^2 \left(E - s_j\right) \Big) . \qquad (35)
\end{aligned}$$

The derivative of the market share of firm f (12) with respect to the brand equity β_j of product j is

$$\frac{d\overline{s}_f}{d\beta_j} = \rho s_j \left(1 - \overline{s}_f\right) - \alpha \overline{s}_f \left(\frac{dm_f}{d\beta_j} - \sum_{g=1}^{F} \frac{dm_g}{d\beta_j} \overline{s}_g \right).$$

Substituting the derivatives (34) and (31), we get

$$\begin{aligned}
\frac{d\overline{s}_f}{d\beta_j} &= \rho s_j \left(1 - \overline{s}_f\right) - \alpha \overline{s}_f \frac{dm_f}{d\beta_j} \left(1 - \overline{s}_f\right) + \alpha \overline{s}_f \sum_{g \neq f} \frac{dm_g}{d\beta_j} \overline{s}_g \\
&= \rho s_j \left(1 - \overline{s}_f\right) - \overline{s}_f \rho \frac{s_j}{D} \left(E - \overline{s}_f\right)\left(1 - \overline{s}_f\right) - \rho s_j \overline{s}_f \frac{\left(1 - \overline{s}_f\right)^2}{D} \sum_{g \neq f} \frac{\overline{s}_g^2}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g} \\
&= \frac{\rho s_j \left(1 - \overline{s}_f\right)}{D} \left(D - \overline{s}_f \left(E - \overline{s}_f\right) - \overline{s}_f \left(1 - \overline{s}_f\right)\left(1 - E\right) \right) \\
&= \frac{\rho s_j \left(1 - \overline{s}_f\right)}{D} \Big(\left(1 - \overline{s}_f\right)^2 E - \overline{s}_f \left(1 - \overline{s}_f\right)\left(1 - E\right) \Big) \\
&= \frac{\rho s_j \left(1 - \overline{s}_f\right)^2}{D} \left(E - \overline{s}_f\right), \qquad (36)
\end{aligned}$$

where for the fourth equality we use (27).

Substituting the derivatives from (31) and (36) as well as the markup expression (5), we obtain

$$\frac{d\pi_f}{d\beta_j} = \frac{\rho s_j}{\alpha D} \left(E - \overline{s}_f\right) \overline{s}_f + \frac{1}{\alpha} \frac{1}{1 - \overline{s}_f} \frac{\rho s_j \left(1 - \overline{s}_f\right)^2}{D} \left(E - \overline{s}_f\right) = \frac{\rho s_j \left(E - \overline{s}_f\right)}{\alpha D}. \qquad (37)$$

The derivative of the profit from product j with respect to the brand equity β_j is

$$\frac{d\pi_j}{d\beta_j} = \frac{dm_f}{d\beta_j} s_j + m_f \frac{ds_j}{d\beta_j}.$$

Substituting (31), (35) and (5), we obtain

$$\begin{aligned}
\frac{d\pi_j}{d\beta_j} &= \frac{\rho s_j}{\alpha D}\left[\left(E-\overline{s}_f\right)s_j + \frac{1}{1-\overline{s}_f}\left(\left(\overline{s}_f - s_j\right)\left(E-\overline{s}_f\right) + \left(1-\overline{s}_f\right)^2\left(E-s_j\right)\right)\right] \\
&= \frac{\rho s_j}{\alpha D}\left[\left(E-\overline{s}_f\right)s_j + \frac{\left(\overline{s}_f - s_j\right)\left(E-\overline{s}_f\right)}{1-\overline{s}_f} + \left(1-\overline{s}_f\right)\left(E-s_j\right)\right] \\
&= \frac{\rho s_j}{\alpha D}\left[\frac{\left(1-\overline{s}_f\right)\left(E-\overline{s}_f\right)s_j + \left(\overline{s}_f - s_j\right)\left(E-\overline{s}_f\right)}{1-\overline{s}_f} + \left(1-\overline{s}_f\right)\left(E-s_j\right)\right] \\
&= \frac{\rho s_j}{\alpha D}\left[\frac{E-\overline{s}_f}{1-\overline{s}_f}\left(\left(1-\overline{s}_f\right)s_j + \left(\overline{s}_f - s_j\right)\right) + \left(1-\overline{s}_f\right)\left(E-s_j\right)\right] \\
&= \frac{\rho s_j}{\alpha D}\left[\frac{1-s_j}{1-\overline{s}_f}\overline{s}_f\left(E-\overline{s}_f\right) + \left(1-\overline{s}_f\right)\left(E-s_j\right)\right]. \qquad (38)
\end{aligned}$$

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