

Best foot forward or best for last in a sequential auction?

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Should a seller with private information sell the best or worst goods first? Considering the sequential auction of two stochastically equivalent goods, we find that the seller has an incentive to impress buyers by selling the better good first because the seller's sequencing strategy endogenously generates correlation in the values of the goods across periods. When this impression effect is strong enough, selling the better good first is the unique pure-strategy equilibrium. By credibly revealing to all buyers the seller's ranking of the goods, an equilibrium strategy of sequencing the goods reduces buyer information rents and increases expected revenues in accordance with the linkage principle.

1. Introduction

■ Is it better to make a strong first impression or build up to a strong finish? For instance, should a government privatize its best firms first to impress buyers, or should it warm up the market by starting with less valuable firms? And should a firm divesting multiple units start with the most profitable units, or should it wait until buyers have learned more from earlier sales? Related choices arise in a wide variety of situations, such as whether to schedule the best papers in a conference first to raise expectations, or schedule them last after the audience is better informed. The traditional counsel to “put one’s best foot forward” might seem appropriate, but so does the admonition to “save the best for last.”

To investigate this conflicting advice we consider the issue of strategic sequencing from two perspectives. First, if a strategy can be committed to, which sequencing strategy offers the highest payoffs? For instance, will revenues be highest if a government commits to privatizing the best or worst firms first? Second, from an equilibrium perspective, which sequencing strategies are credible even without commitment? For instance, if buyers believe that a firm is selling the best

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(or worst) units first, will the firm actually benefit from doing so, or will it prefer to fool buyers by reversing the order?

To analyze these questions, we consider an auction model where the seller wants to maximize revenues from the sale of two goods. This model allows us to consider the particular issue of strategic sequencing in auction environments, such as privatization auctions, and also to use the tools of auction theory to gain insight into the problem more generally. For the auction of each good we follow a standard Milgrom and Weber (1982) model in which all auction participants, including the seller, have some private information about the value of the good. The values of the two goods are identically and independently distributed based on public information. We investigate whether the seller, based on her own private information about the values of the goods, should sell the better or worse good first.

We start by considering the simpler case where the goods are auctioned simultaneously so that no information is released between auctions. In a simultaneous auction, the equivalent of a sequencing strategy is for the seller to rank the two goods. Such information raises the expected price for one good and lowers it for the other good, but we find that on average it generates higher expected revenues than selling the goods in random order. This follows from the “linkage principle” that a policy of publicly revealing information equalizes the knowledge of buyers and thereby leads to more competitive bidding (Milgrom and Weber, 1982). A problem with implementing the linkage principle is that the seller has an incentive to exaggerate, so committing to a policy of truthful revelation can be difficult. Looking at this issue from an equilibrium perspective, we show that ranking the goods can be credible because the ranking provides both favorable and unfavorable information at the same time. The ranking does not fully reveal the seller’s information, but it does reveal some information and therefore increases seller revenues.

Using the results from the simultaneous auction, we then consider the sequential auction where one of the two goods is sold first. The difference is that buyers of the second-period good now observe the first-period price or other information about the first-period good. For instance, in a privatization auction, buyers of a firm sold later see the price of a firm sold first and, if the interval between auctions is sufficient, may also observe the post-privatization performance of the first firm. Because the values of the two goods are independently distributed, information from the first period may seem irrelevant for the second-period good. However, we show that the seller’s sequencing strategy endogenously generates correlation across the two auction periods by truncating the distribution of the second-period good. Because of this endogenous correlation, the first-period information is itself a public signal that, on average, increases the second-period price in accordance with the linkage principle.¹ Therefore, if the seller can commit to a sequencing strategy, revenues are increased by both the rank information and the first-period information.

Looking at the sequential auction from an equilibrium perspective, we find that the endogenous correlation gives the seller an incentive to strategically sequence the goods so as to make the most favorable impression on second-period buyers. If buyers expect a best-foot-forward strategy, unexpectedly selling the worse good first will lead second-period buyers to infer that the second good is also low quality. So the “impression effect” from observing the first-period information penalizes deviation from the best-foot-forward strategy. But if buyers expect a best-for-last strategy, unexpectedly selling the better good first will lead second-period buyers to think the second good is also high quality. So the impression effect encourages deviation from the best-for-last strategy. We find that the best-foot-forward strategy is an equilibrium in the sequential auction whenever the best-for-last strategy is, that it is an equilibrium whenever ranking the goods is an equilibrium in the simultaneous auction, and that it is the unique pure-strategy equilibrium when the impression effect is sufficiently strong.

While either sequencing strategy increases expected revenues by revealing the seller’s ranking of the goods and the first-period price or other information, there may be tension between the

¹ This linkage principle effect has been previously noted in cases where the values of the two goods are identical (Milgrom and Weber, 2000) or exogenously correlated (Hausch, 1986), but we show that endogenous correlation induces the same effect even when the two goods are *ex ante* independent.

sequencing strategy that is an equilibrium and the sequencing strategy that maximizes revenues. In particular, since best for last is never the unique pure-strategy equilibrium, a conflict can arise between “best for last” as the revenue-maximizing strategy and “best foot forward” as the equilibrium strategy. In an example we find that expected revenues are higher from the best-for-last strategy for all parameter values even though best foot forward is the unique pure-strategy equilibrium for a subset of parameter values. If the seller could commit to a strategy, she would therefore prefer best for last. But without commitment, best foot forward is the only credible sequencing strategy.

The issue of how to sequence the sale of goods can arise in any situation where a seller of multiple products has some private information about the quality of the goods. For example, in privatization auctions governments usually sequence the sale of companies over many years rather than following a “big bang” strategy of privatizing firms simultaneously (Roland, 2000). A common explanation for this delay is fear of selling firms at below market value. From the perspective of auction theory, this concern is justifiable whenever the lack of reliable public information about formerly state-owned firms gives privately informed buyers substantial information rents. This article indicates that sequential privatization can increase revenues by credibly and publicly revealing information to all buyers, thereby reducing the value of private information and reducing buyer information rents. The same logic also applies to divestiture auctions.² The restructuring literature has shown that the decision to sell assets implies information about both the value of the assets and the value of the remaining firm (Nanda, 1991). Our results imply that when multiple assets are sold, the sequencing decision also reveals information about the relative values of the assets. This information will decrease the expected price of one asset and increase the expected price of the other, but on average it will increase revenues.

Gupta, Ham, and Svejnar (2003) show that in the mass privatization programs undertaken in the Czech Republic, more profitable firms were auctioned first, a phenomenon that appears to be widespread in transitional economies (Claessens and Djankov, 2002) and that is consistent with our result that the impression effect favors a best-foot-forward strategy. The possible role of sequencing strategies is important for measuring the impact of privatization. For instance, the fact that firms sold earlier tend to outperform firms sold later has been taken to reflect efficiency gains from early privatization (Lopez-de-Silanes, 1997), but it could also reflect the use of a best-foot-forward strategy.

The best-foot-forward strategy may also offer insight into the “declining price anomaly” or “afternoon effect” in which the prices for comparably valued goods fall during the course of a sequential auction (Ashenfelter, 1989; McAfee and Vincent, 1993). If the goods are identical or correlated in value (e.g., bottles of the same vintage of wine), the pattern contradicts the linkage principle, since public information about bids in auctions of earlier goods should lead to higher expected prices for later goods (Milgrom and Weber, 1982, 2000). If the values of the goods are independently distributed (e.g., companies in different industries) there should be no regular price pattern. For nonidentical goods, we find that declining prices may result because the quality of the second good is on average lower than the first good based on the seller’s private information. Under our assumption of independently distributed values, prices always fall under the best-foot-forward strategy. In an extension of the model we examine correlated values, and in this case prices can rise if the correlation is strong enough.

That the sequencing of goods based on hard-to-observe quality differences might be one factor behind declining prices has been previously recognized in the empirical literature. For instance, Ashenfelter and Genesove (1992) consider that condominiums of the same type in a single development may differ subtly in location and that the best ones are likely to be bought first. In their analysis of the market for Picasso prints, Pesando and Shum (1996) note that different impressions vary slightly in condition and that it is common practice among auction houses to offer the better impression first. Ginsburgh (1998) notes that wines of the same vintage vary

² In a sample of divestitures in the 1990s, Boone and Mulherin (forthcoming) find that, contrary to much of the discussion in the restructuring literature, auctions are a common selling mechanism.

slightly in quality due to such factors as fullness of the bottle and consequent air exposure, and that the policy at Christie's auction house is to sell the better wine first.³ Whether the sequencing of goods is an important factor behind the declining price anomaly is unresolved in the literature. Ashenfelter and Genesove (1992), Beggs and Graddy (1997), and Ginsburgh (1998) find that prices still fall after sequencing is controlled for, but Raviv (forthcoming) examines an auction where the seller randomly sells the goods and finds evidence that the declining price anomaly disappears.

Previous theoretical analyses of sequencing consider private-value auctions where the distributions of valuations are known to be different based on public information (Bernhardt and Scoones, 1994; Beggs and Graddy, 1997; Baba, 1998; Benoit and Krishna, 2001; and Elmaghraby, 2003). Our model applies to auctions with a common-value component where, based on public information, the goods have the same distribution of values, but where the auction participants have their own private information about the values. Since the seller makes a sequencing decision based on her own private information, the sequencing strategy can play a role in credibly (although, imperfectly) revealing this information to buyers.⁴

The article proceeds as follows. In Section 2 we present a simple example that illustrates the impression effect. Section 3 introduces the general auction model. Section 4 considers the impact on revenues if the seller can commit to a sequencing strategy. In Section 5 we consider whether sequencing is an equilibrium and analyze the declining price anomaly. Section 6 extends the model to examine exogenous correlation between the seller's signals and shows that best foot forward is the unique pure-strategy symmetric equilibrium if correlation is sufficiently strong. Section 7 presents an expanded example that illustrates the key findings. Section 8 concludes the article.

2. An introductory example

■ The role of the impression effect in strategic sequencing can be seen in a simple common-value auction in which buyers do not have any private information so there are no information rents. Two *ex ante* identical goods, a and b , are sold in two periods by a seller who observes the actual values, V_a and V_b , of the goods. The goods are independently distributed with support in $\{0, 1\}$, and $\Pr[V_i = 0] = \Pr[V_i = 1] = 1/2$ for $i = a, b$. There are two different groups of two or more identical buyers in each period. Buyers in the first period bid the expected value of the good conditional on the seller's strategy, and buyers in the second period bid the expected value of the good conditional on both the seller's strategy and the observed value of the first good. Let V_1 represent the value of the good sold first and V_2 the value of the good sold second.

Figure 1 shows the distribution of values conditional on the seller's sequencing strategy.⁵ Suppose buyers believe that the seller follows the best-foot-forward strategy of selling the better good first when $V_a \neq V_b$. If either $V_a = 1$ or $V_b = 1$, buyers expect the first-period good to be high value, so regardless of what the seller actually does, the first-period price is the probability $\Pr[V_1 = 1] = 3/4$. Any impact of the seller's actions falls on the second-period price. If the seller follows the best-foot-forward strategy, then second-period buyers observe that a high-value good was sold in the first period, so the second-period price is $\Pr[V_2 = 1 \mid V_1 = 1] = 1/3$. If instead the seller deviates and leads with the worse good, then second-period buyers, expecting a best-foot-forward strategy, will infer that if the first good was low value the second good must

³ Pesando and Shun (1996) and Ginsburgh (1998) find that some of the differences in quality are publicly stated by the seller. Our model implies that the comparative component of information released by the seller (including price estimates) is often credible.

⁴ In many auctions the goods are likely to differ based on both public information and the seller's private information. While additional issues are raised by such a mix, the role of the sequencing strategy in credibly revealing comparative information and the potential for conflict between equilibrium and revenue-maximizing strategies remain as long as the seller has any private information.

⁵ The correlation coefficient between V_1 and V_2 is $\rho = 1/3$ in both cases. With binary values nonnegative correlation is equivalent to affiliation, which is the basis for the theoretical results in the next section.

FIGURE 1

ENDOGENOUS CORRELATION FROM SEQUENCING STRATEGIES

	$V_2 = 1$	$V_2 = 0$
$V_1 = 1$	1/4	1/2
$V_1 = 0$	0	1/4

Best foot forward

	$V_2 = 1$	$V_2 = 0$
$V_1 = 1$	1/4	0
$V_1 = 0$	1/2	1/4

Best for last

also be, so the second-period price is $\Pr[V_2 = 1 \mid V_1 = 0] = 0$. Since deviation is not profitable, best foot forward is an equilibrium.⁶

It may seem that the problem is symmetric and that best for last is also an equilibrium. Checking, buyers believe the first-period good is high value only if both goods are high value, so the first-period price is $\Pr[V_1 = 1] = 1/4$. If the seller follows the best-for-last strategy then second-period buyers observe that a low-value good was sold in the first period, so the second-period price is $\Pr[V_2 = 1 \mid V_1 = 0] = 2/3$. However, if the seller deviates and leads with the better good, then the second-period buyers believe the second good must also be of high value, so the price is $\Pr[V_2 = 1 \mid V_1 = 1] = 1$. Since deviation is profitable, best for last is not an equilibrium.

The seller wants to make a favorable impression even though goods a and b are independent because the seller's sequencing strategy endogenously generates correlation across the two periods by truncating the distribution of the second-period good. This impression effect offers insight to a wide range of situations where strategic sequencing arises. For instance, if the audience for a talk is proportional to the expected quality of the talk, then a speaker wishing to maximize attendance at two talks should start with the better talk. Of course, reality is often more complicated—audience members may have different opinions about the talk, their valuations of the talk might depend on the opinions of other audience members, information about the first talk may be imperfectly transmitted to the potential audience for the second talk, etc. Formal auction models offer a well-developed set of tools to capture such complexities. In the following sections, we show that the incentive to create a favorable impression carries over to an affiliated-values auction in which the seller need not be perfectly informed, the buyers have private information that may be stronger than that of the seller, the good has differing values to different buyers, and second-period buyers observe only a noisy signal of the value of the first-period good, such as its price.

3. The model

■ Our auction model is based on Milgrom and Weber (1982), which includes both common-value and private-value features. As in their model, the seller and the buyers all have some private information. Distinct from their model, the seller can choose which of two goods to sell first based on her private information, and buyers in the second period observe some information about the value of the first-period good. We describe the model below.

□ **Goods, signals, and values.** There is one seller who sells two goods indexed by $k \in \{a, b\}$. For each good k the seller observes a private signal $S_k \in \{H, L\} \subset \mathbb{R}$, where $H > L$. Thus the seller can tell if good k is likely to be above average ($S_k = H$) or below average ($S_k = L$).⁷ This information is soft in the sense that the seller cannot credibly reveal it, even though she may like to, except through the sequencing strategy. Let $\Pr[S_k = H] = \lambda \in (0, 1)$.

⁶ We have ignored the cases where both goods are high or low value and the seller just randomizes. Adding these cases, the expected payoff from sticking to best foot forward is $3/4 + (1/4)(1/3) + (1/2)(1/3) + (1/4)0 = 1$ and the expected payoff from deviating is $3/4 + (1/4)(1/3) + (1/2)0 + (1/4)0 = 5/6$.

⁷ By limiting the seller's signal space we can restrict attention to the simplest and most intuitive sequencing strategies. When S has more than two elements (for example, S is a continuum), the seller could have more complicated strategies such as selling the better good first only if the gap between signals is sufficiently large.

For each good k there is a group of $n \geq 2$ buyers.⁸ Each buyer i observes a private signal of the quality of the goods, $X_{ik} \in \mathbf{X} \subset \mathbb{R}$. Let $X_k = (X_{1k}, \dots, X_{nk})$ be the vector of buyer signals for good k . Also, let Z_{jk} be the j th-highest signal among the n buyers of good k , with Z_k being the vector, and let Y_{ik}^j be the j th-highest signal of the bidders other than i , with Y_{ik} being the vector.

The random variables (X_k, S_k) associated with good k are independently and identically distributed across $k \in \{a, b\}$.⁹ The joint density $f(x, s)$ is bounded away from zero and infinity, implying that a buyer is never certain of the seller's information given his own signal. Furthermore $f(x, s)$ is symmetric in its first n arguments, so the distribution of the buyers' private signals does not depend upon the identity of the buyers. In a slight tightening of the Milgrom and Weber (1982) assumptions, we assume $f(x, s)$ displays strict rather than weak affiliation, so that if one player (including the seller) observes a high private signal of the value of a good, other players are strictly more likely to observe high private signals of the value of that good.¹⁰ To simplify notation, throughout the article we use $f(\cdot)$ to represent joint and marginal densities and $f(\cdot | \cdot)$ to represent conditional densities.

The value of good k to buyer i is given by $V_{ik} = V(X_{ik}, \{X_{i'k}\}_{i' \neq i}, S_k)$ for each X_k and S_k , where $V : \mathbf{X}^n \times \{H, L\} \rightarrow \mathbb{R}$ is nondecreasing in its first n arguments and strictly increasing in its last argument. Note that the valuations of all buyers for good k depend on the seller's signal in the same way, and the valuation of each buyer depends on the signals of the other buyers $\{X_{i'k}\}_{i' \neq i}$ in the same way.¹¹ Let $V_k = (V_{1k}, \dots, V_{nk})$ be the vector of buyer valuations for good k . As shown in Milgrom and Weber (1982), our assumption of affiliation and the monotonicity of the function $V(\cdot)$ implies that the random variables (V_k, X_k, S_k) are affiliated. Note that the random variables (V_k, X_k, S_k) are also identically and independently distributed across k .

□ **Seller's strategies, the timing structure, and equilibrium.** For a seller with signals $S_k = H$ and $S_{k'} = L$, where $k \neq k'$, the possible pure-strategy sequencing strategies are to sell the good with the high signal first (best foot forward or BFF) and to sell the good with the low signal first (best for last or BFL). When the signals for the two goods are identical, the seller is indifferent about the sequencing strategy, in which case leading with either good is strategically equivalent and the seller will randomize.¹²

Buyers in the second auction observe the first-period price. We also allow for the possibility that the second-period buyers observe other more informative public signals about the good sold in the first period. For example, later buyers in a privatization auction might observe the post-privatization performance of the firms sold previously.

Our primary interest is the sequential auction with the following timing structure: (i) The seller observes her signals and decides which good to sell first. (ii) The buyers of the good the seller sells first note that their good is being sold first, observe their private signals, and bid for the good. (iii) The buyers of the good the seller sells second note that their good is being sold second, observe their private signals, observe the first-period public signals relating to the first-period auction, and bid for the good. In order to facilitate the understanding of our results, we will

⁸ For simplicity we assume that there are different buyers for each good. This is appropriate when a government privatizes firms in different industries, or a restructuring firm sells off unrelated units. The assumption precludes such strategies as underbidding for the first good so as to lower other buyers' expectations for the second good (Hausch, 1986). It also prevents the buyer of the first good from acquiring an information advantage when bidding for the second good (Luton and McAfee, 1986).

⁹ In an extension we relax the independence assumption for the seller's signal. For a related analysis where the assumption of identical distributions is also relaxed, see Chakraborty and Harbaugh (forthcoming).

¹⁰ Random variables admitting a density are (strictly) affiliated if the log of their joint density is a (strict) supermodular function. A function is supermodular (submodular) if a higher value for any argument does not decrease (increase) the marginal return to a higher value for the remaining arguments.

¹¹ As in Milgrom and Weber (1982), this formulation allows for the possibility of pure common values, where $V_{ik} = V_k$ for all i . It also allows for a private-value component for each buyer, so that buyers may not agree on value even if all private signals are made public.

¹² Strategies that condition on the names of the goods, such as selling good a first if the signals are the same and good b first otherwise, are not considered.

also consider a “simultaneous” timing structure that is identical to the sequential case except that buyers of the “second” good do not receive any additional information about the first auction.

We assume that the identical buyers for each good play a symmetric Bayesian Nash equilibrium of the auction given their correct beliefs about the seller’s strategies and their information, and that the seller’s strategy is sequentially rational given the buyers’ beliefs.

The auction, prices, and bids. In each period the seller employs an English auction to sell the good.^{13,14} Because the goods are stochastically equivalent, the auctions in the two periods are identical except that buyers in the second period have more information and buyers in each period expect the seller’s signal to differ systematically based on the seller’s sequencing strategy. Therefore, to simplify the presentation, we do not develop different notation for the two periods. Instead, we analyze the auction for each good k in a way that allows for these differences.

In addition to their private signals X_k , all buyers of good k also observe which period the good is sold in. If buyers believe the seller is randomly selling the two goods, then they learn nothing from this information. But if buyers believe that the seller is following a BFF strategy, then when good k is sold in the first period the buyers of good k interpret this information as the favorable signal $S_k = \max\{S_a, S_b\}$, and when good k is sold in the second period the buyers of the good interpret this information as the unfavorable signal $S_k = \min\{S_a, S_b\}$. If buyers believe that the seller is following a BFL strategy the signals are reversed. Since we focus attention on only these two pure strategies, we can exploit the symmetry in the model by defining one signal that captures how buyers combine their information about the period the good is sold in with their beliefs about the seller’s strategy. In particular we define the ordinal signal $T_k \in \{\tau_L, \tau_H\}$, $\tau_H > \tau_L$, where

$$\begin{aligned} T_k = \tau_H &\Rightarrow S_k = \max\{S_a, S_b\} \\ T_k = \tau_L &\Rightarrow S_k = \min\{S_a, S_b\}. \end{aligned}$$

This signal partially reveals the seller’s private information S_k and captures the pure-rank effect of the seller’s strategy. In our benchmark case where the goods are sold simultaneously, T_k is the only public signal that buyers of any good receive. But when the goods are sold sequentially, with good k sold in the second period, buyers of good k may also observe additional signals $\Psi_{k'}$ affiliated with the seller’s signal $S_{k'}$ for good k' that is sold in the first period. Let Φ_k denote the different public signals that buyers of good k may receive in different scenarios. Thus, $\Phi_k = T_k$ if the goods are sold simultaneously or if good k is sold first, while $\Phi_k = \{T_k, \Psi_{k'}\}$ if the goods are sold sequentially and good k is sold second.

As noted above, the signal $\Psi_{k'}$ could simply be the first-period price $P_{k'}$. Even though all signals associated with the two goods are independently drawn, the price $P_{k'}$ is correlated with the seller’s signal for good k' and so, via the seller’s sequencing strategy, it is correlated with the seller’s signal for good k . As a result, the price $P_{k'}$ is relevant information for buyers of good k , as long as the buyers believe that the seller is conditioning her sequencing strategy on her private information. Furthermore, due to independence, the seller’s sequencing strategy is the only channel through which the auctions for the two goods are related.

For any buyer i , realizations $X_{ik} = x$, $Y_{ik}^j = y^j$ for $j = 1, \dots, n-1$ of the private signals, and a realization $\Phi_k = \phi$ of the public signal, define the function $v_k(\cdot)$ as

$$v_k(x, y^1, \dots, y^{n-1}, \phi) = E[V_{ik} \mid X_{ik} = x, Y_{ik}^1 = y^1, \dots, Y_{ik}^{n-1} = y^{n-1}, \Phi_k = \phi]. \quad (1)$$

Due to symmetry, $v_k(\cdot)$ does not depend on the identity of the bidder. We focus on the symmetric

¹³ In particular, we consider an ascending-bid auction where the price rises continuously and each bidder has to decide when to drop out from the auction after observing the number of active bidders and when other bidders have dropped out. Dropouts are final. See Milgrom and Weber (1982).

¹⁴ We do not consider reserve prices and entry fees. More generally, we do not consider mechanism design issues but instead take the selling mechanism as given.

equilibrium characterized by Milgrom and Weber (1982). In this equilibrium the bidder with the highest signal will win the auction and pay a price equal to the bid of the second-highest bidder. The second-highest bidder's bid will equal the expected value of the good given that he is tied for the highest bid after observing $n - 2$ bidders with the lowest signals drop out (thereby inferring their signals Z_{3k}, \dots, Z_{nk}), and also given the public signals Φ_k . The price of good k as a function of the private and public signals is therefore

$$P_k(X_k, \Phi_k) = v_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \Phi_k). \quad (2)$$

4. Revenue effects

■ We first investigate the revenue advantages of sequencing strategies rather than randomly selling the goods. Throughout this section we assume the seller can commit to a sequencing strategy and postpone analysis of equilibrium strategies until the next section. We start out by considering the case where buyers do not observe the first-period price or any other information about the first auction. Without such information, the only public signal that the buyers of good k receive is $T_k \in \{\tau_H, \tau_L\}$. Therefore this case is essentially a simultaneous auction where the "period" could be, for instance, the room that the good is auctioned in rather than the time that it is auctioned. Since the two pure strategies BFF and BFL are identical subject to renaming the periods, we will refer to either strategy as a "rank-revealing strategy." We will refer to the strategy of completely randomizing the sale of the goods as the mixed strategy.

Following Milgrom and Weber (1982), *ex ante* the seller always benefits in expectation from revealing information. For each good k , the "period" in which the good is sold is informative for the buyers, as it tells them whether the seller's signal S_k for the good is the maximum or the minimum of two independent draws. Therefore, in accordance with the linkage principle, the seller would like to commit to a rank-revealing strategy. We state this as our first result. It follows immediately from Milgrom and Weber (1982).

Result 1. In the simultaneous auction, a rank-revealing strategy generates higher *ex ante* expected revenues than the mixed strategy.

In the sequential auction, buyers may observe the first period price or other information from the first period auction. Information from the first period provides additional information regarding the realization of the seller's signal in the second period. If buyers believe the seller follows either pure strategy, a high first-period price raises the probability that the seller received two high signals, and therefore raises the estimated value of the second-period good for each buyer. By the linkage principle the expected revenues from the best-foot-forward or best-for-last strategy are therefore higher when buyers observe first-period information compared to the simultaneous auction. On the other hand, for the mixed strategy where the seller does not condition the sequencing on her information, the expected revenues will be the same for the simultaneous and sequential auctions because information about good k' contains no information for the buyers of good k , due to independence of all signals across k . Therefore, compared to the simultaneous case, in the sequential auction the seller has an even greater incentive to commit to a rank-revealing strategy. We collect these observations as our next result which also follows immediately from Milgrom and Weber (1982).

Result 2. (i) Both BFF and BFL in the sequential auction generate higher expected revenues than a rank-revealing strategy in the simultaneous auction. (ii) The mixed strategy generates the same expected revenues in the sequential and simultaneous auctions.

A natural question is whether the expected revenues from BFF and BFL can be unambiguously ranked. Under the BFF strategy, the first-period price signal is more likely to carry positive information about the second-period good. Under the BFL strategy, however, the more valuable good is sold in the second period when this information has been released, thereby reducing buyer information rents for the more valuable good. Consequently, the revenues cannot be ranked in general. In Section 7 we provide an example of an auction where BFL generates higher revenues

for all parameter values, thereby supporting the idea that buyers should be “warmed up” with cheaper goods first.¹⁵

These revenue results are based on the assumption that the seller can commit to a strategy that reveals information. Milgrom and Weber (1982) suggest that an auction house could commit to reveal information by virtue of its reputation, and the same possibility exists in this model. However, as the following section shows, in many cases sequencing the goods is an equilibrium, so reputation and other commitment devices are not necessary.

5. Equilibrium strategies

■ We now consider when a sequencing strategy is an equilibrium. In both the simultaneous and sequential auctions the question is whether, given buyer beliefs about whether the better or worse good is being sold, the seller has an incentive to follow the expectations or instead trick the buyers and reverse the order. Again, it is convenient to start with a simultaneous auction and then analyze the sequential auction.

□ **Simultaneous auction.** Recall that in a simultaneous auction, either BFF or BFL is equivalent to a rank-revealing strategy of indicating which of the goods is better. For a seller with one high signal and one low signal,¹⁶ the payoff from following the rank-revealing strategy is $E[P_k(X_k, \tau_H) | S_k = H] + E[P_{k'}(X_{k'}, \tau_L) | S_{k'} = L]$ and the payoff from deviating is $E[P_k(X_k, \tau_L) | S_k = H] + E[P_{k'}(X_{k'}, \tau_H) | S_{k'} = L]$. Because of the symmetry in the model, the necessary and sufficient condition for an equilibrium can be written as

$$\begin{aligned} E[P_k(X_k, \tau_H) | S_k = H] + E[P_k(X_k, \tau_L) | S_k = L] &\geq \\ E[P_k(X_k, \tau_L) | S_k = H] + E[P_k(X_k, \tau_H) | S_k = L]. \end{aligned} \quad (3)$$

Reversing the order of the goods increases the expected price for one good and decreases it for the other. Our first proposition provides sufficient conditions on the primitives of the model under which the net gain is positive or negative. We find that both the shape of V and the prior probability λ of a favorable seller signal interact to determine whether the net benefit from deviation is positive or negative. When V is supermodular and λ is low, both factors favor sticking with the equilibrium strategy, and when V is submodular and λ is high, both factors favor deviation.

Proposition 1. If $V(x_1, \dots, x_n, s)$ is supermodular (submodular) in (x, s) , then there exists $\bar{\lambda}, \underline{\lambda} \in (0, 1)$ such that if $\lambda \leq \bar{\lambda}$ ($\lambda > \underline{\lambda}$), a rank-revealing strategy is (not) an equilibrium of the simultaneous auction.

Proof. See the Appendix.

To understand the role of λ , note that the sensitivity of buyers' bids to their own information is decreasing in the strength of their priors about the seller's signal. Therefore, since buyer information is correlated with the seller's signal, the seller has an incentive to sell the better good when buyer priors are weakest, and to sell the worse good when buyer priors are strongest. When λ is high, the buyers have strong priors that the higher-ranked good is of high quality but are less certain about the lower-ranked good. Therefore, this “priors effect” gives the seller an incentive to trick buyers and sell the better good when buyers expect the worse good. When λ is low, the priors effect works in the opposite direction. Buyers have strong priors that the lower-ranked good is bad but are less certain about the higher-ranked good. Thus, the seller has an incentive to stick with the equilibrium strategy and sell the better good when buyers are expecting it.

¹⁵ Relatedly, McMillan (1994) notes that the issue of whether to sell rights for large or small regions first was considered in designing spectrum auctions in the United States, with one factor being that the linkage principle favored a small-to-large sequence. However, this situation differs from ours in that the size differences between regions are common knowledge.

¹⁶ When the seller receives identical signals, there is no strategic decision to be made, as the seller is indifferent between sequencing choices, and we assume that, in equilibrium, the seller sells either good first with probability 1/2.

Regarding the shape of V , if V is supermodular, then the seller’s signal and the buyers’ signals are complements in determining buyer valuations. In this case buyers bid more aggressively in response to their own favorable information when they are optimistic about the seller’s signal, so the seller should sell her better good when buyers are expecting her to. A sufficient condition for strict supermodularity is multiplicative separability of buyer values in the seller’s information and the buyers’ own information. Such separability can occur when the seller has information on a common component and the individual buyers have information on a private component. For instance, multiplicative separability could arise in divestiture auctions of two units if buyers for each unit have a private signal about how well they could manage the unit, the divesting firm knows the general productivity of each unit, and the profitability of a unit is a multiplicative function of both factors. Similarly, multiplicative separability could occur in a privatization auction of two firms in separate industries if the buyers for each firm have a private signal about the firm’s profitability and the government knows the likely tax rates in the two industries.

If V is submodular, then the seller’s signal and the buyers’ signals are substitutes in determining buyer valuations. In this case the seller’s incentive to deviate is stronger because buyers bid less aggressively in response to their own favorable information when they are optimistic about the seller’s signal. Submodularity is less common in the literature but can easily be justified in some cases. For instance, if the value of the good is a concave function of the sum of the different signals of the buyers and the seller, then submodularity holds.

A special case is where V is weakly both supermodular and submodular so there is only the priors effect and an equilibrium exists for λ low enough and does not exist for λ high enough. For instance, this is clearly the case in a common-value auction with a perfectly informed seller, since $V(x_1, \dots, x_n, s) = s$. The same result arises in a more general affiliated values auction if V is additively separable in the seller’s information and the buyers’ own information. For instance, in the above divestiture example it might be that profitability of a unit is an additive rather than multiplicative function of the two factors.

□ **Sequential auction.** With the simultaneous auction as a baseline, we now consider equilibrium strategies in the sequential auction where buyers of the second-period good can see information about the first-period good. As shown in the example of Figure 1, endogenous correlation makes such information of interest to buyers of the second-period good. As a result the seller expects that the second-period price will be affected by the quality of the good sold in the first period. We call this expected impact the “impression effect.”

Lemma 1. For both the BFF and BFL strategies, when good k is sold in the second period, observation of the first-period signal $\Psi_{k'}$ by second-period buyers raises (lowers) the expected second-period price if the seller sells a good with a high (low) signal $S_{k'}$ in the first period: for all $\tau \in \{\tau_H, \tau_L\}$ and each $s \in \{H, L\}$ of S_k ,

$$E[P_k(X_k, \tau, \Psi_{k'}) \mid S_{k'} = H, S_k = s] > E[P_k(X_k, \tau) \mid S_k = s] > E[P_k(X_k, \tau, \Psi_{k'}) \mid S_{k'} = L, S_k = s]. \tag{4}$$

Proof. See the Appendix.

The proof depends on the affiliation between the first-period public signal and the first-period buyer signals, the affiliation between these buyer signals and the first-period seller signal, the endogenous correlation between the first- and second-period seller signals, and the affiliation between the second-period buyer signals and the second-period seller signal. A special case of interest is when second-period buyers observe only the first-period price, $\Psi_{k'} = P_{k'}$.

Corollary 1. If buyers of good k only observe the first-period price $P_{k'}$, then for all $\tau \in \{\tau_H, \tau_L\}$ and each $s \in \{H, L\}$ of S_k ,

$$E[P_k(X_k, \tau, P_{k'}) \mid S_{k'} = H, S_k = s] > E[P_k(X_k, \tau) \mid S_k = s] > E[P_k(X_k, \tau, P_{k'}) \mid S_{k'} = L, S_k = s]. \tag{5}$$

If, as in the simultaneous auction, the first-period price is not observed, then the BFF and BFL strategies are equivalent, so either both are equilibria or neither are equilibria. The impression effect adds a boost in favor of the BFF strategy and against the BFL strategy, implying that the equilibrium condition for the former is always less strict than that for the latter. Thus whenever BFL is an equilibrium, BFF must also be an equilibrium, but not the converse. One equilibrium that always exists is where the seller plays a mixed strategy of randomly sequencing the sale of the goods. If buyers expect such randomization, the seller is indifferent between sequencing strategies because the first-period price conveys no information to the buyers.

Proposition 2. (i) If BFL is an equilibrium in the sequential auction, then BFF is an equilibrium in the sequential auction. (ii) If a rank-revealing strategy is an equilibrium in the simultaneous auction, then BFF is an equilibrium in the sequential auction. (iii) If a rank-revealing strategy is not an equilibrium in the simultaneous auction, then BFL is not an equilibrium in the sequential auction. (iv) The mixed strategy is always an equilibrium.

Proof. Exploiting the symmetry between good k and good k' , the equilibrium condition for BFF can be written as

$$E[P_k(X_k, \tau_H) \mid S_k = H] + E[P_k(X_k, \tau_L, \Psi_{k'}) \mid S_{k'} = H, S_k = L] \geq E[P_k(X_k, \tau_H) \mid S_k = L] + E[P_k(X_k, \tau_L, \Psi_{k'}) \mid S_{k'} = L, S_k = H] \tag{6}$$

and that for BFL can be written as

$$E[P_k(X_k, \tau_L) \mid S_k = L] + E[P_k(X_k, \tau_H, \Psi_{k'}) \mid S_{k'} = L, S_k = H], \geq E[P_k(X_k, \tau_L) \mid S_k = H] + E[P_k(X_k, \tau_H, \Psi_{k'}) \mid S_{k'} = H, S_k = L]. \tag{7}$$

The proof of (i)–(iii) follows from (4) by observing that the left-hand side of (6) is greater than the left-hand side of (3) while the right-hand side of (6) is less than the right-hand side of (3), and similarly that the left-hand side of (7) is less than the left-hand side of (3) while the right-hand side of (7) is greater than the right-hand side of (3). The proof of (iv) follows from the fact that if the buyers believe that the seller does not condition the sequencing decision on her information, then, for each realization of her signals, all sequencing strategies for the seller yield the same expected revenues. *Q.E.D.*

Note that it is possible for both pure-strategy equilibria to exist so that the actual outcome will depend on receiver beliefs in the particular context. It is also possible for neither pure-strategy equilibrium to exist, so the seller will randomize the order. The more powerful is the impression effect in (4), the stronger is the incentive in (6) to follow the BFF strategy and the weaker is the incentive in (7) to follow the BFL strategy. The extreme case is where the seller is fully informed of the goods' values and the seller's first-period signal is fully revealed. This situation was examined in the introductory example for a parameterized case where buyers did not have informative signals, and it was shown that BFF was the unique pure-strategy equilibrium. In fact, this result also holds when buyers have informative signals.

Proposition 3. In a common-value auction, if the seller is perfectly informed, $V_k = S_k$, and the seller's signal is revealed between periods, $\Psi_{k'} = S_{k'}$, then BFF is an equilibrium and is the unique symmetric pure-strategy equilibrium.

Proof. Under the BFF strategy, if the first-period seller signal is L , then buyers will infer that the second-period seller signal is also L . Therefore, for $\Psi_{k'} = L$, the second-period price is $E[P_k(X_k, \tau_L, L) \mid S_{k'} = L, S_k = H] = L$. So the equilibrium condition for BFF simplifies to

$$E[P_k(X_k, \tau_H) \mid S_k = H] + E[P_k(X_k, \tau_L, H) \mid S_{k'} = H, S_k = L] \geq E[P_k(X_k, \tau_H) \mid S_k = L] + L, \tag{8}$$

which holds because affiliation implies $E[P_k(X_k, \tau_H) \mid S_k = H] \geq E[P_k(X_k, \tau_H) \mid S_k = L]$, and

because $V_k = S_k \in \{L, H\}$ implies $E[P_k(X_k, \tau_L, H) \mid S_{k'} = H, S_k = L] \geq L$. Conversely, under the BFL strategy, $E[P_k(X_k, \tau_H, H) \mid S_{k'} = H, S_k = L] = H$. So the condition for BFL simplifies to

$$E[P_k(X_k, \tau_L) \mid S_k = L] + E[P_k(X_k, \tau_H, L) \mid S_{k'} = L, S_k = H] \geq E[P_k(X_k, \tau_L) \mid S_k = H] + H, \quad (9)$$

which does not hold because affiliation implies $E[P_k(X_k, \tau_L) \mid S_k = L] \leq E[P_k(X_k, \tau_L) \mid S_k = H]$ and because $V_k = S_k \in \{L, H\}$ implies $E[P_k(X_k, \tau_H, L) \mid S_{k'} = L, S_k = H] < H$. *Q.E.D.*

As discussed in Section 4, in some cases the expected revenues from a BFL strategy might be higher than from a BFF strategy. Therefore, a conflict can arise between equilibrium and revenue-maximizing strategies if the BFL strategy yields higher revenues but is not an equilibrium.¹⁷ In Section 7 we provide an example where BFL yields higher revenues than BFF for all parameter values, but only BFF is an equilibrium for a subset of these values.

To conclude this section we turn to a discussion of expected prices across periods under the different pure strategies. As discussed in the Introduction, empirical evidence indicates that the prices of seemingly identical goods often fall during the course of a sequential auction. While a number of different approaches have been taken to explain this anomaly, they are focused primarily on buyer characteristics and strategies. We find that the “declining-price anomaly” can also arise endogenously out of the seller’s choice of an equilibrium-sequencing strategy when the goods are stochastically equivalent rather than identical as often assumed in the literature. Even though all signals related to the two goods are identically and independently distributed, either sequencing strategy results in prices being correlated over time. As the next result shows, when the seller employs the BFF strategy, prices fall simply because on average the second good is of lower value than the first good based on the seller’s private information. This negative effect outweighs the positive effect on second-period prices from the linkage principle.

Proposition 4. (i) Under the BFF strategy, the expected first-period price is higher than the expected second-period price:

$$E[P_k(X_k, \tau_H)] > E[P_k(X_k, \tau_L, \Psi_{k'})]. \quad (10)$$

(ii) Under the BFL strategy, the expected first-period price is lower than the expected second-period price:

$$E[P_k(X_k, \tau_L)] < E[P_k(X_k, \tau_H, \Psi_{k'})]. \quad (11)$$

(iii) Under the mixed strategy the expected first-period price is equal to the expected second-period price:

$$E[P_k(X_k)] = E[P_k(X_k, \Psi_{k'})]. \quad (12)$$

Proof. See the Appendix.

6. Extension: exogenous correlation

■ The previous section shows how the sender’s sequencing strategy endogenously generates correlation in the values of the first- and second-period goods, thereby creating an impression effect that favors the BFF strategy. If the values of the two goods are exogenously correlated, separate from the sequencing strategy, clearly the impression effect will be stronger. For instance, if the two goods are very likely to be of the same quality, then buyers will be strongly affected by

¹⁷ When neither BFF nor BFL is an equilibrium, there is also a conflict because either strategy generates higher revenues than the mixed strategy.

any information about the first good. Therefore, if the realized values of the goods are of different quality, the seller is particularly ill-advised to sell the worse good first.¹⁸

To examine the impact of exogenous correlation, we extend the model to allow the seller's signals to be correlated with each other but otherwise maintain all of the model's other assumptions. Separate from the impact on the impression effect, we find that the priors effect identified in Proposition 1 is weakened by exogenous correlation and disappears asymptotically as correlation increases. As the correlation increases, buyers become increasingly certain that the seller has either a favorable signal for both goods or an unfavorable signal for both goods, so buyer priors are increasingly similar for the higher- and lower-ranked good. The asymptotic disappearance of the priors effect implies that condition (3) holds with equality, meaning that in the simultaneous auction the seller is indifferent between ordering the goods truthfully or not. Since the impression effect remains strictly positive, BFF is the unique equilibrium in the sequential auction.

Proposition 5. For given λ , if the seller's signals S_a and S_b are sufficiently positively correlated, then BFF is the unique symmetric pure-strategy equilibrium.

Proof. See the Appendix.

Regarding the declining price anomaly, for the case of independent S_a and S_b it was shown that the downward effect on the price path from the first-period good being better under the BFF strategy was larger than the upward effect on the price path from the linkage principle. For the case of perfect exogenous correlation it is known that the linkage principle implies rising prices (Milgrom and Weber, 2000), so continuity arguments therefore imply that the effect of the linkage principle dominates for sufficiently high exogenous correlation.

7. An example

■ We now expand on the introductory example of Section 2 by allowing buyers to have private information that generates information rents. We consider the case where the impression effect is the weakest in that second-period buyers only observe the first-period price rather than a more informative signal of the first-period good's value.¹⁹

Similar to the earlier example, $V_k \in \{0, 1\}$ where $\Pr[V_k = 1] = \gamma$ for $k \in \{a, b\}$. The seller has a signal of each good's value $S_k \in \{L, H\}$, where

$$\Pr[S_k = H \mid V_k = 1] = \Pr[S_k = L \mid V_k = 0] = \alpha \in \left(\frac{1}{2}, 1\right] \quad (13)$$

for $k \in \{a, b\}$. Therefore the probability λ of a good seller signal is $\Pr[S_k = H] = \alpha\gamma + (1 - \alpha)(1 - \gamma)$. For each good there are $n = 2$ buyers who each receive a noisy signal of the quality of the good being sold in that period, $X_k \in \{L, H\}$, where

$$\Pr[X_k = H \mid V_k = 1] = \Pr[X_k = L \mid V_k = 0] = \beta \in \left(\frac{1}{2}, 1\right) \quad (14)$$

for $k \in \{a, b\}$. For simplicity we assume that the buyer and seller signals are independent conditional on the value of the good. We continue to use an English auction, which with two bidders is equivalent to a second-price auction.

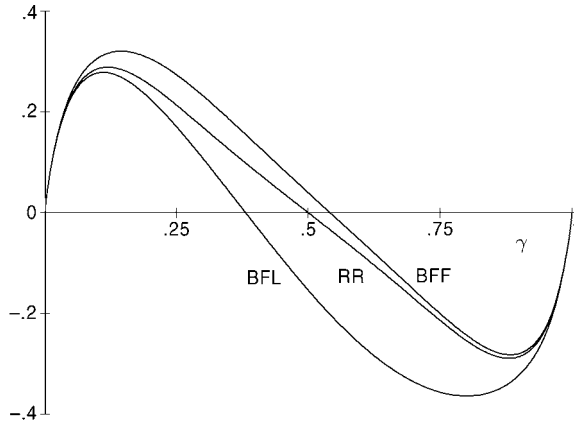
Regarding revenue-maximizing strategies, if the seller can commit to a strategy, then Results 1 and 2 imply that revenues from either BFF or BFL are higher than those from the mixed strategy because the ranking and the first-period price publicly reveal information. For instance, for $\alpha = 1$,

¹⁸ For instance, in the introductory example of Section 2, in the limit as the goods are perfectly correlated, following the BFF strategy when buyers expect it gives a second-period price of one, and deviating gives a second-period price of zero.

¹⁹ In privatization and divestiture auctions, assets are typically sold over a period of many years, so buyers interested in assets sold later are likely to see highly informative signals regarding the values of assets sold earlier.

FIGURE 2

NET REVENUE GAIN FROM FOLLOWING EQUILIBRIUM STRATEGY, $\alpha = 1, \beta = 3/4$



$\beta = 3/4$, and $\gamma = 1/2$, the revenue gain from BFF relative to the mixed strategy is about 4.6%, while the gain from BFL is about 6.1%.²⁰ In fact, in this example, BFL offers slightly higher revenues than BFF for all parameter values.

Regarding what strategies can be an equilibrium without commitment, consider the case where $\alpha = 1$ so that the auction is a pure common-value auction and the value function is both weakly submodular and weakly supermodular, as discussed earlier. Therefore, from Proposition 1, the priors effect implies that a rank-revealing equilibrium exists for λ small enough and does not exist for λ large enough. Noting that when $\alpha = 1, \Pr[S_k = H] = \Pr[V_k = 1]$ so $\lambda = \gamma$, this can be seen in Figure 2. The RR line shows that the net revenue gain from sticking to a rank-revealing strategy rather than deviating is positive for $\gamma < 1/2$ and negative for $\gamma > 1/2$. The impression effect implies that the seller always has more incentive to stick with the BFF strategy and less incentive to stick with the BFL strategy. Consistent with Proposition 2, Figure 2 shows that BFF is an equilibrium for a larger range of γ than BFL is.^{21,22} Therefore, even though a policy of committing to BFL raises more revenue than BFF for all parameter values, for some values of γ only BFF can be implemented as an equilibrium outcome in the absence of commitment.

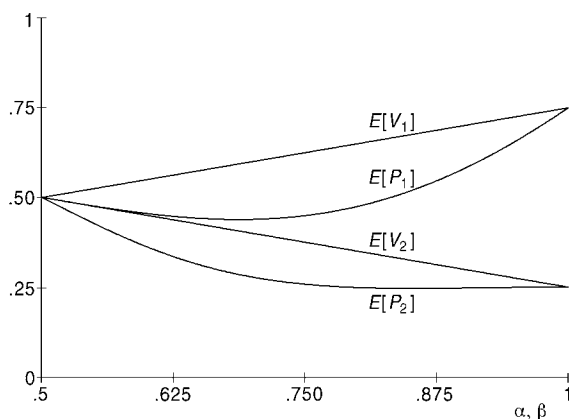
From Proposition 4 we know that the expected second-period price is lower than the expected first-period price when the seller follows the BFF strategy. Figure 3 shows this declining price pattern for $\gamma = 1/2$ when α and β jointly vary from $1/2$ to 1 . For $\alpha = \beta = 1/2$, all signals are completely uninformative so the buyers bid the unconditional expected value of $1/2$ in each period. As the informativeness of the seller's signal increases, the expected values of the goods diverge, with the expected value of the first good (labelled as $E[V_1]$) increasing linearly to $3/4$ and the expected value of the second good (labelled as $E[V_2]$) decreasing linearly to $1/4$. The expected prices for the two periods (labelled as $E[P_1]$ and $E[P_2]$) do not follow this pattern exactly, due to the buyers' information rents. Although the information rents are smaller for the second good due to the linkage principle, as seen by the smaller gap between $E[V_2]$ and $E[P_2]$ than between $E[V_1]$ and $E[P_1]$, the expected prices decline between periods for all parameter

²⁰ Note that for low values of β , neither buyer has much information, while for high values of β , both buyers have highly correlated information. The gains from reducing buyer information rents are highest for intermediate values of β , where these rents are largest.

²¹ The incentive to follow BFF and deviate from BFL was stronger in the introductory example because the actual seller signal from the first period, rather than just the price, was revealed in the second period. In fact, from Proposition 3, BFF is then the unique pure-strategy equilibrium for any γ and any β .

²² Note that the graph is not symmetric. The English auction format makes a high price in the first period a particularly strong signal of second-period quality when buyers expect the seller to follow BFL. The incentive to deviate from BFL is therefore stronger than the incentive to follow BFF.

FIGURE 3

DECLINING PRICE PATTERN UNDER BFF STRATEGY, $\gamma = 1/2$ 

values. In contrast, if the two goods were identical rather than just stochastically equivalent, the linkage principle would imply the opposite pattern of rising prices (Milgrom and Weber, 2000).

8. Conclusion

■ In this article we use an auction model to analyze the credibility and payoff effects of strategic sequencing. Even when the goods being auctioned are *ex ante* independent, we find that the sequencing strategy endogenously generates correlation across periods, so evidence of a high-quality good in the first period, such as a high price, makes a positive impression on second-period buyers. If this impression effect is strong enough, then leading with the better good is the unique pure-strategy equilibrium. When either sequencing strategy is an equilibrium, the strategy credibly reveals the seller's private information about the relative values of the goods and thereby increases seller revenues in accordance with the linkage principle.

Strategic sequencing offers insight into the general issue of how to credibly reveal comparative information (Chakraborty, Gupta, and Harbaugh, 2002; Chakraborty and Harbaugh, 2003; Chakraborty and Harbaugh, forthcoming). While seller statements about the values of their goods are normally suspect, comparative statements can be part of an equilibrium strategy, since they simultaneously reveal both good and bad information. For instance, if a seller provides estimated valuations for a set of goods, the relative information might be credible even if the absolute information is not. Compared to simultaneous auctions, sequential auctions are better at revealing comparative information even when the only signal between periods is the price. The impression effect from observation of the first-period price expands the range in which a pure-strategy equilibrium exists, and thereby expands the range in which the sequencing strategy can credibly reveal comparative information.

Appendix

■ Proofs of Propositions 1, 4, and 5 and Lemma 1 follow.

Proof of Proposition 1. Condition (3) is equivalent to $E[P_k(X_k, \tau_H) - P_k(X_k, \tau_L) \mid S_k]$ being nondecreasing in S_k , which from (2) is equivalent to $E[v_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \tau_H) - v_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \tau_L) \mid S_k]$ being nondecreasing in S_k . We look for sufficient conditions for this to hold and sufficient conditions for it not to hold.

From (1),

$$\begin{aligned} v_k(X_{ik}, Y_{ik}, T_k) &= E[V(X_{ik}, \{Y_{ik}^j\}, S_k) \mid X_{ik}, Y_{ik}, T_k] \\ &= \sum_{s \in \{H, L\}} V(X_{ik}, \{Y_{ik}^j\}, s) \Pr[S_k = s \mid X_{ik}, Y_{ik}, T_k]. \end{aligned}$$

Then

$$\begin{aligned}
 & v_k(X_{ik}, Y_{ik}, \tau_H) - v_k(X_{ik}, Y_{ik}, \tau_L) \\
 &= \sum_{s \in \{H, L\}} V(X_{ik}, \{Y_{ik}^j\}, s) \{\Pr[S = s \mid X_{ik}, Y_{ik}, \tau_H] - \Pr[S = s \mid X_{ik}, Y_{ik}, \tau_L]\} \\
 &= [V(X_{ik}, \{Y_{ik}^j\}, H) - V(X_{ik}, \{Y_{ik}^j\}, L)] \{\Pr[H \mid X_{ik}, Y_{ik}, \tau_H] - \Pr[H \mid X_{ik}, Y_{ik}, \tau_L]\}. \tag{A1}
 \end{aligned}$$

From the properties of conditional probabilities,

$$\begin{aligned}
 \Pr[H \mid X_{ik}, Y_{ik}, \tau_H] &= \frac{f(H, X_{ik}, Y_{ik}, \tau_H)}{f(X_{ik}, Y_{ik}, \tau_H)} \\
 &= \frac{\sum_{S_{k'} \in \{H, L\}} f(\tau_H \mid H, S_{k'}) f(X_{ik}, Y_{ik} \mid H) f(S_{k'}, H)}{\sum_{S_{k'} \in \{H, L\}} \sum_{S_k \in \{H, L\}} f(\tau_H \mid s, s') f(X_{ik}, Y_{ik} \mid S_k) f(S_{k'}, S_k)} \\
 &= \frac{\lambda_1 f(X_{ik}, Y_{ik} \mid H)}{\lambda_1 f(X_{ik}, Y_{ik} \mid H) + (1 - \lambda_1) f(X_{ik}, Y_{ik} \mid L)}, \tag{A2}
 \end{aligned}$$

where

$$\lambda_1 \equiv \Pr[S_k = H \mid T_k = \tau_H] = 1 - (1 - \lambda)^2,$$

and we have used the independence of signals associated with goods k and k' in the second line.

Similarly,

$$\Pr[H \mid X_{ik}, Y_{ik}, \tau_L] = \frac{\lambda_2 f(X_{ik}, Y_{ik} \mid H)}{\lambda_2 f(X_{ik}, Y_{ik} \mid H) + (1 - \lambda_2) f(X_{ik}, Y_{ik} \mid L)}, \tag{A3}$$

where

$$\lambda_2 \equiv \Pr[S_k = H \mid T_k = \tau_L] = \lambda^2.$$

Define the likelihood ratio of the densities of (X_{ik}, Y_{ik}) conditional on S_k as

$$\ell(X_{ik}, Y_{ik}) \equiv \frac{f(X_{ik}, Y_{ik} \mid H)}{f(X_{ik}, Y_{ik} \mid L)} \tag{A4}$$

and define the function

$$h(\ell) \equiv \frac{2\lambda(1 - \lambda)\ell}{[\lambda_1 \ell + (1 - \lambda_1)][\lambda_2 \ell + (1 - \lambda_2)]}. \tag{A5}$$

Noting that

$$h(\ell(X_{ik}, Y_{ik})) = \Pr[H \mid X_{ik}, Y_{ik}, \tau_H] - \Pr[H \mid X_{ik}, Y_{ik}, \tau_L],$$

it follows from (A1)–(A5) that if

$$E[\{V(Z_{2k}, Z_{2k}, \dots, Z_{nk}, H) - V(Z_{2k}, Z_{2k}, \dots, Z_{nk}, L)\} h(\ell(Z_{2k}, Z_{2k}, \dots, Z_{nk})) \mid S_k] \tag{A6}$$

is nondecreasing in S_k , then a rank-revealing strategy is an equilibrium, and if it is decreasing it is not an equilibrium. Supermodularity (submodularity) of V implies that the first expression in braces inside the expectation is nondecreasing (nonincreasing) in each of its first n arguments. Further, by affiliation, the likelihood ratio $\ell(Z_{2k}, Z_{2k}, \dots, Z_{nk})$ is nondecreasing in each argument. Therefore, to show existence (nonexistence) for supermodular (submodular) V , it suffices to show that $h(\ell)$ is nondecreasing (increasing) in ℓ . First consider existence. It can easily be verified that $h'(\ell) \geq 0$ if and only if

$$\ell \geq \sqrt{\frac{(1 - \lambda)^2(1 - \lambda^2)}{(1 - (1 - \lambda)^2)\lambda^2}}. \tag{A7}$$

Let $\bar{\lambda}$ be such that

$$\sqrt{\frac{(1 - \bar{\lambda})^2(1 - \bar{\lambda}^2)}{(1 - (1 - \bar{\lambda})^2)\bar{\lambda}^2}} = \sup_{x, y^1, \dots, y^{n-1} \in \mathbf{X}^n} \ell(x, y^1, \dots, y^{n-1}).$$

Since the left-hand side of the expression above is continuous and monotonically decreasing in $\bar{\lambda}$, equal to zero at $\bar{\lambda} = 1$, and approaching infinity as $\bar{\lambda}$ goes to zero, and since the right-hand side is bounded away from infinity by the full support assumption, such a $\bar{\lambda} \in (0, 1)$ exists by the intermediate value theorem.

Regarding nonexistence, note that $h'(\ell) < 0$ if and only if the inequality in (A7) is reversed and strict. Let $\underline{\lambda}$ be such that

$$\sqrt{\frac{(1-\underline{\lambda})^2(1-\underline{\lambda}^2)}{(1-(1-\underline{\lambda})^2)\underline{\lambda}^2}} = \inf_{x, y^1, \dots, y^{n-1} \in \mathbf{X}^n} \ell(x, y^1, \dots, y^{n-1}).$$

Since the right-hand side is greater than zero by the full support assumption, by the same argument as above such a $\underline{\lambda} \in (0, 1)$ exists. *Q.E.D.*

Proof of Lemma 1. Recall that when the only signal that buyers of good k observe is T_k , the expected price of good k conditional on the seller's signal $S_k = s \in \{H, L\}$, given a realized value $\tau \in \{\tau_H, \tau_L\}$ of T_k , is

$$E[P_k(X_k, \tau) | S_k = s] = E[v_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \tau) | S_k = s].$$

Now, from (1),

$$\begin{aligned} v_k(X_{ik}, Y_{ik}^1, \dots, Y_{ik}^{n-1}, T_k) &= E[V_{ik} | X_{ik}, Y_{ik}^1, \dots, Y_{ik}^{n-1}, T_k] \\ &= E[v_k(X_{ik}, Y_{ik}^1, \dots, Y_{ik}^{n-1}, T_k, \Psi_{k'}) | X_{ik}, Y_{ik}^1, \dots, Y_{ik}^{n-1}, T_k]. \end{aligned}$$

By independence of the random variables related to good k from good k' , the density of $\Psi_{k'}$ conditional on X_{ik}, Y_{ik}, T_k , and $S_{k'}$ depends only on $S_{k'}$. Using this we obtain

$$v_k(X_{ik}, Y_{ik}^1, \dots, Y_{ik}^{n-1}, T_k) = E[E[v_k(X_{ik}, Y_{ik}^1, \dots, Y_{ik}^{n-1}, T_k, \Psi_{k'}) | S_{k'}] | X_{ik}, Y_{ik}^1, \dots, Y_{ik}^{n-1}, T_k].$$

By strict affiliation of $S_{k'}$ with $\Psi_{k'}$ and the monotonicity of v_k in $\Psi_{k'}$, we have

$$\begin{aligned} E[v_k(x, y^1, \dots, y^{n-1}, (\tau, \Psi_{k'})) | S_{k'} = H] &> v_k(x, y^1, \dots, y^{n-1}, \tau) \\ &> E[v_k(x, y^1, \dots, y^{n-1}, (\tau, \Psi_{k'})) | S_{k'} = L]. \end{aligned}$$

Therefore,

$$\begin{aligned} E[P_k(X_k, \tau) | S_k = s] &= E[v_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \tau) | S_k = s] \\ &> E[E[v_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, (\tau, \Psi_{k'})) | S_{k'} = L] | S_k = s] \\ &= E[v_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, (\tau, \Psi_{k'})) | S_{k'} = L, S_k = s] \\ &= E[P_k(X_k, (\tau, \Psi_{k'})) | S_{k'} = L, S_k = s]. \end{aligned}$$

Similarly,

$$\begin{aligned} E[P_k(X_k, \tau) | S_k = s] &= E[v_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \tau) | S_k = s] \\ &< E[E[v_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, (\tau, \Psi_{k'})) | S_{k'} = H] | S_k = s] \\ &= E[v_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, (\tau, \Psi_{k'})) | S_{k'} = H, S_k = s] \\ &= E[P_k(X_k, (\tau, \Psi_{k'})) | S_{k'} = H, S_k = s]. \end{aligned}$$

Q.E.D.

Proof of Proposition 4. We start with the proof of (i). Note first that the expected second-period price under the BFF strategy is higher when second-period buyers directly observe the seller's first-period signal than when they only observe the first-period price:

$$E[P_k(X_k, (\tau_L, \Psi_{k'}))] \leq E[P_k(X_k, (\tau_L, S_{k'}))], \tag{A8}$$

as the signal $\{\tau_L, S_{k'}\}$ contains more information about S_k than the signal $\{\tau_L, \Psi_{k'}\}$ (see Theorem 13 in Milgrom and Weber (1982)).

Furthermore, for fixed x, y^1, \dots, y^{n-1} and $\tau = \tau_L$,

$$\begin{aligned} v_k(x, y^1, \dots, y^{n-1}, (\tau_L, s')) &= E[V_{ik} | X_{ik} = x, Y_{ik}^1 = y^1, \dots, Y_{ik}^{n-1} = y^{n-1}, T_k = \tau_L, S_{k'} = s'] \\ &< E[V_{ik} | X_{ik} = x, Y_{ik}^1 = y^1, \dots, Y_{ik}^{n-1} = y^{n-1}, S_{k'} = s'] \\ &= E[V_{ik} | X_{ik} = x, Y_{ik}^1 = y^1, \dots, Y_{ik}^{n-1} = y^{n-1}] \\ &= v_k(x, y^1, \dots, y^{n-1}, \emptyset), \end{aligned}$$

where the inequality follows from strict affiliation of T_k with the other random variables related to good k , the next equality follows from the fact that $S_{k'}$ is independent of S_k (and contains information about S_k only in conjunction with τ_k), and the last equality is definitional where we use the notation $\Phi_k = \emptyset$ to denote the case where the buyers of good k receive no public signal. Thus,

$$E[P_k(X_k, (\tau_L, S_{k'}))] = E[v_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \tau_L, S_{k'})] < E[v_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \emptyset)] = E[P_k(X_k, \emptyset)]. \quad (A9)$$

Finally, note that

$$E[P_k(X_k, \emptyset)] = E[v_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \emptyset)] < E[v_k(Z_{2k}, Z_{2k}, \dots, Z_{nk}, \tau_H)] = E[P_k(X_k, \tau_H)] \quad (A10)$$

as

$$\begin{aligned} v_k(z_2, z_2, \dots, z_n, \emptyset) &= E[V_{ik} \mid X_{ik} = z_2, Y_{ik}^1 = z_2, \dots, Y_{ik}^n = z_n] \\ &< E[V_{ik} \mid X_{ik} = z_2, Y_{ik}^1 = z_2, \dots, Y_{ik}^n = z_n, T_k = \tau_H] \\ &= v_k(z_2, z_2, \dots, z_n, \tau_H), \end{aligned}$$

where the inequality follows from strict affiliation. From (A8)–(A10) we conclude that (i) holds.

The proof of (ii) is similar and that of (iii) follows immediately from symmetry and the inability of the mixed strategy to reveal information. *Q.E.D.*

Proof of Proposition 5. Because S_k and $S_{k'}$ are identically distributed, their joint distribution can be represented with two parameters. In particular, let $\Pr[S_k = H, S_{k'} = H] = \lambda - c$, $\Pr[S_k = L, S_{k'} = L] = 1 - \lambda - c$, and $\Pr[S_k = H, S_{k'} = L] = c$. As before, $\Pr[S_k = H] = \lambda$. If S_k and $S_{k'}$ are independent, then $c = \lambda(1 - \lambda)$. For fixed λ , as correlation increases, c decreases monotonically with $c = 0$ for correlation equal to one. We continue to assume that conditional on S_k and $S_{k'}$ the distribution of X_k depends only on S_k and that the value of the good for buyers of good k does not depend on signals associated with good k' .

First note that all expected values such as the function $v_k(\cdot)$ are continuous in the correlation parameter c so that expected prices given the seller's signal are also continuous in the correlation parameter c . Furthermore, all these expressions are well defined in the extreme case of perfect correlation where $c = 0$. Next, replicating the arguments in the proof of Proposition 1 in the case of correlation, we obtain expressions (A1) to (A3) with the difference that now $\lambda_1 = \lambda + c$ whereas $\lambda_2 = \lambda - c$. Consequently, in the case $c = 0$, we obtain

$$\Pr[H \mid X_{ik}, Y_{ik}, \tau_H] = \Pr[H \mid X_{ik}, Y_{ik}, \tau_L], \quad (A11)$$

so that the seller's incentive to follow (or deviate) from the rank-revealing strategy exactly disappears at $c = 0$. Furthermore, note that the proof of the impression effect in Lemma 1 did not rely on independence of S_k and $S_{k'}$, but only on the independence of X_k from $S_{k'}$ given S_k , implying that the impression effect remains strictly positive for all $c \in [0, \lambda(1 - \lambda)]$. Since the impression effect is strict, by the same arguments as in Proposition 2, we conclude that BFF is a strict equilibrium (and BFL is not an equilibrium) at $c = 0$. By continuity of relevant expected values in c , the result then follows for c small enough. *Q.E.D.*

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